Dynamics

A galaxy is made up of stars in the same way that a crystal is made up of atoms, and the galaxy's large-scale properties reflect the forces that act between its component stars in the same way that the structure of a crystal reflects the nature of interatomic forces. Galaxies and crystals have strongly contrasting large-scale properties because a galaxy is held together by a force that has very different properties from a typical interatomic force.

For most purposes we may neglect the physical size of stars and consider them to be point particles that interact only through gravity. Gravity is a uniquely long-range force, so the net force on an individual star depends directly on the locations of every other star in the galaxy. By contrast, the net force on an atom in a crystal depends only on the location of up to a few dozen near neighbors.

Given that there are in excess of 10^{11} stars in a galaxy such as our own Milky Way, progress in STELLAR DYNAMICS would be slow if we had to know the locations of every star before we could calculate the motion of a single star. Fortunately, the problem can be simplified by arguing that the net force on a star does not depend sensitively on the precise locations of the billions of stars that are rather distant from it, so an excellent estimate of the contribution these stars make to the net force can be obtained by mentally smearing the mass of these stars into a smooth continuum of density. Mathematically, if in some region there are ν stars per unit volume, then we calculate the gravitational attraction from a smooth distribution of matter of density $\rho = \langle m \rangle \nu$ units of mass per unit volume, where $\langle m \rangle$ is the average mass of a star. The required gravitational attraction is easy to calculate if we first determine the gravitational potential that is generated by the density distribution ρ.

Replacing individual stars by a smoothed-out distribution of mass yields an indifferent estimate of the part of the force on a star that arises from the star's near neighbors. In particular, it completely neglects the fact that the star sometimes experiences short intense periods of acceleration when a neighboring star passes unusually close. This neglect of 'collisions' can be made good at a subsequent stage of the analysis, when collisional effects are added in to the 'collisionless dynamics' that are determined by approximating the system's mass distribution by a smooth one.

The longer a stellar system evolves, the more important collisional phenomena become. As a general rule, the time required for collisional effects to become important in a system of N stars is $\sim N/8 \ln N$ times the mean time it takes a star to cross the system. This time is substantially larger than the age of the universe in the case of a typical galaxy but significantly shorter than its age in the case of a typical STAR CLUSTER.

In this article we review the basic principles that govern the dynamics of galaxies and star clusters. More information on most topics will be found in the monograph [2], which will not be explicitly cited again.

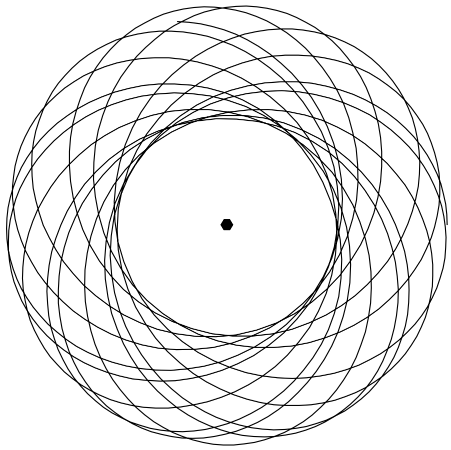

Figure 1. A typical orbit in a spherical potential.

Collisionless dynamics

A stellar system is usefully thought of as a collection of orbits in the smoothed-out potential of the system, each of which is populated by a certain number of stars. Consequently, a primary task of stellar dynamics is to understand what orbits can arise in a given gravitational potential.

The easiest case to consider is a spherical potential. Then every star moves in the plane perpendicular to the star's ANGULAR-MOMENTUM vector. Within this plane the star describes a rosette (figure 1). The orbit is best quantified by its angular momentum, $\boldsymbol{L}$, and its 'radial action' J_r, which quantifies the vigor of the star's radial excursions: a circular orbit has zero radial action, $J_\mathrm{r} = 0$. The direction of the angular-momentum vector $\boldsymbol{L}$ defines the orbital plane, while the magnitude of the vector, L, controls the size and eccentricity of the orbit – if J_r/L is small, the orbit is nearly circular, and the orbit is physically small if L is small and large if L is large.

Few galaxies have spherical potentials, but many have potentials that are nearly axisymmetric. How do the orbits in a spherical potential change if we squash the potential along some axis, so that it remains axisymmetric about that axis? In a squashed potential, orbits are not confined to planes, but we can obtain a reasonably accurate picture of them by considering them to be confined to precessing planes. That is, the orbit is approximately confined to a plane that is inclined at some angle to the potential's symmetry axis, and slowly rotates about this axis. On the plane the orbit remains a rosette. The shape of this rosette is controlled by J_r and a number called I_3 in the same way that the shape of a rosette in a spherical potential was controlled by J_r and L. The inclination of the precessing orbital plane to the potential's symmetry axis is determined by the ratio I_3/L_z of I_3 to the star's angular-momentum about the potential's symmetry axis.

The potentials of many galaxies deviate significantly from axisymmetry, so let us ask what happens if we squash an axisymmetric potential along an axis perpendicular to

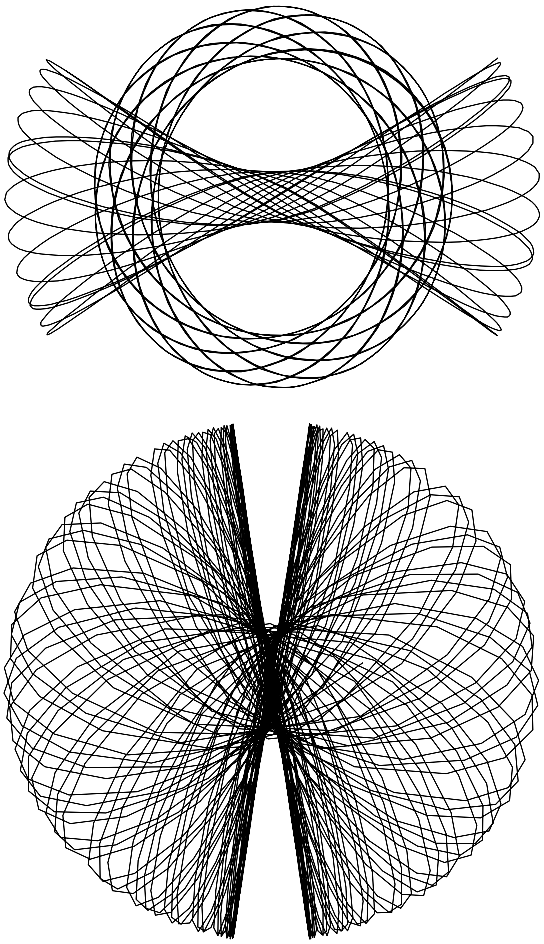

Figure 2. Three orbits in a triaxial potential. The contours of constant gravitational potential are elongated horizontally with axis ratio 0.9.

its symmetry axis. Figure 2 shows that the consequences of deforming a potential from axisymmetric to triaxial symmetry can be dramatic. While some orbits deform only slightly, others strongly amplify the deviation of the potential from axisymmetry.

When the potential of a galaxy is triaxial, it will usually tumble around one of its principal axes. The orbits shown in figure 2 are in a potential whose axes are fixed rather than tumbling. Figure 3 shows one important consequence of letting the potential tumble. The rate at which the potential tumbles defines a characteristic radius, the corotation radius, r_{CR}, at which a star that moves around with the potential moves on a circular orbit. At radii much smaller than r_{CR}, the rotation of the potential's figure is unimportant. At radii larger than about $r_{\mathrm{CR}}/10$ the orbits are profoundly modified by the figure rotation, and at radii close to r_{CR} a large fraction of the orbits are chaotic. Chaotic orbits such as that shown in figure 3 look rather ragged and have a propensity to change their character suddenly and irreversibly. It is likely that the

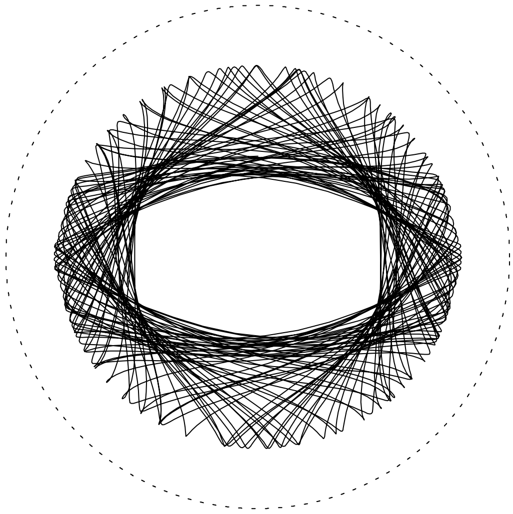

Figure 3. A chaotic orbit in a rotating barred potential. The dashed curve shows the potential's corotation radius.

steady accumulation of such changes drives the evolution of some galaxies, for example LENTICULAR GALAXIES.

Entropy and equilibrium

In stellar dynamics we seek statistical information in the sense that we would like to know the probability of the occurrence of any given configuration—in which given numbers of stars are on each orbit. In ordinary statistical physics the probability P that a given quantum state occurs can usually be obtained from the Gibbs distribution, $P \propto \mathrm{e}^{-\beta E}$, where $\beta = 1/kT$ is the inverse temperature and E is the energy of the state. Can this distribution be carried over into stellar dynamics?

The answer to this question is 'no'. In fact, the concept of thermodynamic equilibrium ultimately fails for any self-gravitating system, including every stellar system. The reason for this failure is very interesting and can be understood by considering a self-gravitating system of mass M and characteristic radius R. The energy of the system is then of order $U = -GM^2/2R$, and, by the 'virial theorem', the mean-square random velocity of the system's constituent particles (atoms or stars), $\langle v^2 \rangle$, is of order $\langle v^2 \rangle = GM/R$. As in ordinary kinetic theory, we identify the system's temperature as

$$T = \frac{\langle m \rangle \langle v^2 \rangle}{3k} = \frac{GM\langle m \rangle}{3kR} = -\frac{2U}{3Nk}$$

where k is Boltzmann's constant and N is the number of particles (atoms or stars) in the system. Now imagine putting the system into thermal contact with a slightly cooler reservoir of heat. Heat will flow out of the system into the reservoir. By conservation of energy, the system's internal energy U will become more negative and, by the equation above, T will increase. This behavior is quite the reverse of what we observe in every-day life: when heat flows out of a coffee cup, it cools down rather than heats up.

We can show that this anomalous behavior of self-gravitating systems drives them away from thermal equilibrium by considering such a system to be made up of two parts: the core and the envelope. If the system is very centrally concentrated, the core is orders of magnitude denser than the envelope and the core's equilibrium is very little affected by the presence of the envelope, so the relation between internal energy and temperature that we derived above can be applied to the core alone. The envelope is confined by a mixture of the 'external' gravitational field of the core and its own self-gravity, so its temperature and internal energy will be connected by a relation that lies somewhere between that derived above for a perfectly self-gravitating system and that appropriate for a classical gas, in which U is positive and $T = \frac{2}{3}U/Nk$. In general we may write $T = \alpha U/Nk$, where $-\frac{2}{3} \leq \alpha \leq \frac{2}{3}$ depending on the degree to which the envelope is self-gravitating.

If initially the core and the envelope are in thermal equilibrium, they must have a common temperature T. Consider a thermal fluctuation that transfers a small quantity of heat, δU, from core to envelope. This transfer causes the temperature of the core to rise by $\delta T_{\rm core} = 2\,\delta U/3Nk$ (because the energy of the core changes by $-\delta U$), while the temperature of the envelope will rise by $\delta T_{\rm env} = \alpha\,\delta U/Nk$. Since the envelope must be to some extent self-gravitating, $\alpha < \frac{2}{3}$ and $\delta T_{\rm core} > \delta T_{\rm env}$. Consequently, after the fluctuation, the core is hotter than the envelope, and the temperature gradient between core and envelope starts to drive a steady flow of heat from core to envelope. This flow increases the temperature difference between core and envelope, and a 'gravithermal catastrophe' ensues in which $T_{\rm core}$ rises inexorably.

The sequence of events that we have just described prevents any classical self-gravitating system, whether it be a star or a galaxy, from achieving thermal equilibrium. None the less, the concept of thermal equilibrium is invaluable in understanding the structure of a star, because in a star there is usually a well-defined temperature at any given radius since individual atoms or photons do not stray far between collisions. In a stellar system, by contrast, stars move at constant energy through wide ranges in radius, and the distribution of stellar velocities at a given radius is not characteristic of any particular temperature.

Although the analogy between the structure of a star and a stellar system is imperfect, the theory of STELLAR EVOLUTION does provide valuable insights into the dynamics of stellar systems. Stars start out in life relatively homogeneous. Gradually the gravithermal catastrophe causes the core to contract and heat up, and the envelope to expand and cool, with the observational consequence that the star becomes a RED GIANT. All stellar systems are at some point on this trek towards every greater central condensation. It is a journey without end because one can show that, no matter what its current configuration, the entropy of a stellar system can be increased by a transfer of energy from the dense interior to the more rarefied exterior.

Dynamics of star clusters

What varies from system to system is the rate at which such transfers are proceeding and the physical processes responsible for them. The systems that evolve in closest analogy with a star are star clusters. In these systems, which contain from a few hundred to perhaps a hundred thousand stars, collisional processes are important in two contexts. First, they effect a transfer of heat from the contracting core to the expanding envelope. Second, they enable energy to be released in the core in much the same way that in a stellar core nuclear reactions release energy. The mechanism of this energy release is as follows. When a single star approaches a BINARY STAR closely, it exchanges energy with the binary and will often form a triple star in which all three stars are bound. Subsequently, two of the three stars will move towards one another, releasing energy which is used to eject the third star at speed. Hence, the net result of the formation of the triple star is the replacement of a relatively slow-moving free star by a much faster one. Over time, this faster star will tend to lose energy to stars that it encounters as it moves through the cluster. Binary stars, being more massive than most single stars, tend to sink to the core. Moreover, the rate at which any given binary star will suffer close encounters with single stars is highest in the dense environment of the core. So most of the energy that is liberated by binary–star encounters heats the core rather than the envelope. It follows that a star cluster resembles a star in that its core is heated and its envelope transmits a flux of thermal energy outwards.

A star cluster's core responds to heating by expanding, and thus reducing the rate at which binary–star encounters heat it. Consequently, oscillations in the density of the core can arise, in which expansion and decreased heating is followed by contraction and increased heating, and then the cycle repeats [6].

Dynamics of elliptical galaxies

ELLIPTICAL GALAXIES differ from GLOBULAR CLUSTERS principally in scale: whereas a globular cluster contains of order 10^5 stars, a massive elliptical galaxy contains in excess of 10^{12} stars. In addition to being much more massive than globular clusters, elliptical galaxies are physically larger too. In fact, a massive elliptical galaxy may have several thousand globular clusters bound to it, with ample space for them to move through the galaxy just like stars. The vast number of stars in an elliptical galaxy, and the huge volume within which they are distributed combine to render collisional effects entirely unimportant. So whereas collisional effects are constantly moving a globular cluster down a well-defined evolutionary path, an elliptical galaxy spends most of its time stuck in a particular configuration. What physical processes got it into the configuration in which it is now frozen?

The answer to this question is not entirely clear, but undoubtedly two important processes are mergers and cannibalism.

In the absence of collisional effects, an individual star can change the energy of its orbit only when the overall galactic gravitational potential is changing. This fact implies that the evolution of a collisionless galaxy is concentrated into brief episodes of violent change: revolution rather than evolution. During one of these episodes of change, rapid change in the overall gravitational potential causes individual stars to gain and lose energy fairly rapidly. This vigorous exchange of energy between stars in turn drives the rapid evolution of the overall potential.

An episode of rapid change in the gravitational potential can be initiated by a close encounter with another galaxy. If the intruding galaxy comes close enough and moves slowly enough, the two galaxies become a binary galaxy within which the gravitational potential is constantly changing as the galaxies swing around each other while moving alternately closer together and further apart.

As the gravitational potential changes, some stars gain energy and others lose it, but overall more energy is picked up than lost by stars. The net gain of energy by stars is mirrored by a net loss of energy from the relative orbit of the two galaxies. Hence the galaxies spiral together and eventually merge (see GALAXIES: INTERACTIONS AND MERGERS).

The time required for the galaxies to merge completely depends on their relative initial masses. If the two galaxies are initially of comparable size, merging is complete by the time the galaxies have orbited around each other two or three times. If one galaxy is much less massive than the other, more orbits are required for merging—in order of magnitude, the number of orbits required is equal to the mass of the bigger galaxy divided by the mass of the smaller. When this ratio is large, the larger galaxy seems to eat the smaller, and one speaks of 'cannibalism'.

The configuration of the galaxy that emerges from a merging event depends significantly on the mass ratio of the merger. If the mass ratio is close to unity, the merger product tends to be triaxial in shape and at most slowly rotating. Larger mass ratios are more likely to produce axisymmetry and significantly rotating merger products [7].

Nuclear black holes

It is likely that all substantial galaxies contain massive central BLACK HOLES—we know for certain that our own Milky Way has a black hole of mass $\sim 2 \times 10^6 M_\odot$, and that our nearest giant neighbor, the ANDROMEDA GALAXY M31, has an even more massive nuclear black hole. The presence of nuclear black holes in galaxies that merge has important consequences for the dynamics of the merger remnant. First, and most conspicuously, the irregular and time-varying gravitational potential of the merging galaxies feeds a significant fraction of any gas in the system onto the black holes, which are then liable to become ultraluminous as they accrete some of this abundant gas supply. In the final stage of the merger, when the nuclei of the two galaxies have spiralled very close together, the black holes can have a significant impact on the distribution of stars in their vicinity. Indeed, the merger leads to the formation of a binary black hole that initially moves through a centrally concentrated sea of background stars. One by one the stars gain energy from the binary black hole and move out to larger radii. Over time the density of the stellar sea ceases to be centrally concentrated. In consequence of losing energy to background stars, the orbit of the binary black hole shrinks and shortens in period until it begins to be relativistic in the sense that its period is not extremely long compared with the time light requires to cross it. When this stage is reached, the radiation of GRAVITATIONAL WAVES by the orbit becomes the dominant mechanism of energy loss. As the orbit shortens further in period, the intensity of the radiated gravitational waves and the rate of orbital decay rapidly accelerate. The merger of the holes must then ensue, surely one of the most awesome events the universe is ever called upon to witness.

Over a long time-scale, the final, merged, black hole can profoundly modify the enveloping galaxy [5, 8]. The galactic merger will frequently have left the latter triaxial. Its triaxiality is sustained by 'box' orbits like that shown in the top panel of figure 2. A star on such an orbit is liable to pass very close to the galactic center, where the black hole resides. If on some passage it comes very close to the black hole, it will be deflected through a large angle and emerge on a very different orbit, like that shown in the bottom panel of figure 2. Clearly, when a sufficient number of stars have been transferred in this way from elongated orbits to fat orbits, the galaxy will become less elongated overall. Moreover, this is liable to be a run-away process: transfers of stars from elongated to fat orbits make the overall galactic potential less elongated, and this reduction in the elongation of the potential reduces the elongation of the orbits of other stars, that have passed nowhere near the black hole.

The details of these processes—the ejection of stars from galactic centers by binary black holes and the probably catastrophic erasure of triaxiality following scattering of high-energy stars by a merged black hole—have not been fully worked out [9], but it is likely they account for the observed fact that lower-luminosity elliptical galaxies have cuspy and probably axisymmetric centers, while more luminous systems have more nearly homogeneous and probably triaxial centers [1].

Dynamics of disk galaxies

Many, perhaps essentially all, stars form in centrifugally supported disks of gas. Consequently, the dynamics of disks of gas and stars is of vital importance for galactic dynamics (see also DISK GALAXIES).

Imagine a disk of stars in which every star is on a circular orbit. This is an equilibrium configuration, but it turns out to be a violently unstable one that will quickly break up into rings of enhanced density separated by

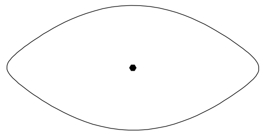

Figure 4. In a suitably rotating frame of reference the rosette orbit of figure 1 closes. The filled point marks the centre of the potential.

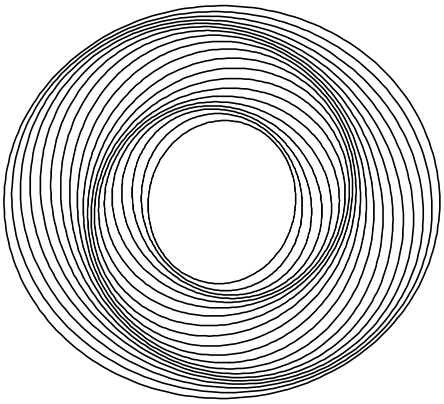

Figure 5. When elliptical orbits are arranged so that the orientation of each major axis changes smoothly with radius, spirals of enhanced density are generated.

density depressions. To avoid rapid fragmentation, the stars of a disk must have random velocities in addition to rotating around the galactic center, and the higher the surface density of the disk, the larger the random velocities of stars need to be if fragmentation is to be avoided.

Since the stars of a stable disk have random velocities, they have to be on non-circular orbits, so they must be on rosette-like orbits such as the one shown in figure 1. If we view such an orbit from a frame of reference that rotates at the right frequency, ω_p, the orbit looks closed as in figure 4. If we populate such an orbit with many stars uniformly distributed in phase around the orbit, we have an elliptical hoop of stars, whose long axis rotates steadily at the angular velocity ω_p—we say that the hoop 'precesses' with frequency ω_p. Notice that the precession frequency ω_p is entirely distinct from the frequency Ω at which individual stars circulate around the hoop—Ω is generally much greater than ω_p. Figure 5 shows what happens if we superimpose many hoops of stars, each with a slightly different radius and initial orientation of its long axis. Spiral arms of enhanced density appear.

Figure 5 describes the essence of how the arms of SPIRAL GALAXIES are constructed. What makes the dynamics of these systems extremely complex is the fact that, in an axisymmetric potential, the precession frequency ω_p varies with the radius of the hoop. In fact, the precession frequency of the innermost orbits in the figure will generally be higher than that of the outermost orbits shown, with the consequence that the spiral pattern will gradually wind up into an ever tighter spiral.

In reality, the spiral arms will contribute a significant non-axisymmetric component to the gravitational potential, and we need to ask how this component affects the precession frequencies of individual hoops. The non-axisymmetric component of the potential applies torques to each ring, and these torques can either increase or decrease the hoop's precession frequency. The idea behind the normal mode theory of spiral structure is to find a configuration of the hoops—essentially a rule, $\phi_0(a)$, for the initial orientation of a hoop's major axis as a function of semimajor axis length, a—such that the torques speed up the slowest hoops and slow down the fastest ones, so that all hoops rotate at the same rate, and the spiral pattern that they form rotates like a rigid body. In these circumstances, ω_p is called the 'pattern speed' of the spiral arms.

Determining the function $\phi_0(a)$ is a mathematically challenging problem and the theory of spiral structure has not often been approached by this direct route. The usual approach is to start from the insight that a self-gravitating disk of stars is a medium that is capable of supporting waves, in the same way that the surface of a pond is. If we disturb the stars at some radius, r_0, we will change the way they contribute to the overall surface density near r_0, and thus we will change the gravitational potential in which other, previously undisturbed stars move. Hence, the motion of these other stars will be disturbed, and they will in turn disturb yet other stars. In fact, our disturbance will spread from radius r_0 like a ripple over a pond.

When waves move in a confined region, such as a teacup or a piano wire, resonant frequencies arise. Waves with these frequencies can travel from a source, and, after possibly multiple reflections from the boundaries, return to the point of origin with the same phase regardless of the number of times that they have been reflected. Because the waves always return with the same phase, they interfere constructively and achieve a significant net amplitude. The resonant frequencies of a simple system such as a piano wire can be exactly determined by finding the frequencies at which waves that are reflected numbers of times interfere constructively. The resonant frequencies of a teacup cannot be found exactly in this way, but good approximations to them can be found thus.

The 'density-wave' theory of spiral structure interprets spiral structure as generated by resonant spiral waves and yields approximate values for the possible pattern speeds and shapes of spiral arms. A significant complication of the theory arises because a typical galactic disk does not have well-defined boundaries like those of a teacup or a piano wire. Boundaries of a sort emerge once one chooses a pattern speed ω_p for the desired spiral pattern. These boundaries are defined by radii at which a star moving on a nearly circular orbit in the undisturbed axisymmetric potential of the galaxy can resonate with a disturbance of frequency ω_p. The inner boundary is the radius of the 'inner Lindblad resonance' (ILR). At this radius the star rotates faster than ω_p, so it regularly overtakes the spiral wave pattern, and the frequency at which it experiences the wave coincides with the frequency, ω_r, of its natural oscillations in radius. The outer boundary is the edge of an annulus in which spiral disturbances cannot propagate. This annulus is centered on the 'corotation resonance', which is the radius at which a star could move on a circular orbit at frequency ω_p. Spiral structure arises when leading spiral waves propagate out towards corotation and are reflected and amplified into trailing waves at the edge of the corotation annulus. The amplified trailing waves then propagate in towards the ILR, where they are refracted into leading waves that move back out to corotation, completing the cycle.

A piano wire is not fundamentally changed by the passage along it of waves. A stellar disk, by contrast, is significantly modified by spiral waves. Such waves increase the random velocities of stars that are in resonance with them. This increase in random velocities is a reflection of the tendency of trailing spiral arms to transport angular momentum out from the ILR and thus to enable stars at the ILR to move inwards. In fact, spiral arms achieve for stellar disks something very similar to what mergers achieve for elliptical galaxies: an increase in the central concentration and entropy of the system. Since the ability of a disk to sustain spiral arms decreases as the random velocities of stars increase, spiral structure will eventually die away in a purely stellar disk. Spiral galaxies such as our own have sustained strong spiral structure for a Hubble time because they have star-formation rates that are fairly constant in time. Young stars have small random velocities and so enhance the disk's ability to support spiral arms.

If one releases from an axisymmetric equilibrium a disk in which stars have very small random velocities, the disk very quickly becomes strongly non-axisymmetric. The non-axisymmetry is not primarily due to spiral structure, however, but to a tumbling bar. The dynamics of a tumbling bar is analogous to that of the spiral arms shown in figure 5: large numbers of orbits are elongated with major axes that rotate at a common frequency, ω_p. A bar differs from a spiral disturbance in that (i) individual orbits tend to be much more strongly elongated in a bar, and (ii) in a bar the major axes are all aligned, irrespective of the radius of the orbit. A large proportion of spiral galaxies, including the Milky Way, have bars at their centers. In a strong bar, a significant fraction of all stars have a chance to pass arbitrarily close to the galactic nucleus, where a massive black hole is expected to lurk. In close analogy to what happens in a triaxial elliptical galaxy, deflection of stars by the gravitational potential of the black hole can lead to the dissolution of the bar [10].

Dynamics of the Milky Way

We live far out in the disk of a BARRED SPIRAL GALAXY, and from this vantage point can study many of the physical processes described above [1]. Figure 6 is a plot of the random velocities of groups of solar-neighborhood stars versus the age τ of the stellar group: the open circles show the group's rms velocity σ_r in the direction to the Galactic center, and the full points show the rms velocity σ_z perpendicular to the Galactic plane. The unambiguous increase in rms velocity with age demonstrates that, although collisional processes are unimportant for the Milky Way, the solar neighborhood is evolving dynamically. We believe that spiral waves drive the increase in σ_r. These waves cannot be directly responsible for the increase in σ_z, however. Clouds of molecular gas with masses in the region of $10^5 M_\odot$ are thought to be responsible for this increase: the gravitational fields of these objects deflect stars that have picked up significant random motion within the plane into the direction perpendicular to the plane, so that they henceforth make significant excursions above and below the plane.

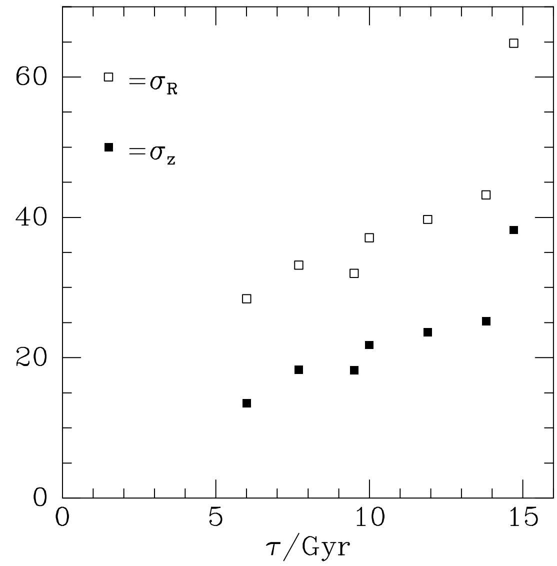

Figure 6. Velocity dispersion versus age for stars in the solar neighborhood. Data from [11].

Figure 6 shows that the random velocities of stars are highly anisotropic in the sense that the random velocity perpendicular to the plane is not much larger than a half of that in the radial direction. The velocity dispersion in the tangential direction, σ_ϕ, lies between these two extremes, and there is an interesting connection between the ratio σ_ϕ/σ_r and the way the speed of a circular orbit varies with galactocentric radius.

Figure 7 shows the difference between the Sun's velocity in the direction of Galactic rotation and the mean velocity in this direction of various stellar groups as a function of the random velocity, S^2, of each stellar group. We see that the Sun is moving faster around the Galactic centre than any stellar group, and the amount by which a stellar group lags the rotation of the Sun tends to increase with the group's random velocity. The physical origin of this observation is that the hotter a stellar disk is, the more slowly it needs to spin to be in centrifugal equilibrium in a given gravitational potential; a hot stellar disk has significant pressure support. Each stellar group forms an

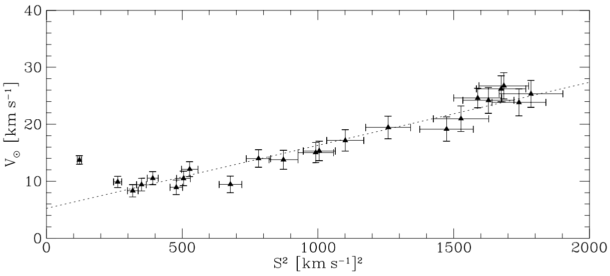

Figure 7. The velocity of the Sun in the direction of Galactic rotation relative to the mean velocity of different stellar groups. S^2 is the mean-square random velocity of each group. Data from [3].

independent disk, which spins at its own rate. The Sun happens to belong to one of the coolest, fastest-spinning disks, and within that disk happens just now to be one of the fastest-circulating stars (see also SUN AS A STAR).

When we look at stars that lie 1 or 2 kpc from the Sun, we see that the mean velocity of the stars at a given place varies from point to point around the Galaxy. This effect mainly reflects the fact that the Galactic disk is a swirling accretion disk, a sort of vortex, in which stars stream around the Galactic center at a speed that varies with radius. In fact, historically observations of this effect were a primary source of data from which the Galactic potential's run of circular speed with radius was inferred. Unfortunately, the data are not easy to interpret, in part because, as we have just seen, the speed with which a population of stars circulates depends on the population's velocity dispersion, and in part because, as we shall now see, the pattern of stellar flow within the Galaxy is in reality more complex than simple circular motion.

If we identify a group of stars, we can measure its 'velocity-dispersion tensor', which is the 3×3 matrix whose elements σ_{ij}^2 are the averages $\langle v_i v_j \rangle$ of products of velocity components. The diagonal elements of this matrix are just the squares of the velocity dispersions that we have just been discussing: $\sigma_{rr}^2 = (\sigma_r)^2$. If the Galaxy were axisymmetric, the off-diagonal elements would vanish. In fact the component $\sigma_{r\phi}^2$ is significantly non-zero for every group of solar-neighborhood stars. Responsibility for this observational result probably lies with two agents. For groups of stars that have small random velocities and therefore stay near the Sun, the non-zero values of $\sigma_{r\phi}^2$ are probably caused by spiral structure. In the case of a group of stars with large ($\sim$50 km s^{-1}) velocities relative to the Sun, the non-zero value of $\sigma_{r\phi}^2$ is probably caused by the bar at the Galactic center. The length of this bar is somewhat uncertain, but probably lies near 3 kpc, rather less than half the Sun's distance from the Galactic center, R_0. Stars with larger velocities relative to the Sun travel on their orbits to sufficiently small Galactocentric radii that they are profoundly influenced by the non-axisymmetric potential of the bar.

Figure 8 is a plot against radius r of the mass $M(r)$ contained within a sphere of radius r around the Galactic

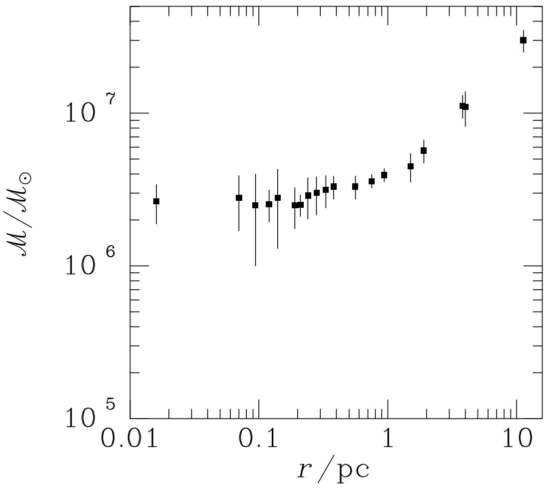

Figure 8. The mass $M(r)$ contained in a sphere of radius r around the Galactic center. Data from [4].

center. This plot was made by studying the kinematics of gas clouds and stars that lie near the Galactic centre and then applying simple dynamical theory. For radii greater than about a parsec, $M(r)$ decreases with r, but, at smaller r, M levels out at a value of order $2 \times 10^6 M_\odot$. The natural interpretation of this result is that a black hole of this mass sits at the Galactic center and that stars make a comparable contribution to M only at radii larger than about a parsec.

Bibliography

[1] Binney J and Merrifield M 1998 *Galactic Astronomy* (Princeton, NJ: Princeton University Press)

[2] Binney J and Tremaine S 1987 *Galactic Dynamics* (Princeton, NJ: Princeton University Press)

[3] Dehnen W and Binney J J 1998 *Mon. Not. R. Astron. Soc.* **298** 387

[4] Genzel R, Eckart A, Ott T and Eisenhauer F 1997 *Mon. Not. R. Astron. Soc.* **291** 219

[5] Gerhard O E and Binney J J 1985 *Mon. Not. R. Astron. Soc.* **216** 467

[6] Goodman J 1987 *Astrophys. J.* **313** 576

[7] Weil M and Hernquist L 1995 *Astrophys. J.* **431** L79

[8] Norman C A, May A and van Albada T S 1985 *Astrophys. J* **296** 20

[9] Rees M J and Hut P 1992 *Mon. Not. R. Astron. Soc.* **259** 27

[10] Sellwood J A and Moore E M 1999 *Astrophys. J.* **510** 125

[11] Strömgren B 1987 *The Galaxy (NATO ASI)* ed G Gilmore and R Carswell (Dordrecht: Reidel) p 229

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