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Andrew Liddle

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The Cosmological Constant and its Interpretation

The cosmological constant was first introduced into the equations of general relativity by Einstein himself, who later famously criticized this move as his ‘greatest blunder’. His main motivation had been to allow cosmological models featuring a static universe, but this possibility swiftly became redundant with Edwin Hubble’s discovery of the expansion of the universe. Despite this, it has periodically been invoked by astronomers to explain various phenomena and currently is a key part of the standard cosmological model, which seeks to explain both the evolution of large-scale universe and also the development of structures, such as galaxies, within it.

The most straightforward description of the cosmological constant is that it represents the energy of empty space. Usually physical laws only concern the *difference* in energy between two states (nature favouring evolution with force directed towards the state of lowest potential energy) rather than the absolute value of energy, but relativity demands that all energy gravitates and so the zero point of energy must be specified. The Einstein equation can be written

$$G_{\mu\nu} = g_{\mu\nu}\Lambda + 8\pi GT_{\mu\nu}$$

where $G_{\mu\nu}$ is the Einstein tensor measuring the curvature of space–time (which gives the gravitational attraction), $T_{\mu\nu}$ is the energy–momentum tensor measuring the properties of matter, $g_{\mu\nu}$ is the metric of space–time and Λ is the cosmological constant. Throughout the speed of light will be set to $c = 1$. If $\Lambda = 0$, then an absence of matter ($T_{\mu\nu} = 0$) leads to an absence of space–time curvature, but if Λ is non-zero we have gravity associated with the vacuum.

Although Einstein regretted the introduction of the cosmological constant, which certainly reduces the elegance of the equation, once it has been introduced it becomes rather hard to argue that it should not be there. There is no fundamental principle to exclude it, and indeed as we will see modern particle physics requires its consideration.

Cosmological models

Cosmological models including a cosmological constant are described in detail under COSMOLOGY: STANDARD MODEL. The Friedmann equation describes the evolution of the scale factor of the universe, $R(t)$, according to

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{k}{R^2},$$

where ρ is the density of matter, k measures the spatial curvature and the dot indicates time derivative. If preferred, the cosmological constant can be included as a

form of matter by defining $\rho_\Lambda = \Lambda/8\pi G$. The effect of the cosmological constant can be seen more clearly from the acceleration equation,

$$\frac{\ddot{R}}{R} = \frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}$$

where p is the pressure. While the density ρ always decelerates the expansion, a positive cosmological constant favors acceleration and hence acts as a repulsion. Indeed, in the limit where the cosmological constant dominates, the solution is an exponential expansion

$$R(t) \propto \exp(\sqrt{\Lambda/3}t).$$

Studying the way in which Λ enters these equations indicates that it can be considered as a fluid with equation of state $p_\Lambda = -\rho_\Lambda$.

There is a characteristic scale for the density, known as the ‘critical density’, which gives rise to a spatially-flat universe, $k = 0$. On defining the Hubble parameter, which measures the expansion rate, as $H \equiv \dot{R}/R$, the critical density is

$$\rho_c = \frac{3H^2}{8\pi G}.$$

It is often useful to consider the densities of the different components as fractions of this, by defining the density parameter $\Omega = \rho/\rho_c$ and the cosmological constant density parameter as

$$\Omega_\Lambda \equiv \frac{\rho_\Lambda}{\rho_c} = \frac{\Lambda}{3H^2}.$$

The condition for a flat universe is therefore that the sum of the Ω over all types of material is equal to unity.

In general the cosmological constant adds considerably to the possible phenomenology of big bang models. For example, there are models where the cosmic repulsion is so strong that there is no big bang, with the universe instead contracting and then bouncing. There can be open universe models which recollapse and closed universe models which expand forever; it is even possible to construct closed universe models where an observer can (if they wait long enough) see right round the universe. Another possibility is a long ‘loitering’ phase where the universe stays almost constant in size for a protracted period. However, the main focus has been on models which expand from an initial singularity and where the cosmological constant is negligible at early stages, only beginning to manifest itself at recent cosmological epochs.

Observational evidence for Λ

Observations in recent years have established a standard cosmological model where the present density of the universe is dominated by a cosmological constant. It is fair to say that cosmologists have been reluctant to include the cosmological constant but have been forced into it by the quality of a variety of observations, which paint a consistent picture depending on its presence. The main supporting evidence falls into two classes. One is

direct probing of the expansion rate of the universe by measuring the properties of distant objects, in this case supernovae. The second is to determine the density of matter in the universe and compare with the total density as inferred from the geometry of the universe.

Distant supernovae of type I, believed to be fueled by the collapse of an accreting white dwarf to a neutron star, are an excellent probe of the expansion rate of the universe because their absolute brightness is believed to be nearly the same for each explosion, with empirical corrections available to improve the uniformity of the sample even more. This means that they can be used to study the dependence of the luminosity distance (see COSMOLOGY: STANDARD MODEL) on redshift. This relation is known as the HUBBLE DIAGRAM, with supernovae now able to map this out to redshifts approaching unity. This relationship can be related to the expansion history of the universe $R(t)$, which the Friedmann equation shows is sensitive to the presence of a cosmological constant. Constraints on the densities of the cosmological constant and of matter are shown in figure 1 as the full and broken contours, and almost all the preferred region has a positive cosmological constant (and indeed a currently accelerating universe).

The second observational route concerns the matter budget of the universe. Recent cosmic microwave background anisotropy measurements have given strong evidence that the universe is spatially flat, or at least very nearly so (also shown in figure 1 as the dark shaded region). This requires that the total density of the universe, including the cosmological constant, must add to the critical density. However, there is plentiful evidence that the density of matter is well below this. For example, studies of clusters of galaxies show that there is about eight times as much dark matter as protons and neutrons, the latter being constrained by the formation of light elements to be about 4% of the critical density. Alternatively, the observed clustering properties of galaxies are known to be reproduced only if the dark matter density is around 30% of critical. Finally, direct estimates of the amount of matter providing gravitational attraction always fall well below the critical density. There is therefore a substantial shortfall in reaching the critical density required by the cosmic microwave background observations, a gap which must be filled by a form of material which does not clump gravitationally. The cosmological constant fits the bill perfectly.

Accordingly, the currently favoured cosmological model features a cosmological constant at around two-thirds the critical density, with matter making up the remaining third. The latter is mostly dark matter, but includes also the conventional matter from which we are made as well as the cosmic radiation backgrounds.

In summary, current observations strongly favor cosmological models with a flat spatial geometry ($k = 0$)

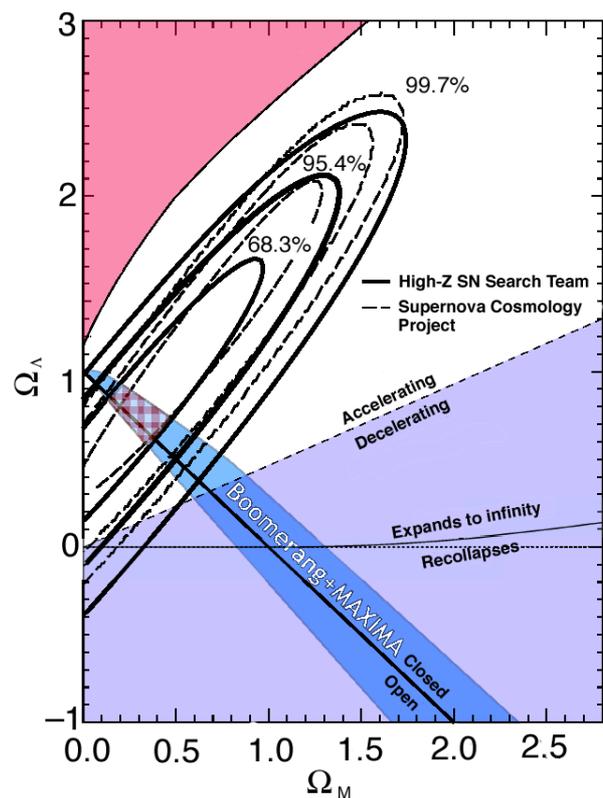


Figure 1. Constraints on cosmology from supernovae and the microwave background. The horizontal axis is the density of matter (including dark matter) as a fraction of the critical density, while the vertical axis is the density of the dark energy. The contour lines show the favored region from two separate studies of distant supernovae. The shaded diagonal band shows the region allowed by measurements of the anisotropies of the cosmic microwave background by the Boomerang and Maxima experiments. In combination, these allow only the small checked region, where the cosmological constant density is about twice that of the matter. (Figure courtesy Brian Schmidt; see also <http://cfa-www.harvard.edu/cfa/oir/Research/supernova/HighZ.html>.)

and which are accelerating at present. The simplest interpretation of the observed acceleration is that the universe possesses a cosmological constant.

The cosmological constant in particle physics

Explaining the observed magnitude of the cosmological constant is regarded as one of the outstanding problems in particle physics. This is because particle physics theory indicates that the magnitude of the cosmological constant should be huge, vastly greater than is observed. The reason is that in quantum physics one expects a zero-point energy to be associated with every kind of particle, and therefore that there should be an energy density associated with the vacuum.

It is not hard to see that the vacuum energy density is equivalent to a cosmological constant. If all observers

are to agree that a given state is the vacuum, then its energy–momentum tensor must be Lorentz invariant. The only tensor with this property is a multiple of the metric tensor $g_{\mu\nu}$, and hence we can write

$$T_{\mu\nu}^{\text{vacuum}} = -\frac{\Lambda}{8\pi G} g_{\mu\nu},$$

which ensures that the vacuum energy density does act precisely as a cosmological constant.

To assess the magnitude of the problem, we should first convert the observed value of the cosmological constant into particle physics units. Its equivalent mass density $\rho_\Lambda = \Lambda/8\pi G$ can be taken to be about two-thirds the critical density, giving

$$\rho_\Lambda \approx 6 \times 10^{-27} \text{ kg m}^{-3} \Rightarrow \rho_\Lambda \approx (0.002\text{eV})^4.$$

This energy scale, written in electronvolts, is tiny compared with known characteristic energy scales of particle physics (for example the proton mass–energy is around 10^9 eV).

In quantum mechanics, unlike in classical physics, the vacuum state is a seething state where particles are constantly coming in and out of existence. There is a zero-point energy (the analog of the residual energy left in a gas at a temperature of absolute zero) associated with each possible particle state, which can be added up to give an estimate of the zero-point energy. Unfortunately the most simplistic calculations give an infinite answer, as each momentum state gives an equal contribution and there are an infinity of them; this indicates that the theory used to compute the contributions cannot be valid to infinite energies.

However, to what energies should the calculation be believed? According to present understanding, we are limited by the Planck energy, at which point gravity would have to be described by a quantum theory. The cosmological constant should therefore be approximately equal to the Planck energy density. Unfortunately, this is a massive $(10^{28} \text{ eV})^4$, larger than the observed value of the cosmological constant by a factor of around 10^{120} (earning this calculation the description as ‘the worst order-of-magnitude estimate in history’). This situation can be improved a little by conceding that actually we do not really understand physical laws above the energy scales probed by large particle accelerators such as CERN, and indeed a popular theory known as supersymmetry predicts that at higher energies a new theory comes into play that would ensure no contributions to the cosmological constant from energies much above those currently probed. Particle accelerators operate up to energies of around 10^{12} eV, so plausibly we can reduce our estimate to $(10^{12} \text{ eV})^4$, reducing the embarrassment to a mere factor of 10^{60} !

It is currently believed that superstring theory (or its more elaborate cousin M-theory) is a strong candidate as

a theory of quantum gravity. In principle it should therefore make much more definite predictions for the cosmological constant, as it is not subject to the cut-offs described above. However, current incarnations of superstring theory have not been developed sufficiently to allow unambiguous predictions to be made at this stage.

The historical cosmological constant problem was to explain why the cosmological constant appeared to be zero, despite the prediction that it should be huge, and investigations focused on various cancellation mechanisms and symmetry principles which might justify why the precise value of zero might be favored. However, the astronomical observations just described have, if anything, made the problem worse, as they demand an explanation for why the cosmological constant is both tiny *and* non-zero. No compelling, or even just plausible, explanation has yet arisen.

Dark energy/quintessence

While a cosmological constant can indeed give the observed accelerated expansion of the universe, it is not the only explanation. Since the early 1980s, it has been popular to suppose that the universe underwent a period of acceleration during its extremely early stages, known as INFLATION. Such a period of inflation cannot continue forever, because that would spoil the outstanding successes of the hot big bang model, such as nucleosynthesis and the origin of the cosmic microwave background. Early universe inflation therefore cannot be driven by a pure cosmological constant, but rather must be caused by a transient phenomenon mimicking a cosmological constant.

Much effort has gone into modeling inflationary expansion, and the standard mechanism is to assume that the universe is temporarily dominated by the potential energy of a scalar field. A scalar field describes a collection of spin-zero particles (which are popularly used in particle physics models to describe the breaking of symmetries), with the potential energy measuring the combination of the mass density of the particles and their binding energy. Under the right circumstances, such a scalar field can drive a temporary period of accelerated expansion, which comes to an end with the scalar particles decaying into conventional matter (see INFLATION for full details).

These ideas are an equally valid mechanism for driving acceleration in the present universe and provide an alternative to a pure cosmological constant that many cosmologists find attractive. The term ‘dark energy’ is now widely used to indicate any method of driving accelerated expansion, encompassing both the pure cosmological constant and any other method of obtaining an acceleration. This phrase is on no account to be confused with dark matter (see DARK MATTER: ITS

NATURE), which has entirely different properties. The specific use of a scalar field to drive the acceleration is usually referred to as ‘quintessence’ (the other four ‘essences’ of the universe being the baryons, the photons, the neutrinos and the dark matter). In the case of quintessence, it is presumed that any pure cosmological constant must be identically zero, for some as-yet-undiscovered reason.

This more general setting allows the possibility that the density of the dark energy may not be strictly constant, as in the case of the cosmological constant. This can be quantified via the equation of state parameter w_Q , defined by

$$p_Q = w_Q \rho_Q$$

where the subscript Q indicates that we refer to the density and pressure of the dark energy component. A pure cosmological constant has $w_Q = -1$, whereas to drive an accelerated expansion requires only $w_{\text{tot}} < -1/3$, where w_{tot} is the combined equation of state of the dark energy and the other matter components (including dark matter) weighted by their densities. With observations indicating roughly twice as much dark energy as other matter, accelerated expansion will result provided that w_Q is less than about -0.5 .

The quintessence energy density appears in the Friedmann equation, which for a flat universe reads

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3} (\rho + \rho_Q),$$

where we will have

$$\rho_Q \propto R^{-3(1+w_Q)}.$$

If the density of dark energy is varying, the solution for the scale factor $R(t)$ will be different to that obtained with a pure cosmological constant, and this provides possible observational signatures. It modifies the relation between luminosity distance and redshift, as probed by distant supernovae, and instead of using those data to constraint the densities of a cosmological constant and conventional matter, one can assume a flat universe and attempt to simultaneously constrain the density of dark energy and its equation of state. Another effect is that the altered expansion rate will change the growth rate of density perturbations in the dark matter, modifying the structure formation process. Finally, the angular diameter distance to the last scattering of the cosmic microwave background will be modified, leading to a shift in the angular size of features (the positions of the acoustic peaks).

Several studies of these effects have been made, and as yet no evidence has been found for variation in the density of dark energy. The pure cosmological constant remains a viable explanation of the observational data, and observations require that the equation of state w_Q be less than about -0.7 (at 95% confidence). Upcoming

observations, perhaps including a satellite-based supernova search, have the capacity to significantly strengthen this constraint and hence may detect a time variation of the dark energy if it is sufficiently large.

If the dark energy is permitted to have time variation, it must also be able to support spatial variations, in the form of density perturbations. In principle these can interact with and affect perturbations in the dark matter, modifying the process of structure formation which may lead to further observational tests. However in practice these effects are highly suppressed in realistic models and it is likely to be very hard to determine the properties of dark energy through studying these effects.

If we appeal to the same physical mechanism both to drive the inflationary acceleration of the early universe and the present-day acceleration, does that mean that the same scalar field might be responsible for both? In most models the answer is no, because of the vastly different energy scales involved; we are using the same physical mechanism, but the two fields are not connected. However, it is possible to contrive models where the same field is responsible; Peebles and Vilenkin called such models ‘quintessential inflation’. As the same scalar field has multiple duties to attend to, it has many more observational requirements that it must satisfy and so such scenarios can be quite strongly constrained. However, fundamental particle physics indicates we should expect many scalar fields to exist in nature, so there seems little motivation to force one of them into doing all the cosmological work. Accordingly, most attention has been directed at models where early universe inflation and the present dark energy are separate phenomena.

The cosmological constant coincidence

In addition to the daunting task of understanding the numerical value of the cosmological constant, it poses a further problem known as the ‘coincidence problem’. It is an apparent coincidence that we are developing our understanding of cosmology during the fairly brief epoch (brief by cosmological standards—a few billions of years in actuality) where the cosmological constant has a density similar to that of other matter. The ratio of densities of the cosmological constant to dark matter goes as the cube of the scale factor; when the universe was a tenth its present size the cosmological constant was utterly negligible, while once the universe has doubled its present age it will be completely dominant.

One of the advantages of the dark energy hypothesis is that, in enabling the density of the effective cosmological constant to vary, it may open routes to explaining this coincidence. Unfortunately, however, no compelling model has yet arisen, despite considerable interest in so-called ‘tracker models’ where a quintessence field responds to, and in some cases mimics, the behavior of conventional matter.

An alternative route is to invoke the ANTHROPIC PRINCIPLE, which states that we should expect to see cosmological parameters take on values which give the universe which, out of all possible universes, is most amenable to the development of life. This approach can in principle explain both the numerical value and the coincidence, as a very large value of the cosmological constant will prevent galaxies from forming, while there is some indication that a significant value may enhance the number of galaxies produced. In this approach, we require fundamental physics to allow a wide range of possible values of the cosmological constant (which may be realized in widely separated regions of the universe, or even perhaps just in possible quantum states of the universe), and the precise value which arises in our part of the universe has no fundamental significance.

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