Astronomy 20

Homework # 6

Posted on the class webpage on November 23, 2004

Due in class on Friday, December 3, 2004

1. Consider a young star cluster in which stars have been made according to the standard IMF: $dN/dm \sim m^{-(1+x)}$, where $x = 1.35$, and ranging from $m_{min} = 0.08 M_\odot$ to $m_{max} = 80 M_\odot$. Assume that the scaling relation between mass and luminosity $L \sim M^4$ applies for all masses.

(a) What is the average stellar mass? Derive a formula for it, for a general power-law IMF, and compute the actual $\langle m \rangle$ for the parameters given above.

(b) Ditto for the average stellar luminosity.

(c) If the cluster mass is $10^3 M_\odot$, what is its absolute bolometric magnitude?

(d) What fractions of the total mass are contributed by the stars above and below the solar mass?

(e) Ditto for the luminosity.

2. An approximate formula for the two-body relaxation time in a stellar system is:

$$t_r = \frac{R}{V} \frac{N}{12 \ln N/2}$$

where R is the typical size of the system, V the typical stellar velocity, and N the number of stars in the system. Derive the relaxation times for:

(a) An open cluster with $R = 10$ pc, $V = 5$ km/s, and $N = 10^3$ stars.

(b) A globular cluster with $R = 1$ pc, $V = 10$ km/s, and $N = 10^5$ stars.

(c) An elliptical galaxy with $R = 10$ kpc, $V = 300$ km/s, and $N = 10^{11}$ stars.

(d) Compare these with the mean crossing times ($t_{cr} = 2R/V$) for these systems, and the age of our Galaxy ($\sim 1.5 \times 10^{10}$ yr).

3. Consider a red giant in a globular cluster with $R = 1$ pc, $V = 10$ km/s, and $N = 10^5$ stars. The radius of the giant is 1 a.u.

(a) What is the chance that it will collide with another star during a single crossing time? (Assume that most other stars have much smaller radii, and approximate the cluster as a sphere of uniform density.)

(b) If there are 1,000 red giants in this cluster, what is the mean time between their collisions with other stars? How many have collided over the entire history of the cluster?

4. (a) Using the virial theorem, estimate the binding energy of a globular cluster containing 10^5 stars of an average mass $M_* \approx 0.5 M_\odot$, with an effective "mean" radius of $R = 3$ pc. What is the mean velocity of stars?

(b) Now consider a binary in the cluster core, where both components have masses of 0.5 $M_\odot$, in a circular orbit with a diameter of 10 a.u. Compute the binding energy of this binary, and compare it with the mean binding energy per star in the cluster as a whole. Is this a "soft" or a "hard" binary?