Ay 20 - Lecture 9

Post-Main Sequence Stellar Evolution

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in order to keep it a reasonable size.

Main Sequence and the Range of Stellar Masses

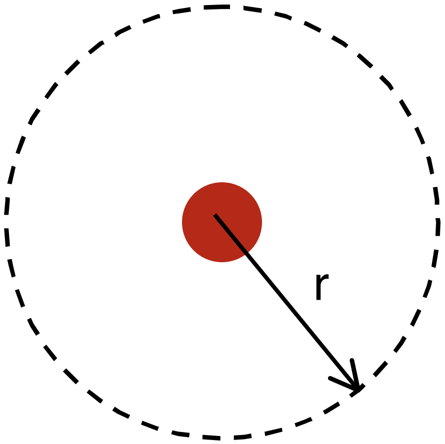
- MS is defined as the locus where stars burn H into He in their cores
- Objects which cannot reach the necessary [T,□] to ignite this fusion, because of their ***low mass*** ($M_\star < 0.08\ M_\odot$) are called **brown dwarfs** (however, they may burn the trace amounts of primordial deuterium)
- Not obvious why should stars form a (nearly) 1-dim. family of objects with the mass as the dominant param.
- The ***high-mass end*** of the stellar family is set by the **Eddington limit**

The Eddington Limit

Radiation is important:

- *inside* stars, as a source of **energy transport**
- *outside* stars and other sources, from its effect on surrounding gas

Consider force which photons exert on surrounding gas when a fraction of them are absorbed:



Source of radiation:

- Luminosity L
- Spherically symmetric emission
- Energy flux at distance r: $L/4\pi r^2$
- Each photon has momentum p=E/c
- Momentum flux: $L/4\pi c r^2$

(From P. Armitage)

If the source is surrounded by gas with opacity κ, then in traveling a distance ds the fraction of radiation absorbed is:

$$\frac{dI}{I} = -\kappa \rho\, ds$$

$$\uparrow$$

column density of gas

Can therefore interpret κ as being the **fraction** of radiation absorbed by unit column density of gas. Force exerted by radiation on that gas is then:

$$f_{rad} = \frac{\kappa L}{4\pi c r^2}$$

outward force

Force due to gravity on that gas (unit mass):

$$f_{grav} = \frac{GM}{r^2}$$

inward, toward star of mass M

(From P. Armitage)

Radiation pressure balances gravity when:

$$f_{rad} = f_{grav}$$

$$\frac{\kappa L}{4\pi c r^2} = \frac{GM}{r^2}$$

$$L = \frac{4\pi c G M}{\kappa}$$

If L is larger than this value the pressure due to radiation exceeds the gravitational force at all radii, and gas will be blown away.

Critical luminosity is called the **Eddington limit**. Depends upon:

- the mass of the star
- the opacity of the gas surrounding the star / source

(From P. Armitage)

Using the value for the opacity appropriate for Thomson scattering:

$$
\begin{aligned}
L_{Edd} &= \frac{4\pi c G M}{\kappa} \\
&= 6.3\times10^{4}\left(\frac{M}{\mathrm{g}}\right)\mathrm{erg\ s^{-1}} \\
&= 1.25\times10^{38}\left(\frac{M}{M_{sun}}\right)\mathrm{erg\ s^{-1}} \\
&= 3.2\times10^{4}\left(\frac{M}{M_{sun}}\right)L_{sun}
\end{aligned}
$$

Only important for sources that are much more luminous that the Sun.

So… why is there a Solar wind?

(From P. Armitage)

Applications: 1) Massive stars

A rough formula for the luminosity of very massive stars immediately after formation (`zero-age main sequence') is:

$$\frac{L}{L_{sun}} \approx 1.2 \times 10^{5} \left(\frac{M}{30\ M_{sun}} \right)^{2.4}$$

Using M_{sun}=1.989 x 10^{33} g and L_{sun}=3.9 x 10^{33} erg s^{-1}:

$$L = 1.6 \times 10^{-45} M^{2.4} \text{ erg s}^{-1}$$ (with M in grams)

Compare with formula for Eddington limit:

$$L_{Edd} = 6.3 \times 10^{4} M \text{ erg s}^{-1}$$

L = L_{Edd} for M = 2.6 x 10^{35} g ~ 130 Solar masses

Radiation pressure is an important effect for massive stars.

(From P. Armitage)

Implications / speculations

a) The most massive stars known have masses of around 100 Solar masses - perhaps radiation pressure sets the limit to how massive a star can form?

b) Stars today form out of gas that also contains dust, so the opacity is larger than the Thomson value. Radiation pressure is therefore important for less luminous (less massive) stars too.

c) Stars don’t form from spherically symmetric collapse, so the `limit’ can be evaded.

d) Perhaps massive stars don’t form from one collapsing cloud, but instead form from **collisions** of smaller stars?

(From P. Armitage)

Applications: 2) Feeding black holes

Gas flowing toward black holes produces radiation as the gravitational potential energy is released. Write this as:

$$L_{accretion} = \square \dot{M} c^2$$

accretion luminosity

gas inflow rate (the accretion rate): units g s^{-1}

radiative efficiency
of the accretion process = the *fraction* of the rest mass energy of the gas that is radiated

For a black hole accreting matter through a disk, the radiative efficiency $\square = 0.1$ or thereabouts.

(From P. Armitage)

Setting the accretion luminosity equal to the Eddington limit gives us the **maximum** rate at which a black hole can accrete gas:

$$L_{Edd} = L_{accretion}$$

$$\frac{4\pi cGM}{\kappa} = \epsilon \dot{M} c^2$$

$$\dot{M} = \frac{4\pi G}{\kappa \epsilon c} M = 1.4 \times 10^{18} \left(\frac{M}{M_{sun}} \right) \mathrm{g\ s^{-1}}$$

assume:

- $\kappa = 0.4\ \mathrm{cm^2\ g^{-1}}$
- $\epsilon = 0.1$

In alternative units, can write this as:

$$\dot{M} = kM = 2.2 \times 10^{-8} \left(\frac{M}{M_{sun}} \right) M_{sun}\ \mathrm{yr^{-1}}$$

↑

a constant

(From P. Armitage)

How fast can a black hole grow?

Assume that a black hole grows as fast as it can - always at exactly the Eddington limit. Then:

$$\dot{M} = kM$$

$$\frac{dM}{dt} = kM$$

$$\int \frac{dM}{M} = \int k\,dt \Rightarrow M = M_0 e^{kt}$$

M_0 is the mass of the hole at the initial time t = 0

Substituting the value we derived previously for k:

$k = 7 \times 10^{-16}\ s^{-1}$

$$M = M_0 e^{t/\tau}$$

where τ, the time scale for the black hole to grow by a factor of e, is 4.5 x 10^7 years

(From P. Armitage)

Stellar Evolution

Main-sequence evolution: star burns H in core, core composition slowly changes from H to He. Small changes in the external properties (L, T_e, R)

Main-sequence lifetime is strongly mass-dependent, more massive stars:

- sustain higher core temperatures
- have higher rates of nuclear fusion
- are more luminous and exhaust H fuel more quickly

$$L \propto M^{3.5} \Rightarrow t_{ms} \propto M^{-2.5}$$

Star leaves the main sequence when it stops burning hydrogen in the core. Normally leads to *expansion* of the envelope, and the formation of a giant. Depending upon mass, final outcome is a white dwarf, a neutron star, or a black hole.

(From P. Armitage)

Stellar Lifetimes on the Main Sequence

- The duration of a star's MS lifetime depends on the amount of hydrogen in the star's core and the rate at which the hydrogen is consumed, i.e., luminosity
- Core H burning ends when the star converts a certain fraction of its total mass into He, the so-called ***Schönberg-Chandrasekhar limit***,

$$(M_{core} / M_{\star}) \approx 0.37 \, (\square_{e} / \square_{core})^{2} \approx 0.08 - 0.1$$

- Since the M-L relation is so non-linear, $L \sim M^{\square}$, with $\square \sim 3 - 5$, **the more massive a star, the shorter is its MS lifetime**
- The same principle applies to post-MS stages of stellar evolution

Stellar Lifetimes on the Main Sequence

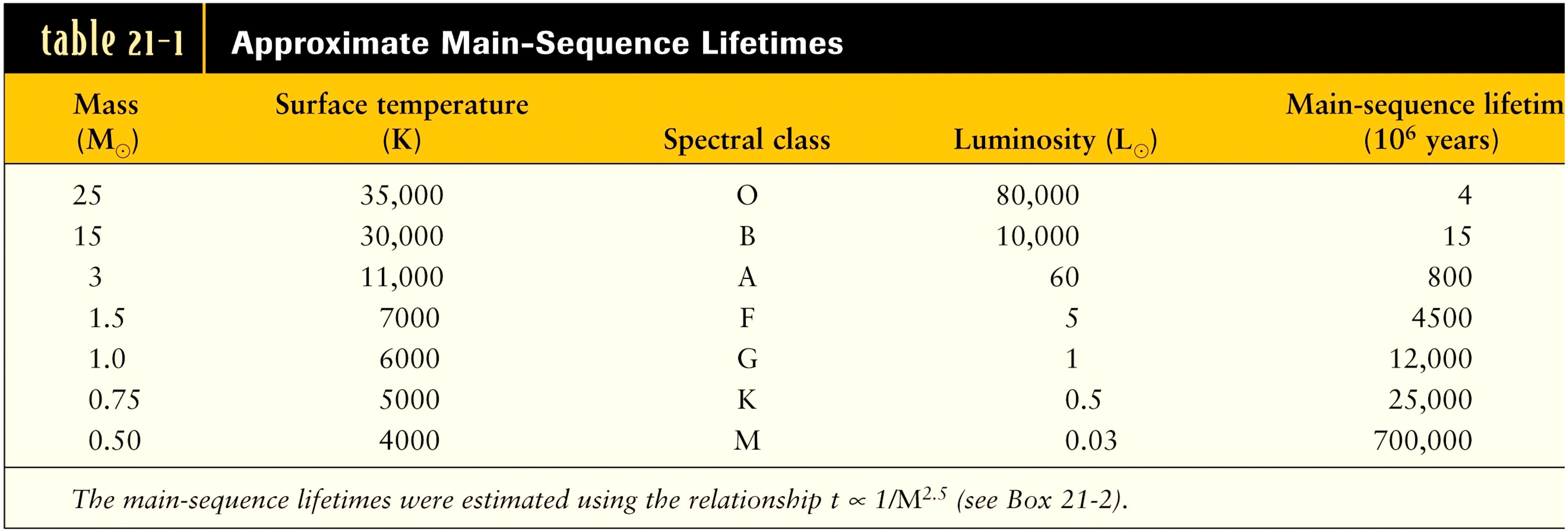


table 21-1 **Approximate Main-Sequence Lifetimes**

Mass ($M_\odot$)	Surface temperature (K)	Spectral class	Luminosity ($L_\odot$)	Main-sequence lifetim (10^6 years)
25	35,000	O	80,000	4
15	30,000	B	10,000	15
3	11,000	A	60	800
1.5	7000	F	5	4500
1.0	6000	G	1	12,000
0.75	5000	K	0.5	25,000
0.50	4000	M	0.03	700,000

The main-sequence lifetimes were estimated using the relationship $t \propto 1/M^{2.5}$ *(see Box 21-2).*

Stellar Evolution is a Sequence of Different Energy Production (Nuclear Fusion) Mechanisms

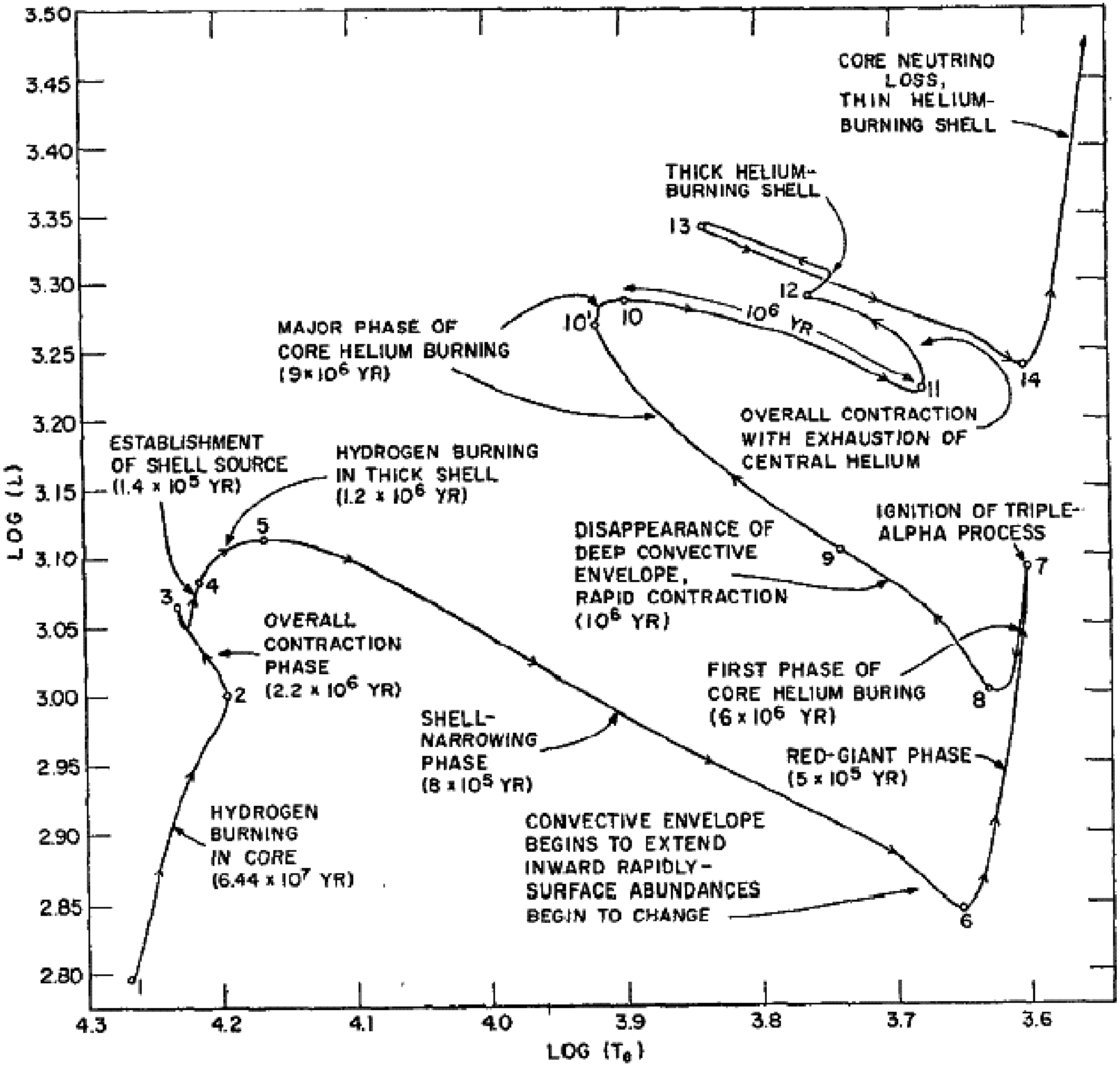
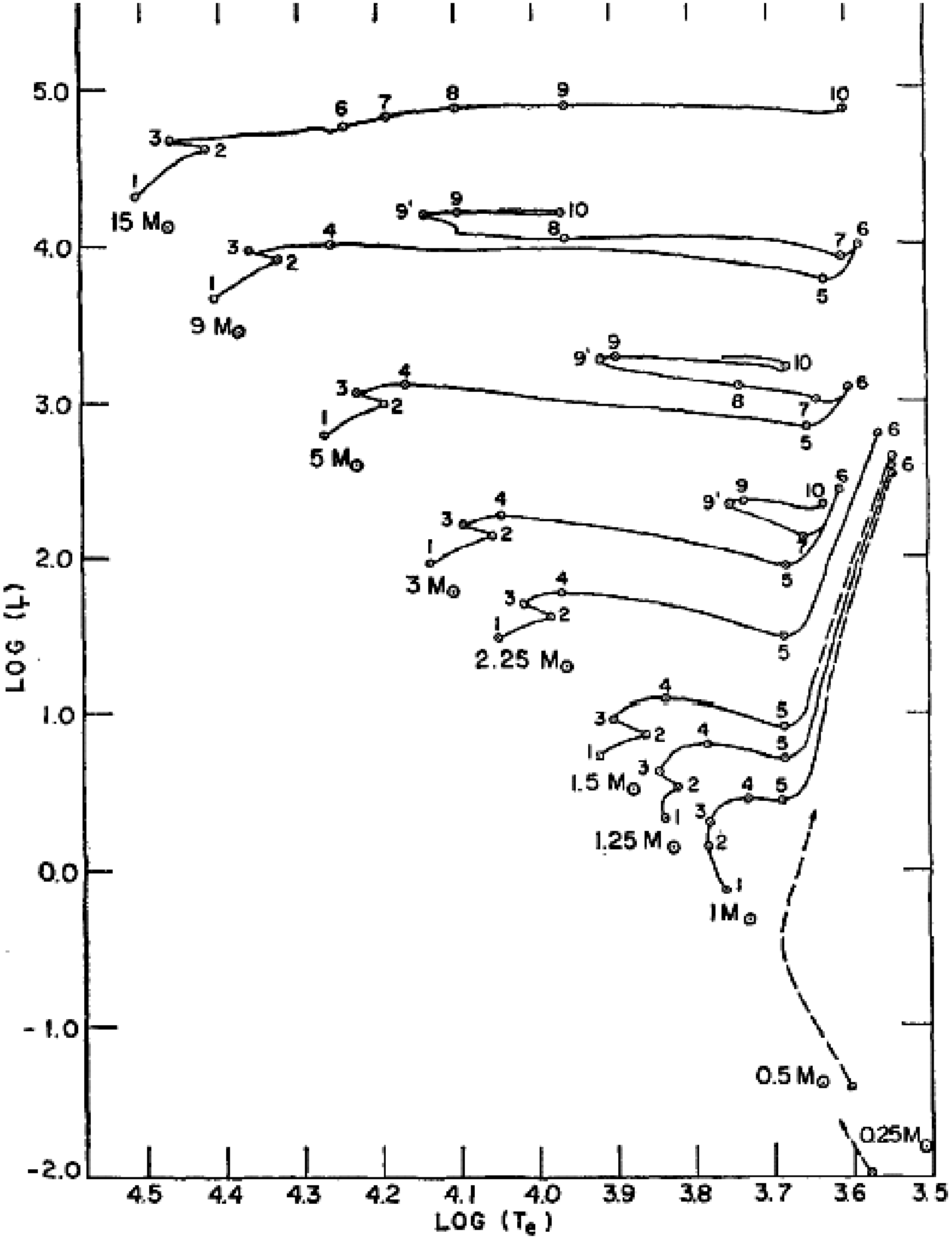


FIG. 1. The path of a metal-rich $5M_\odot$ star in the Hertzsprung-Russell diagram.

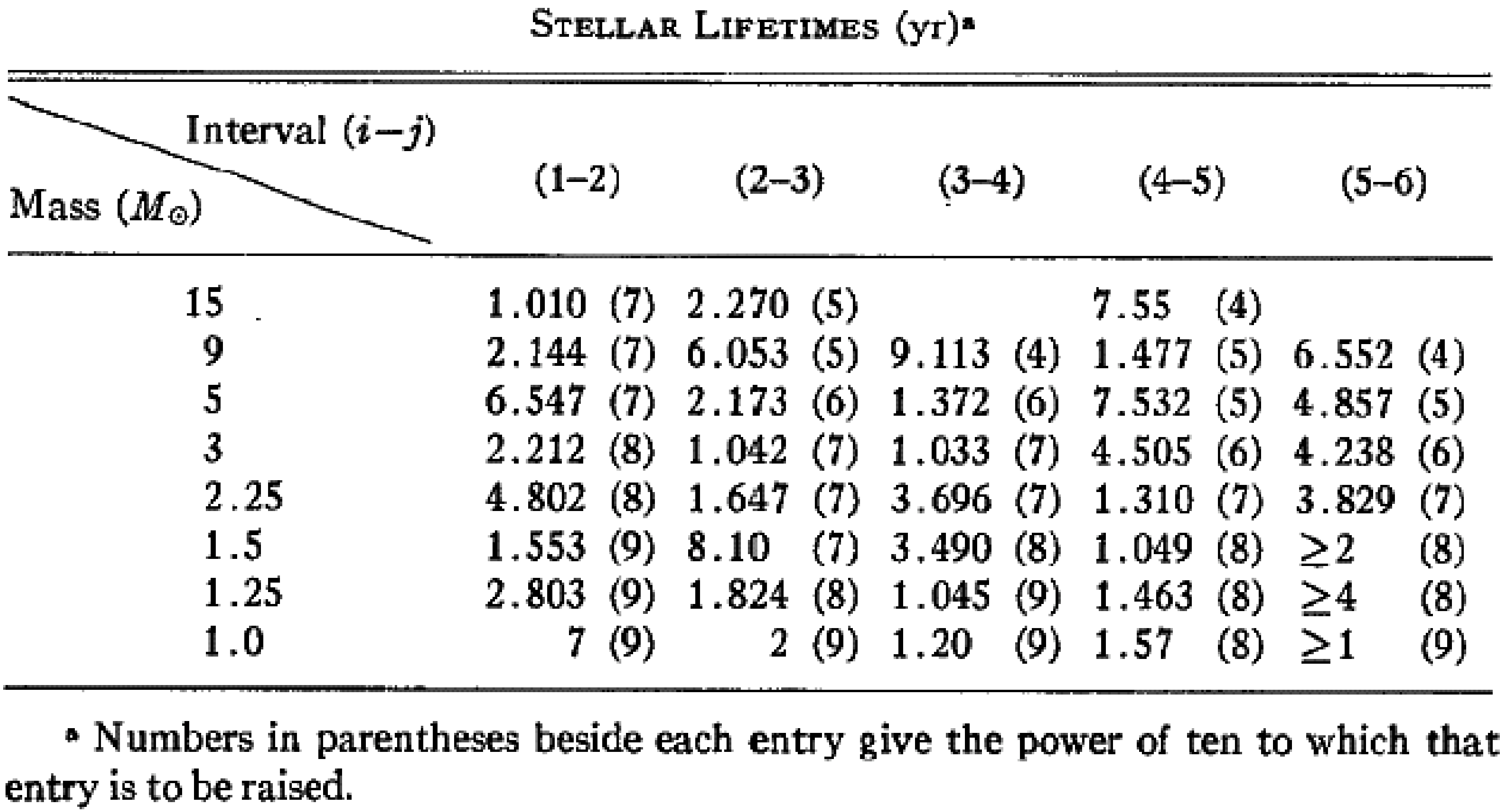
(From Iben)

Stars of Different Mass Evolve Off the MS Along Different Paths in the HRD, but the Physics is Basically the Same



(From Iben)

Stellar Lifetimes



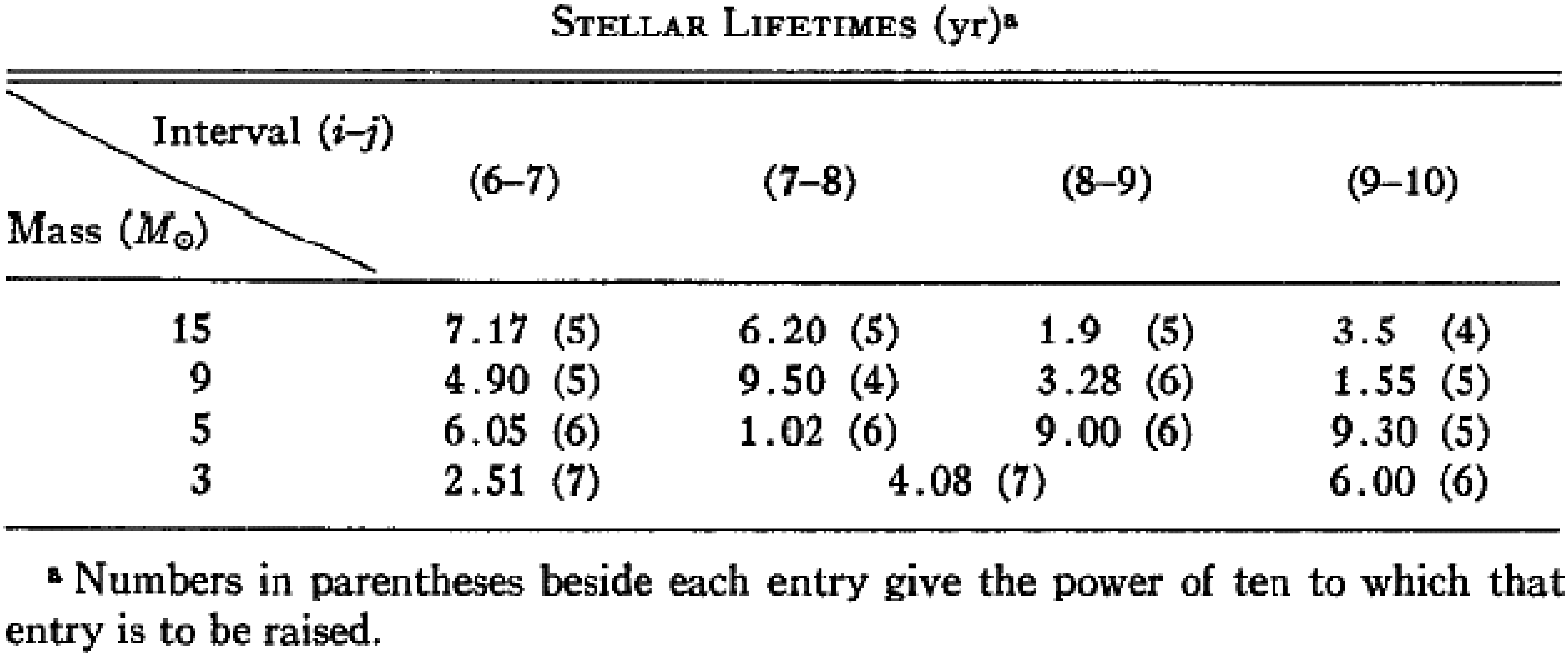
STELLAR LIFETIMES (yr)[a]

Mass ($M_\odot$) \ Interval ($i-j$)	(1–2)	(2–3)	(3–4)	(4–5)	(5–6)
15	1.010 (7)	2.270 (5)		7.55 (4)	
9	2.144 (7)	6.053 (5)	9.113 (4)	1.477 (5)	6.552 (4)
5	6.547 (7)	2.173 (6)	1.372 (6)	7.532 (5)	4.857 (5)
3	2.212 (8)	1.042 (7)	1.033 (7)	4.505 (6)	4.238 (6)
2.25	4.802 (8)	1.647 (7)	3.696 (7)	1.310 (7)	3.829 (7)
1.5	1.553 (9)	8.10 (7)	3.490 (8)	1.049 (8)	≥2 (8)
1.25	2.803 (9)	1.824 (8)	1.045 (9)	1.463 (8)	≥4 (8)
1.0	7 (9)	2 (9)	1.20 (9)	1.57 (8)	≥1 (9)

[a] Numbers in parentheses beside each entry give the power of ten to which that entry is to be raised.

(From Iben)

Stellar Lifetimes



Stellar Lifetimes (yr)[a]

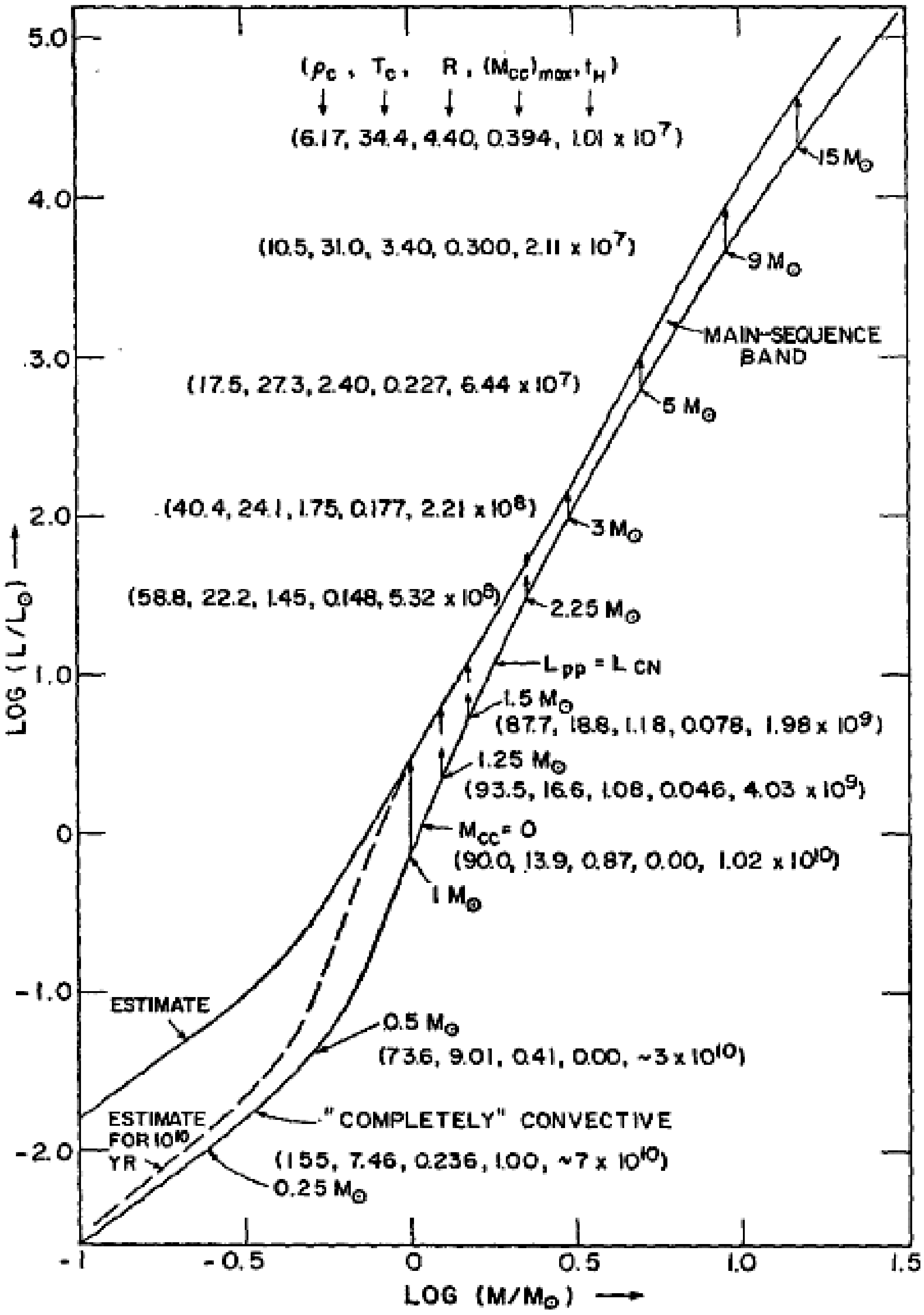
Interval (i–j) Mass ($M_\odot$)	(6–7)	(7–8)	(8–9)	(9–10)
15	7.17 (5)	6.20 (5)	1.9 (5)	3.5 (4)
9	4.90 (5)	9.50 (4)	3.28 (6)	1.55 (5)
5	6.05 (6)	1.02 (6)	9.00 (6)	9.30 (5)
3	2.51 (7)	4.08 (7)		6.00 (6)

[a] Numbers in parentheses beside each entry give the power of ten to which that entry is to be raised.

Generally, later stages of stellar evolution last shorter, because there is less fuel to burn, and because of the non-linearity of the M-L relation (which may be driven by the highly non-linear depencence of the TNR rates on [T,□]

Life on the MS

- Burning H into He changes the chemical composition and lowers the pressure
- Core shrinks, heat up and burns H at a higher rate
- Luminosity increases, which causes the outer envelope to swell up
- The MS stars begin on the lower edge of the MS band and move up and to the right



Very Low Mass Stars: Red Dwarfs

- Mass < 0.4 $M_\odot$
- Their structure is all convection zone, H and He is mixed throughout the star
- The star burns H slowly
- Will never build up a He core, and never ignite He
- Could perhaps survive on the MS for a 100 Gyr!
- Then just fade as a WD/BD

The End of the MS Phase

- On the MS, a star is in a hydrostatic equilibrium, and its core is sufficiently hot to fuse H into He
- Now the star has two chemically distinct zones, a core of inert He surrounded by an H envelope - the core of a MS star is not sufficiently hot for He burning
- When the core becomes pure He, a new evolutionary phase stars - the ascent to the ***Red Giant Branch*** (RGB)
- Without energy generation, the core cannot support itself against gravitational collapse and so it begins to shrink; as it collapses it heats up
- This heat is transferred to a thin shell of H around the core which reaches a temperature in which H fusion can occur

Becoming a Red Giant

- As the core continues to collapse, the temperature in the H fusing shell continues to rise and thus the luminosity in the shell increases as does the pressure
- The entire star is no longer in a hydrostatic equilibrium, and the envelope begins to expand
- As they expand these outer layers cool - the star becomes redder, while its luminosity increases: the star slowly ascends the RGB
- This imbalance will continue until the star again finds a source of core energy generation, i.e., He fusion

Why do stars become red giants?

No simple and fully accepted explanation of this phenomenon. Good plausibility argument:

Suppose the core contraction at the end of hydrogen burning occurs on a timescale *shorter* than the Kelvin-Helmholtz time of the whole star. Then:

Energy conservation: $\prod + U = \text{constant}$

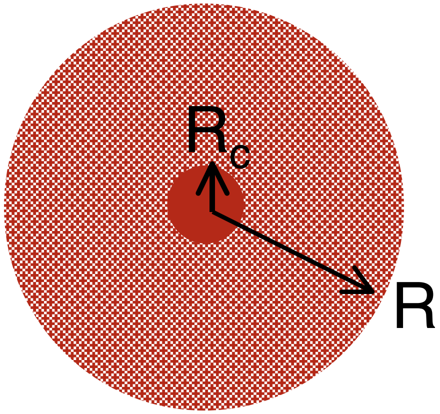
Virial theorem: $\prod + 2U = \text{constant}$

…must both hold. Only possible if □ and U are conserved separately.

(From P. Armitage)

Structure of the star:

- core, radius R_c, mass M_c
- envelope, radius R (total stellar radius), mass M_{env}



For $M_c >> M_{env}$:

$$|\Omega| \simeq \frac{GM_c^2}{R_c} + \frac{GM_c M_{env}}{R}$$

Now assume the division between core and envelope is fixed, and differentiate with respect to time:

$$0 = -\frac{GM_c^2}{R_c^2}\frac{dR_c}{dt} - \frac{GM_c M_{env}}{R^2}\frac{dR}{dt}$$

$$\frac{dR}{dR_c} = -\left(\frac{M_c}{M_{env}}\right)\left(\frac{R}{R_c}\right)^2$$

Envelope expands as the core contracts

(From P. Armitage)

Helium Flash: The End of the RGB

- H fusion leaves behind He ash in the core of the star which cannot begin to fuse until the temperature of the core reaches 100 million K. How a star begins He fusion depends on its mass:
- M > 3 solar mass stars contract rapidly, their cores heat up, and He fusion begins gradually
- Less massive stars evolve more slowly and their cores contract so much that degeneracy occurs in the core
- When the temperature is hot enough He fusion begins to make energy and the T rises, but pressure does not increase due to degeneracy
- Higher T increases He fusion even further resulting in a runaway explosion: the **Helium Flash** which for a few minutes can generate more luminosity than an entire galaxy. The flash does not destroy the star: the envelope absorbs the energy

Structure of a Red Giant

- Now the core has heated to T ~ 10^8 K, which is the threshold temperature for the fusion of He into C
- The star is in a quasi-static equilibrium. The lifetime of a star as a Red Giant is about 10% of its MS lifetime

- The luminosity generated by the core fusion of He into C is far greater than the shell luminosity associated with the fusion of H into He

Low-mass stars go through two distinct red giant stages

A low-mass star becomes

- A red giant (RG) when shell H fusion begins
- A horizontal branch (HB) star when core He fusion begins
- An asymptotic giant branch (AGB) star when the He in the core is exhausted and shell He fusion begins

Dredge-ups bring the products of nuclear fusion to a giant star’s surface

- As a low-mass star ages, convection occurs over a larger portion of its volume
- This takes heavy elements formed in the star’s interior and distributes them throughout the star

The End Phases of Stellar Evolution

- The evolution and eventual fate of stars in the late stages of their lives are critically dependent on the amount of matter they have at birth:
- Stars with initial masses of *less* than about 8 solar masses end their lives as ***white dwarfs.*** The star sheds its RG envelope, which becomes a ***planetary nebula***, and the inert, degenerate core cools passively.
- Stars with initial masses *greater* than about 8 solar masses end their lives by exploding as ***supernovae.*** The stellar remnants are ***neutron stars*** or ***black holes.***

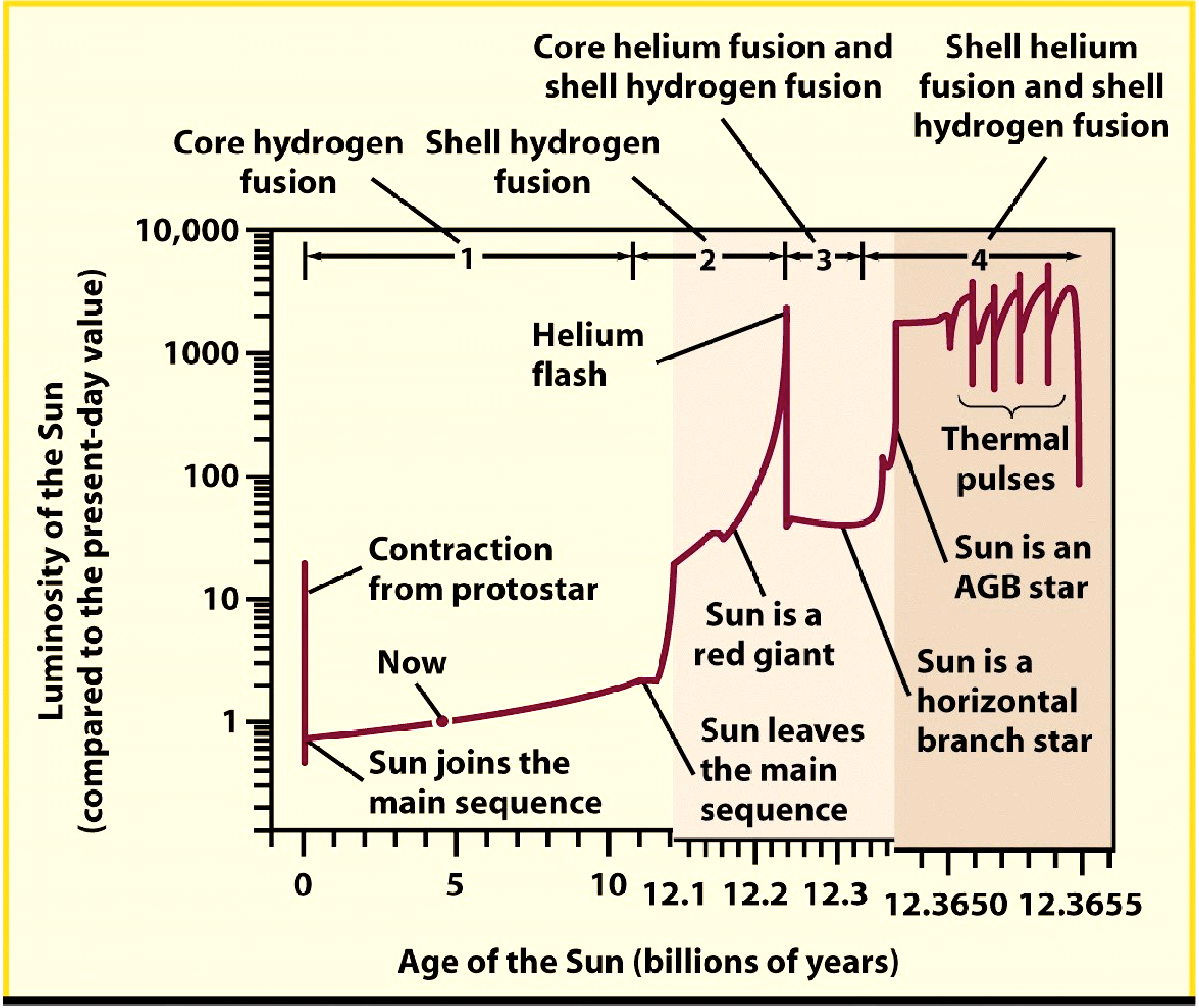
Planetary Nebula Formation

- A RG brightens by a factor of between 1,000 and 10,000. The outer, hydrogen-rich envelope swells up to a few au radius, with T ~ 2,000 - 3,000 K
- A strong stellar wind begins to blow from the star's surface (akin to the Sun's solar wind, but much stronger), and, in the course of the star's RG life, carries away most of the H envelope
- During the final shedding of its envelope, when the mass loss is the greatest, the star becomes unstable and pulsates, with periods ~ few months to > 1 yr. Such stars are called ***long-period variables***.
- The envelope material ejected by the star forms an expanding shell of gas that is known as a ***planetary nebula (PN)***

Formation of Planetary Nebula

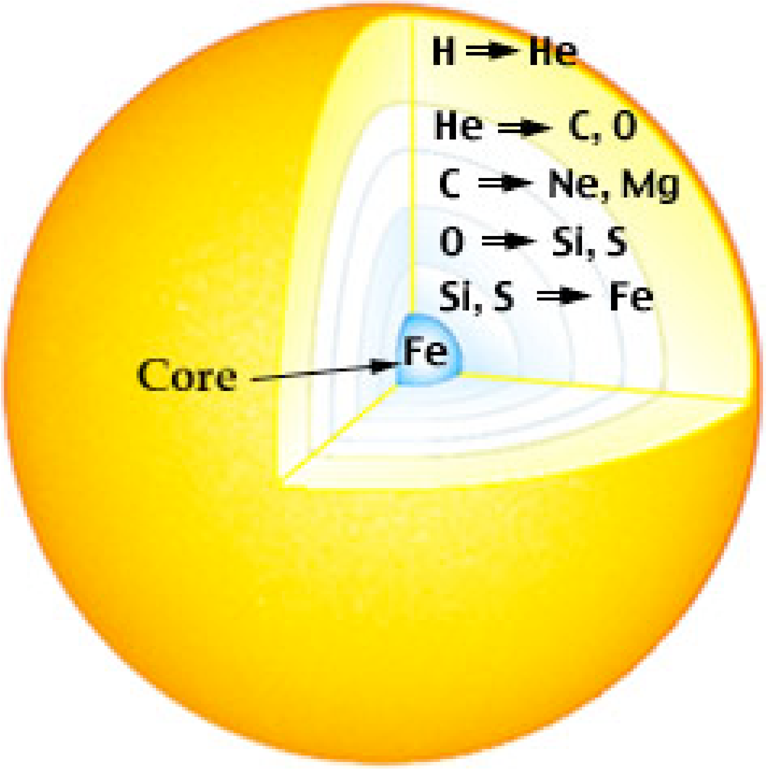
Planetary nebulae typically have masses $\sim 0.2\ M_\odot$, although some are considerably more massive. They expand $V \sim 10 - 20$ km/s, and plow into the surrounding *interstellar medium,* contributing to its chemical enrichment.

Their central stars (degenerate cores of progenitors) provide photoionization for the nebulae.



Core hydrogen fusion
Shell hydrogen fusion
Core helium fusion and shell hydrogen fusion
Shell helium fusion and shell hydrogen fusion
1
2
3
4
10,000
1000
100
10
1
Luminosity of the Sun (compared to the present-day value)
Helium flash
Thermal pulses
Contraction from protostar
Now
Sun joins the main sequence
Sun is a red giant
Sun leaves the main sequence
Sun is an AGB star
Sun is a horizontal branch star
0
5
10
12.1
12.2
12.3
12.3650
12.3655
Age of the Sun (billions of years)

Highly Evolved Star Structure



- In the last stages of its life, a high-mass star has an Fe-rich core surrounded by concentric shells hosting the various thermonuclear reactions
- The sequence of thermonuclear reactions stops here, because the formation of elements heavier than Fe requires an input of energy rather than causing energy to be released

High-mass stars create heavy elements in their cores

- A high mass star undergoes an extended sequence of thermonuclear reactions in its core and shells: C fusion, Ne fusion, O fusion, and Si fusion, all the way to Fe

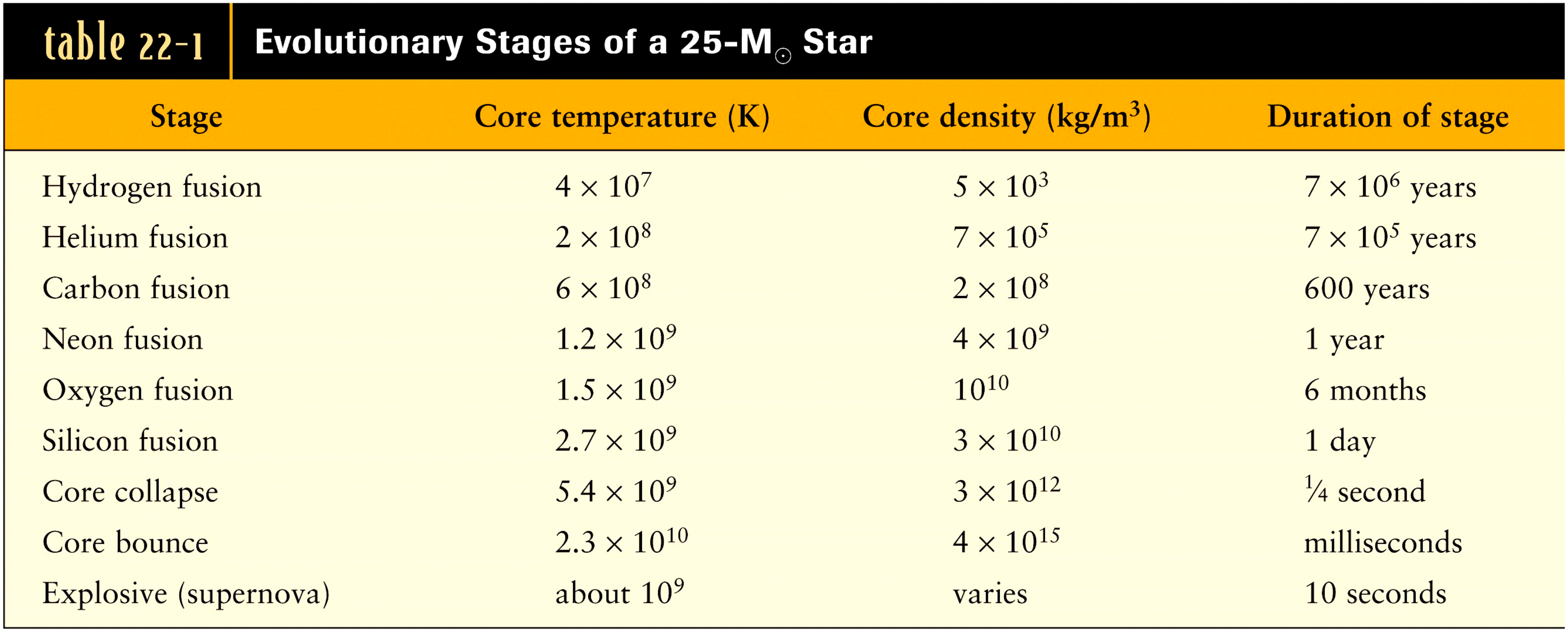


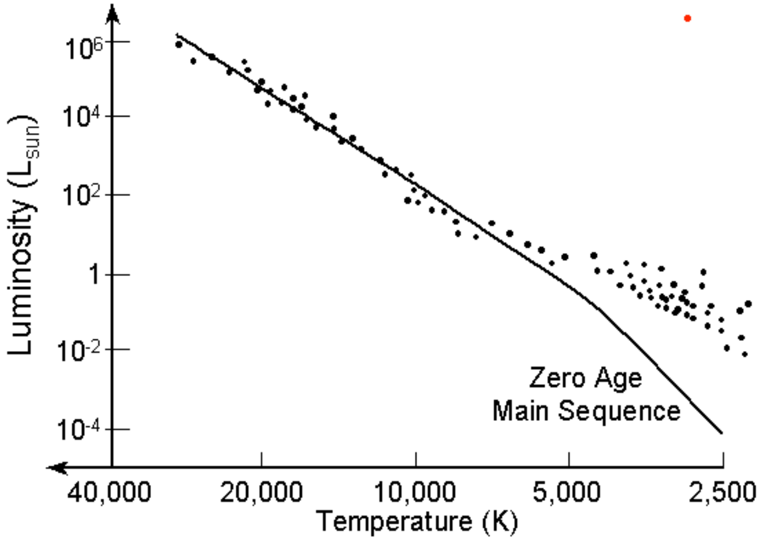
table 22-1 **Evolutionary Stages of a 25-$M_\odot$ Star**

Stage	Core temperature (K)	Core density (kg/m^3)	Duration of stage
Hydrogen fusion	4×10^7	5×10^3	7×10^6 years
Helium fusion	2×10^8	7×10^5	7×10^5 years
Carbon fusion	6×10^8	2×10^8	600 years
Neon fusion	1.2×10^9	4×10^9	1 year
Oxygen fusion	1.5×10^9	10^{10}	6 months
Silicon fusion	2.7×10^9	3×10^{10}	1 day
Core collapse	5.4×10^9	3×10^{12}	¼ second
Core bounce	2.3×10^{10}	4×10^{15}	milliseconds
Explosive (supernova)	about 10^9	varies	10 seconds

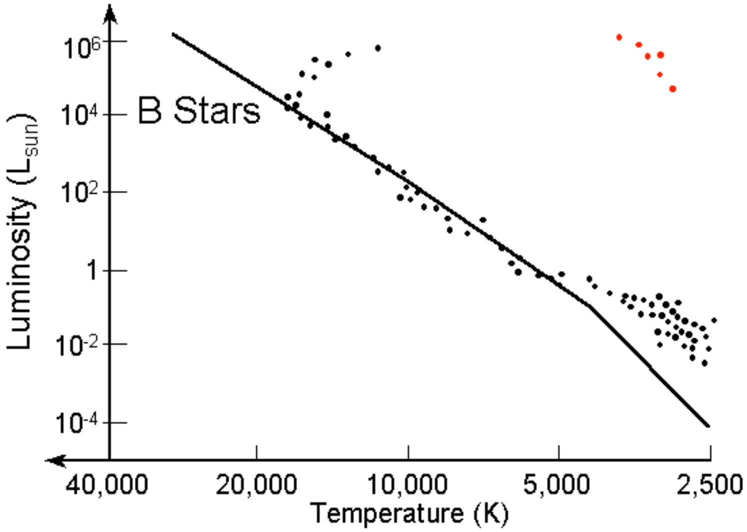
Testing Stellar Evolution

- The problem: stellar evolution happens on billion-year time scales
- The solution: use H-R Diagrams of star clusters with a wide range of ages
  - Clusters contain 100's to 1000's of stars with a broad mass range
  - All stars are at the same distance, so it is easy to measure their relative luminosities
  - They have the same age, have the same chemical composition
- Each cluster thus provides a snapshot of what stars of different masses look like at the same age and composition (coeval populations)

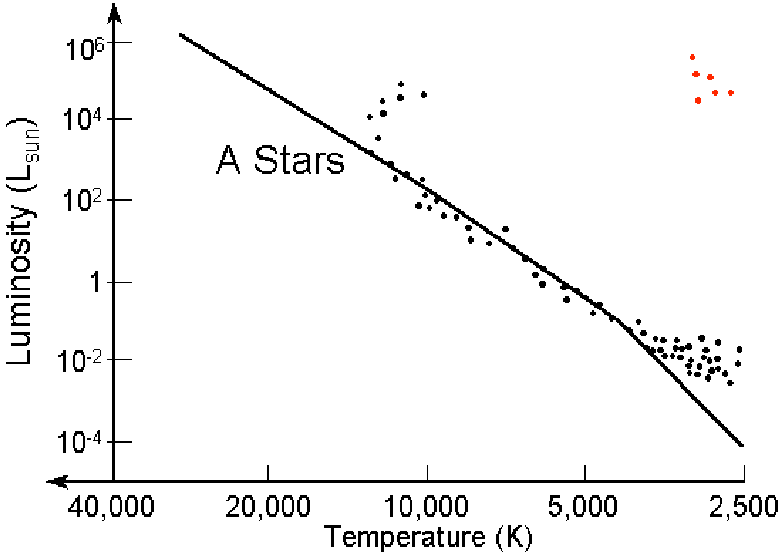
Age: ~1 Myr



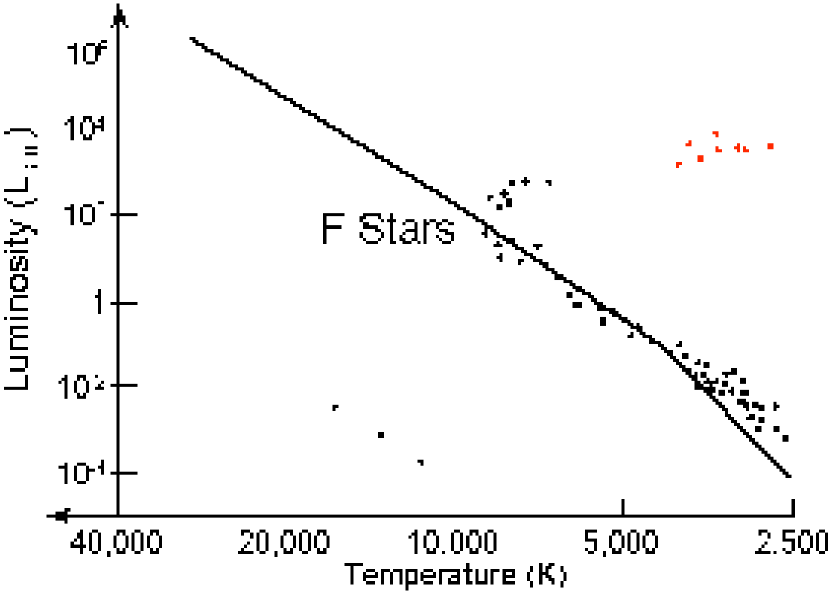
Age: ~10 Myr

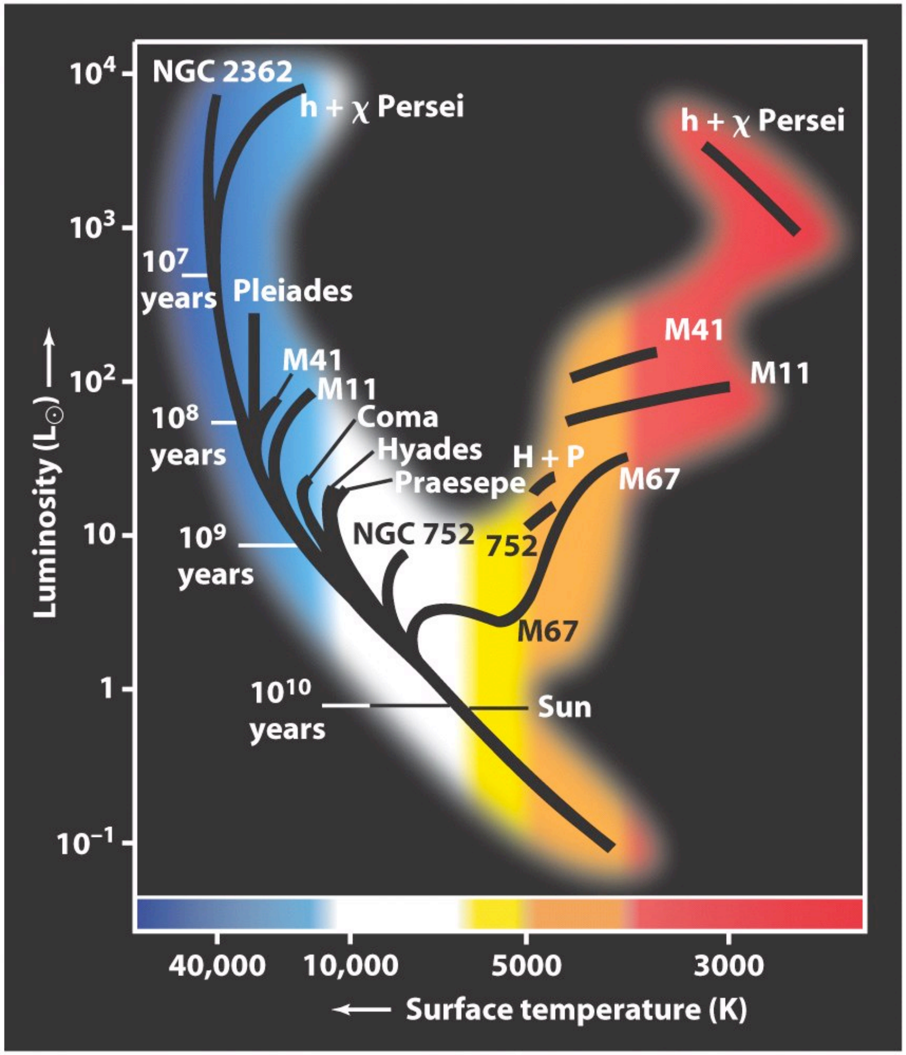


Age: ~100 Myr

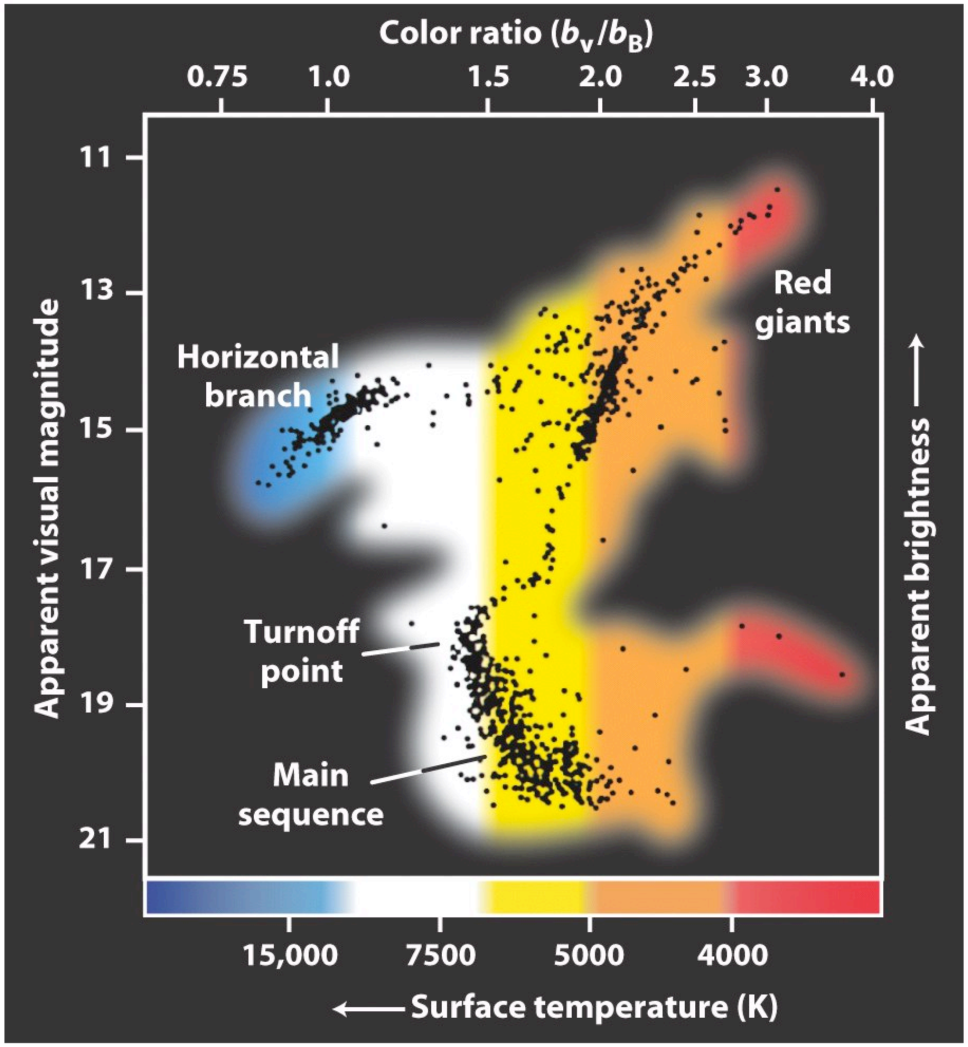


Age: ~1 Gyr



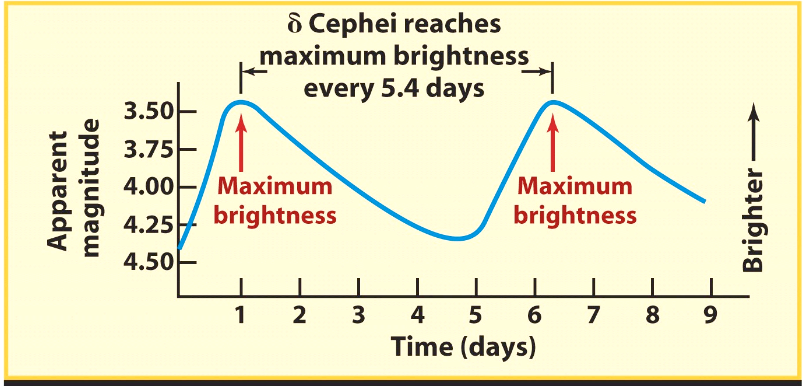


10^4
10^3
10^2
10
1
10^{-1}
Luminosity ($L_\odot$) →
NGC 2362
h + χ Persei
h + χ Persei
10^7 years
Pleiades
M41
M11
10^8 years
Coma
Hyades
Praesepe
H + P
M41
M11
M67
10^9 years
NGC 752
752
M67
10^{10} years
Sun
40,000
10,000
5000
3000
← Surface temperature (K)

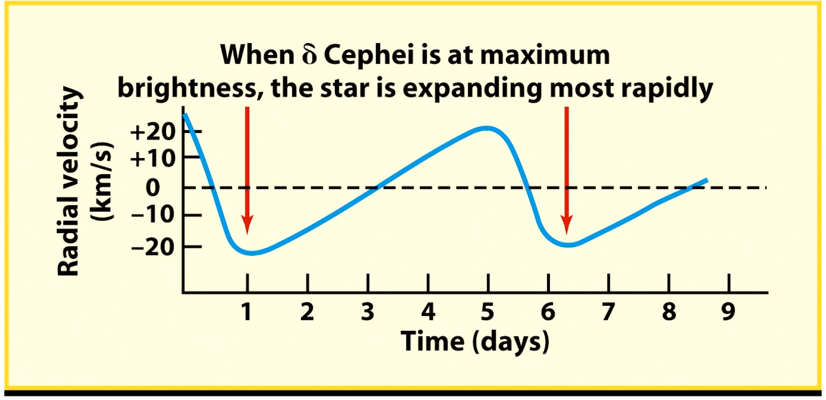


Color ratio (b_V/b_B)
0.75
1.0
1.5
2.0
2.5
3.0
4.0
11
13
15
17
19
21
Apparent visual magnitude
Apparent brightness →
Horizontal branch
Red giants
Turnoff point
Main sequence
15,000
7500
5000
4000
← Surface temperature (K)

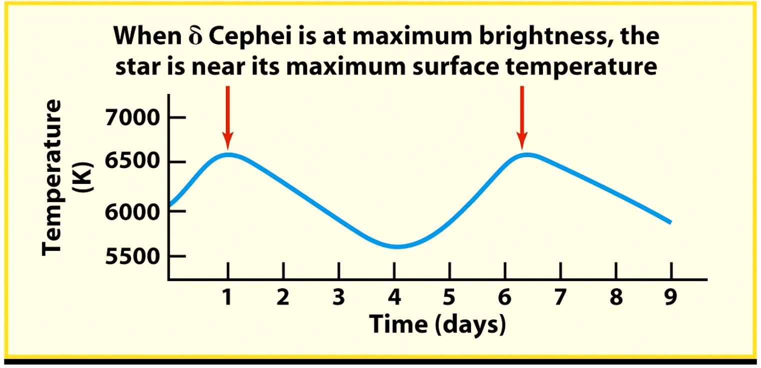
- Cepheid variables are high-mass pulsating variables
- RR Lyrae variables are low-mass, metal-poor pulsating variables with short periods
- Long-period variable stars also pulsate but in a fashion that is less well understood



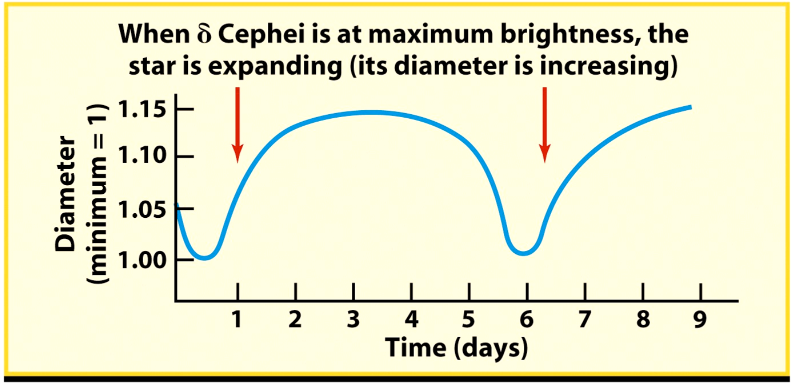
The light curve of δ Cephei (a graph of brightness versus time)



Radial velocity versus time for δ Cephei (positive: star is contracting; negative: star is expanding)

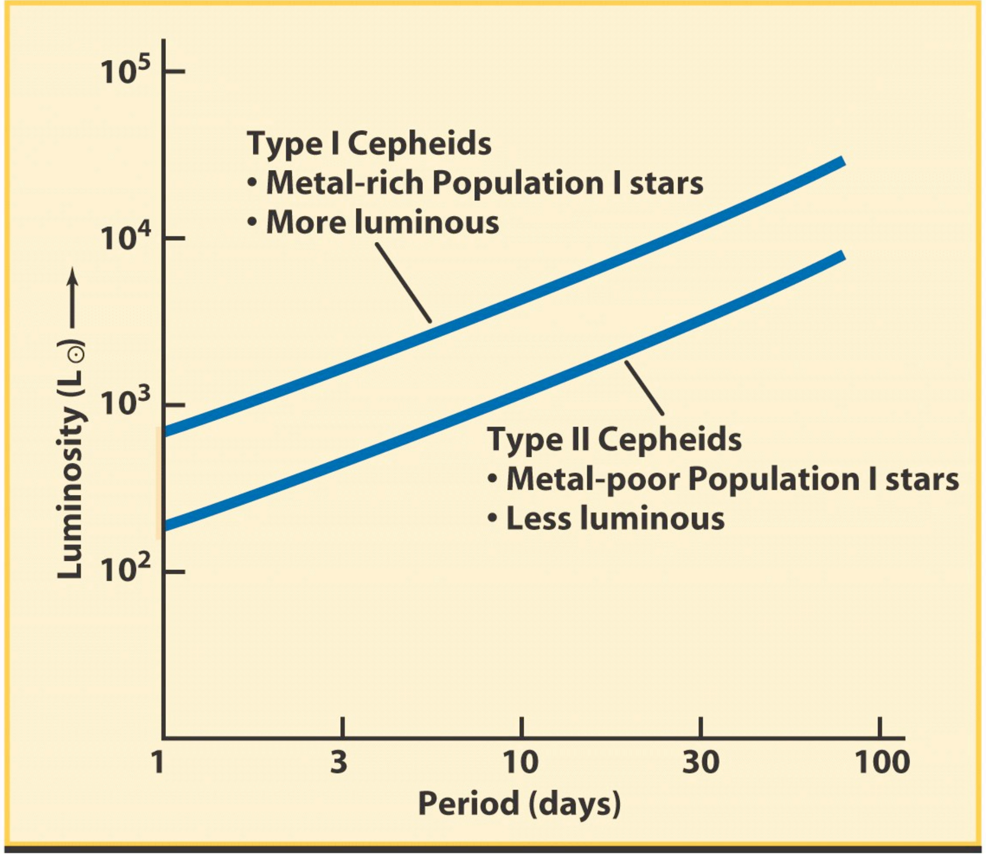


Surface temperature versus time for δ Cephei

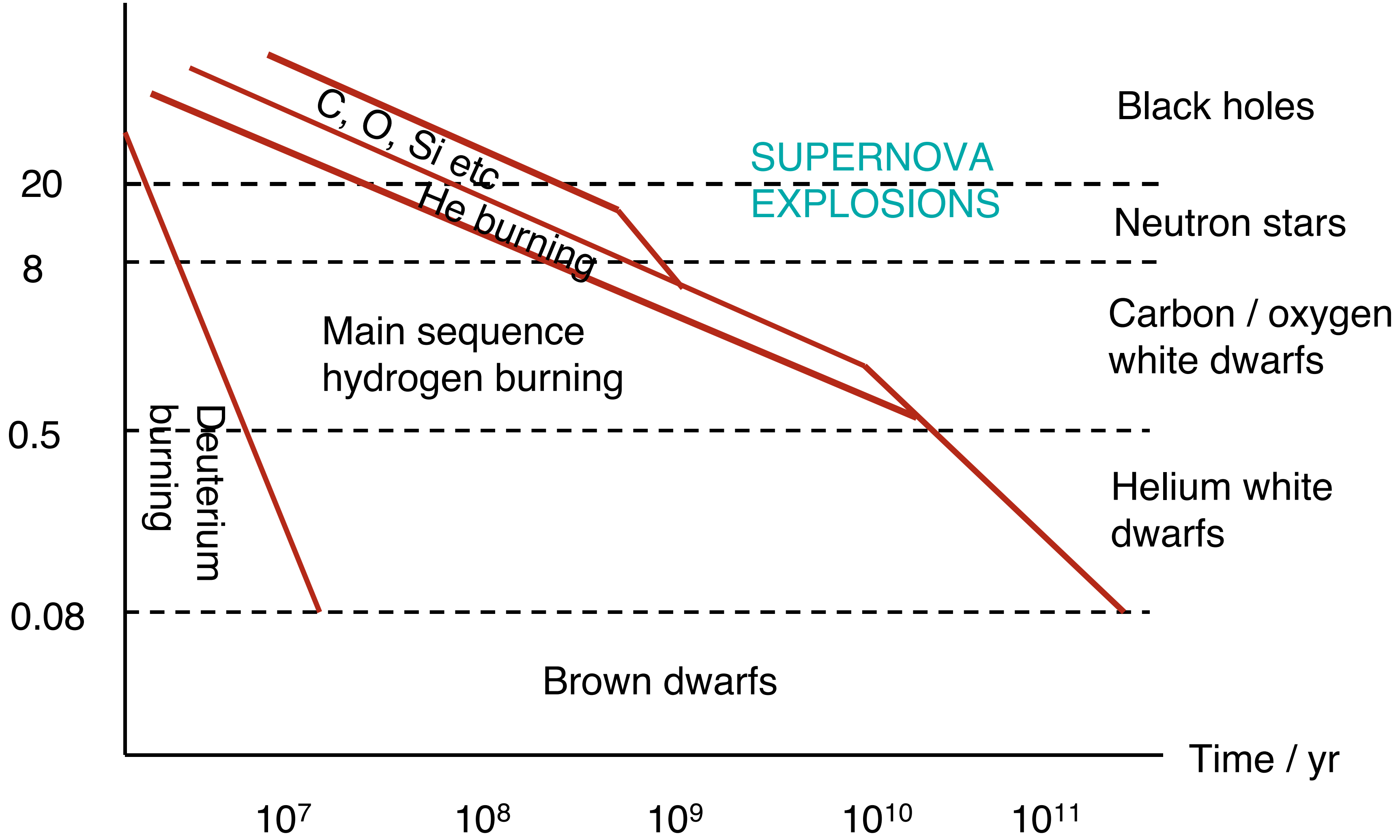


Diameter versus time for δ Cephei

There is a direct relationship between Cepheid periods of pulsation and their luminosities



Overview



(From P. Armitage)