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Correlation Function and Power Spectra in Cosmology

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The galaxy correlation function is a measure of the degree of clustering in either the spatial ($\xi(r)$) or the angular distribution ($w(\theta)$) of galaxies. The power spectrum is the Fourier transform of the correlation function. The spatial two-point or autocorrelation function is defined as the excess probability, compared with that expected for a random distribution, of finding a pair of galaxies at a separation r_{12} (see figure 1):

$$\mathrm{d}P = \bar{n}^2(1 + \xi(r_{12}))\,\mathrm{d}V_1\,\mathrm{d}V_2$$

where $\bar{n}$ is the mean galaxy density. On small scales, $0.1\ h^{-1}$ Mpc $\leq r \leq 10\ h^{-1}$ Mpc, the spatial correlation function is well described by a power law form $\xi(r) = (r_0/r)^\gamma$, with slope $\gamma \sim 1.8$ and a correlation length $r_0 \sim 5\ h^{-1}$ Mpc. The angular galaxy correlation function can also be written as a power law, with $w(\theta) = A\theta^{1-\gamma}$, where A depends upon the depth of the galaxy sample, due to dilution of the clustering signal as a result of projection effects. The power spectrum $P(k)$ is related to the two-point correlation function by:

$$\xi(r) = \frac{1}{2\pi^2}\int \mathrm{d}k\,k^2 P(k)\frac{\sin(kr)}{kr}.$$

The scale or wavelength λ of a fluctuation is related to the wavenumber k by $k = 2\pi/\lambda$.

The power spectrum is the quantity predicted directly by theories for the formation of large scale structure. In the case of a density field in which the fluctuations are drawn from a Gaussian distribution, the power spectrum gives a complete statistical description of the fluctuations.

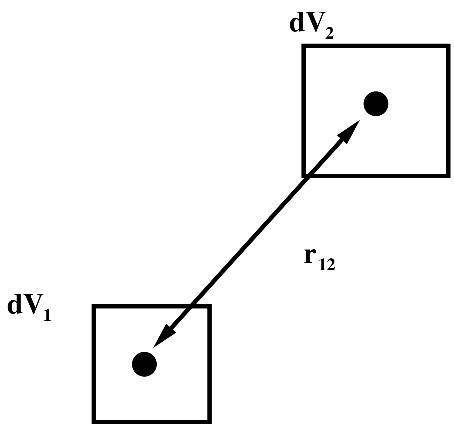

Figure 1. The two-point correlation function describes the excess probability, compared with a random distribution of galaxies, of finding a galaxy in an element of volume $\mathrm{d}V_2$ at distance r_{12} away from a galaxy in $\mathrm{d}V_1$.

The power spectrum of density fluctuations

The primordial power spectrum

The standard paradigm for the formation of large scale structure in the universe is that initially small fluctuations in density are amplified by gravity. The origin of these fluctuations is uncertain, one possible explanation being that they are quantum fluctuations boosted to macroscopic scales by INFLATION. The amplitude of fluctuations on different length scales or equivalently on different mass scales is described by the power spectrum. The primordial power spectrum is usually assumed to have a power law dependence on scale; $P(k) \propto k^n$. A popular choice is the scale-invariant spectrum with spectral index $n = 1$, proposed independently by Zeldovich and Harrison. In this case, fluctuations on different length scales correspond to the same amplitude of fluctuation in gravitational potential.

The shape of the power spectrum

The rate at which fluctuations grow on different scales is determined by an interplay between self-gravitation, pressure support and damping processes. These effects lead to a modification of the form of the primordial power spectrum that is expressed in terms of a transfer function $T(k, z)$

$$P(k, z) = A(z)k^n T(k, z).$$

The normalization factor $A(z)$ is determined observationally.

A key concept in the theory of gravitational instability is the Jeans length. Fluctuations on scales larger than the Jeans length are unstable against contraction under gravity, whilst fluctuations on smaller scales are supported against further collapse by pressure forces.

Fluctuations on scales larger than the size of the horizon grow through self-gravity. As the universe grows older, the horizon expands and encompasses density fluctuations on progressively larger scales. Fluctuations that enter the horizon when the density of radiation is driving the expansion of the universe are effectively frozen until the matter and radiation densities become equal. The radiation density effectively acts as a pressure that prevents the further collapse of any perturbation in the matter density. The universe is expanding too rapidly for the matter fluctuations to collapse under gravity. The horizon scale at the epoch when the matter and radiation densities are equal is imprinted upon the power spectrum as the scale at which the spectrum turns over from the primordial shape, $P(k) \propto k$ (see figure 2).

After the redshift at which the matter and radiation densities are equal, fluctuations made of collisionless matter i.e. nonbaryonic dark matter that is not coupled through electromagnetic interactions to the radiation, can grow gravitationally. Baryonic or collisional matter is still coupled to the radiation, which provides a pressure force that resists gravitational collapse. The Jeans mass in baryons falls by many orders of magnitude when the matter and radiation decouple at a redshift of around $z \sim 1100$.

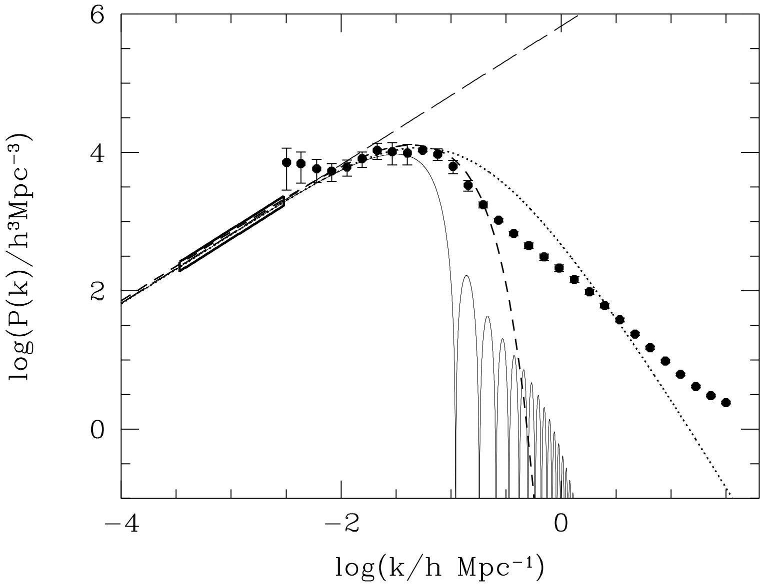

Figure 2. Examples of power spectra for universes with the critical density in mass. The long dashed line shows the Harrison–Zeldovich form of the primordial power spectrum, $P(k) \propto k$. The dotted line shows the power spectrum in a universe with the critical density in cold dark matter, the solid line shows the power spectrum when baryons contribute all the critical density, whilst the short dashed line shows a universe in which all the mass is in the form of massive neutrinos. The amplitude of the power spectra has been set to agree with the constraints from the COBE satellite measurement of temperature fluctuations in the cosmic microwave background, indicated by the box at $\log k \sim -3$. The points show a measurement of the power spectrum of galaxy clustering.

During the process of recombination, when large fractions of neutral hydrogen form locking up the electrons that were responsible for scattering the radiation, Silk damping of baryonic density fluctuations can occur. The mean free path of photons between scattering events with electrons increases during recombination, and the photons can diffuse out of overdense regions, smoothing out fluctuations in the baryon distribution.

Fluctuations in collisionless dark matter are damped by the free-streaming of fast moving particles out of overdensities into neighboring underdense volumes. The scales up to which the density fluctuations are erased depends upon the random velocities of the dark matter particles. Hot dark matter particles (a suitable candidate would be a massive neutrino) remain relativistic until the matter and radiation densities are equal, with free-streaming erasing fluctuations up to the mass scale of large super-clusters. Cold dark matter (possible candidates include axions and weakly interacting massive particles) becomes non-relativistic at much earlier times and fluctuations on essentially all scales are preserved.

Normalization of the power spectrum

Two methods are commonly employed to normalize the power spectrum of density fluctuations. The first uses the amplitude of temperature anisotropies in the COSMIC MICROWAVE BACKGROUND radiation, which can be related to density fluctuations at the last scattering surface. The COBE detection of temperature fluctuations on angular scales around 10° constrains the power spectrum amplitude on very large scales, $k \sim 0.001\ h\ \mathrm{Mpc}^{-1}$. However, some of this signal could arise from gravity waves.

The second method is to count the number of hot x-ray emitting clusters in the local universe. The abundance of these rare objects is very sensitive to the amplitude of density fluctuations on scales around $8\ h^{-1}$ Mpc, which corresponds to a mass of roughly $10^{15}\ h^{-1} M_{\odot}$ in a critical density universe. A source of uncertainty here is the conversion from x-ray temperature to cluster mass.

Constraining the power spectrum

The shape and amplitude of the power spectrum of density fluctuations contains information about both the amount and nature of matter in the universe (figure 2). Direct measurements of the mass power spectrum can be obtained from the dynamics of galaxies in surveys that cover the full sky or from the distortion of images of faint galaxies due to the gravitational lensing of their light by the intervening dark matter distribution. A popular approach to constraining the power spectrum of mass fluctuations is to measure the power spectrum of galaxy clustering. Several large galaxy surveys were completed in the 1990s and a consistent view of galaxy clustering is beginning to emerge; that has influenced the development of models for structure formation. There are a number of effects that make the inference of the mass power spectrum from the galaxy power spectrum difficult; these are listed below. These effects are best understood on large scales where the fluctuations are still relatively small.

Nonlinear evolution of density fluctuations
The evolution of a small amplitude density fluctuation on a particular scale can be followed independently of fluctuations that may exist on other length scales using linear perturbation theory. In the later stages of collapse, fluctuations on different length scales become coupled and the subsequent evolution is nonlinear. Higher order perturbation methods or numerical N-body simulations have to be employed to study the final collapse of the fluctuation. Nonlinear evolution of density fluctuations changes the shape of the power spectrum.

Galaxy formation and bias
There is no reason to assume that the distribution of light in the universe traces exactly the distribution of mass. Galaxy formation involves dissipative processes, such as the radiative cooling of hot gas, as well as gravity, and it is plausible that the efficiency of galaxy formation is related to the density of dark matter. On large scales where density fluctuations are small, a commonly used model is to assume the existence of a linear bias b between fluctuations in mass and in the galaxy distribution:

$$\left(\frac{\delta\rho}{\rho}\right)_{\rm galaxies} = b\left(\frac{\delta\rho}{\rho}\right)_{\rm mass}.$$

In this case, the galaxy power spectrum is simply related to the mass power spectrum by $P_{\rm galaxies}(k) = b^2 P_{\rm mass}(k)$. Computer simulations of galaxy formation and the clustering of dark matter show that this model is inadequate on intermediate and small scales, and that the bias b is actually scale dependent (see UNIVERSE: SIMULATIONS OF STRUCTURE AND GALAXY FORMATION).

Peculiar motions of galaxies
Three-dimensional surveys use the redshift of spectral lines to determine the radial distance to a galaxy. The redshift has a contribution both from the Hubble expansion and from local inhomogeneities in the gravitational field around the galaxy, which give rise to an additional or peculiar velocity. The pattern of clustering inferred from the redshift of the galaxies is thus distorted compared to that obtained if the true radial distances to the galaxies are available. This is turn changes the shape of the power spectrum of galaxy clustering. On small scales the power spectrum is damped due to the motion of galaxies inside virialized structures; on large scales the power is enhanced by coherent flows of galaxies onto large scale structures.

Redshift surveys that will be completed in the first decade of the new millennium will contain up to a million galaxies, an order of magnitude more than existing surveys, thus making possible high precision measurement of the power spectrum on large scales.

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