

Astronomy 20

Homework # 5

Posted on the class webpage on November 12, 2004

Due in class on Friday, November 19, 2004

1. Matter near the surface of a star feels both the gravitational attraction and the radiation pressure. The former is more effective on protons and the later on electrons, but the electrostatic force binds them together. The critical luminosity for a given mass at which the two forces balance is called the *Eddington luminosity*, L_E .

- (a) Derive the formula for L_E , by assuming the Thompson cross section for the $(e + p)$ plasma:

$$\sigma_T = \left(\frac{8\pi}{3}\right) \left(\frac{e^2}{m_e c^2}\right)^2$$

Express it as a scaling relation in solar units.

- (b) Now assume that for the stars of solar mass and higher, the following scaling relation applies: $L \sim M^3$. What are the highest mass and luminosity a star could have?
2. (a) Consider a neutron star with a mass just over the Chandrasekhar limit. Estimate its radius.
 - (b) The star is in a contact binary. Assume for simplicity that the accreting matter starts at infinity with a zero net energy. Estimate the fraction of its rest mass it has to radiate away in order to reach the surface of the neutron star.
 - (c) What is the maximum accretion rate in this system, assuming an Eddington-limited accretion?
 - (d) Derive the expression for the growth of this neutron stars in time. How long will it take to double its mass, starting from the Chandrasekhar limit?

3. According to Hawking, a black hole with the mass M_\bullet emits radiation as a blackbody of a temperature:

$$T = \frac{hc^3}{14\pi GkM_\bullet}$$

- (a) Derive a formula for the b.h. luminosity as a function of its mass, assuming that its effective area is $4\pi R_S^2$, where R_S is the Schwarzschild radius.
- (b) By setting $L = c^2 dM_\bullet/dt$, derive and solve the differential equation for the evaporation time of a black hole with an initial mass of M_0 .
- (c) Evaluate the evaporation times for black holes with the initial masses of 10^{-5} grams (primordial?), $3 M_\odot$ (\sim stellar remnants), and $10^8 M_\odot$ (\sim quasars). Compare them with the present age of the universe, ~ 13.5 Gyr.