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WIMPs and MACHOs

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WIMPs and MACHOs

WIMP is an acronym for weakly interacting massive particle and MACHO is an acronym for massive (astrophysical) compact halo object. WIMPs and MACHOs are two of the most popular DARK MATTER candidates. They represent two very different but reasonable possibilities of what the dominant component of the universe may be.

It is well established that somewhere between 90% and 99% of the material in the universe is in some as yet undiscovered form. This material is the gravitational glue that holds together galaxies and clusters of galaxies and plays an important role in the history and fate of the universe. Yet this material has not been directly detected. Since extensive searches have been done, this means that this mysterious material must not emit or absorb appreciable electromagnetic radiation in any known waveband. Thus it is called dark matter. The nature of this material is one of the biggest unsolved problems in science. It is important to identify the dark matter, but since it is easy to invent substances that could fill intergalactic space and yet have escaped detection, there are a very large number of dark matter candidates. Thus several generic classes of dark matter candidates have been suggested, and each dark matter search experiment concentrates on one of these classes. The most important dark matter classes, in terms of detection efforts, are neutrinos, WIMPs, MACHOs and axions.

For example, NEUTRINOS are weakly interacting particles that were almost certainly created in great abundance during the big bang. These fill the Galaxy, moving freely through the Galaxy and even the Earth, and yet are almost impossible to detect. This is because they can be sensed only through the very small electroweak interaction. If each neutrino had a mass of several electronvolts they would contribute enough mass to make up the bulk of the dark matter. For various reasons, it is unlikely that neutrinos of the type discovered in particle accelerators and nuclear reactors on Earth make up much of the dark matter.

However, it is very possible that some as yet undiscovered weakly interacting particle was created during the big bang and today remains in large enough abundance to make the dark matter. The masses typically required for these particles are in the range 1 GeV–1 TeV, and these hypothetical dark matter particles are called WIMPs. There are hundreds of elementary particles that fall into this class of dark matter particles, including supersymmetric particles such as neutralinos, photinos, higgsinos or sneutrinos, and new heavy neutrinos.

This hypothetical WIMP is well studied and attempts to detect these particles have been mounted both by creating them in accelerators and by sensing them in underground detectors as they pass through the Earth. However, there is a large class of astronomical

objects that could be the dark matter and still escape detection. For example, if the Galactic halo were filled with Jupiter mass objects ($10^{-3}M_{\odot}$) they would not have been detected by emission or absorption of light. Brown dwarf stars with masses below $0.08M_{\odot}$ or the black hole remnants of an early generation of stars would be similarly invisible. Thus these objects are examples of MACHOs. Other examples of this class of dark matter candidates include primordial black holes created during the big bang, neutron stars, white dwarf stars and various exotic stable configurations of quantum fields, such as non-topological solitons.

An important difference between WIMPs and MACHOs is that WIMPs are non-baryonic and MACHOs are typically (but not always) formed from baryonic material. As discussed in the article on big bang nucleosynthesis (see UNIVERSE: THERMAL HISTORY), baryonic material probably cannot make up all of the dark matter, although it could make up most of the dark matter in the halos of spiral galaxies such as the Milky Way. There is preliminary, although controversial, evidence for the existence of large numbers of MACHOs, but because they probably cannot make up all the dark matter, the search for WIMPs continues unabated.

WIMP thermal relics as dark matter

Among the particle dark matter candidates an important distinction is whether the particles were created thermally in the early universe, or whether they were created non-thermally in a phase transition. Thermal and non-thermal relics have a different relationship between their relic abundance Ω and their properties such as mass and couplings, so the distinction is especially important for dark matter detection efforts. For example, the WIMP class of particles can be defined as those particles that are created thermally, while dark matter axions come mostly from non-thermal processes. Light neutrinos are also thermally created relics, but because of their very small mass have a different history.

In thermal creation one supposes that early on, when the universe was at very high temperature, thermal equilibrium obtained, and the number density of WIMPs (or any other particle species) was roughly equal to the number density of photons (particles of light)¹. This is just equipartition of energy among all possible degrees of freedom. As the universe cooled the number density of WIMPs and photons decreased together. When the temperature finally dropped below the WIMP mass, however, creation of WIMPs became very rare while annihilation still proceeded. Thus in equilibrium, the number density of WIMPs dropped exponentially: $\exp(-m_{WIMP}/T)$. If equilibrium were maintained until

¹ There is a small difference between the number density of photons and neutrinos since photons obey Bose–Einstein statistics and neutrinos obey Fermi–Dirac statistics.

today there would therefore be very few WIMPs left, but at some point the WIMP density dropped low enough that the probability of one WIMP finding another to annihilate became small. Note, we must assume that an individual WIMP is stable if it is to become the dark matter. The WIMP number density stopped dropping at this point and we are left with a substantial number of WIMPs today. Detailed evolution of the Boltzmann equation that describes this process can be done for an accurate prediction, but roughly the predicted relic density today of WIMPs is inversely proportional to their interaction strength. The remarkable fact is that, for a relic density equal to the known dark matter density, the interaction strength must be that expected for particles with electroweak-scale interactions: thus the 'W' for 'weakly' in 'WIMP'. There are several theoretical problems with the standard model of particle physics that are solved by new electroweak-scale physics such as SUPER-SYMMETRY. Thus these theoretical problems may be clues that the dark matter does indeed consist of WIMPs. Said another way, any stable particle that annihilates with an electroweak-scale cross section is bound to contribute to the dark matter of the universe. It is interesting that theories such as supersymmetry, invented for entirely different reasons, typically predict just such a particle.

The fact that thermally created dark matter has weak-scale interactions also means that it may be within reach of accelerator experiments such as LEP and LHC at CERN, and the proton collider experiments at Fermilab. Thus many accelerator searches for exotic particles are also searches for the dark matter of the universe. Also, because of the weak-scale interactions, WIMP–nuclear interaction rates are within reach of many direct and indirect detection methods, as discussed below.

Supersymmetry and dark matter

Supersymmetry is a new hypothetical symmetry of nature that relates bosons and fermions. If supersymmetry exists in nature then every known particle should have a supersymmetric partner. Bosonic ordinary particles have fermionic superpartners with the same name except with the suffix 'ino' added, while fermionic ordinary particles have bosonic (scalar) superpartner names with the prefix 's' added. Examples of proposed supersymmetric particles include photinos, higgsinos, Z-inos, squarks and selectrons. Some supersymmetric particles have the same quantum numbers as each other and therefore can mix together producing particles that are not exact partners of any standard model particle. For example, the photino, Higgsino and Z-ino can mix into arbitrary combinations called the neutralinos.

In most models, the lightest supersymmetric particle (LSP) is stable, and since supersymmetric particles have electroweak-strength interactions, the LSP makes an excellent dark matter candidate. Typically the neutralino

is the LSP so most investigations of WIMP dark matter have concentrated on the neutralino. However, there are many possible supersymmetric models and many free parameters in the models, so precise predictions of supersymmetric relic abundance and supersymmetric particle detectability are not possible. Typically experiments attempt to probe a range of model parameters. So far no concrete evidence of any supersymmetric partner exists. If even one supersymmetric partner is found, the theory predicts that they all must exist.

Note that the parameters that determine the relic abundances also determine all the particle production and rare decay cross sections, as well as the rate in various detectors. Thus once these parameters are specified or measured, one can compare the model predictions with experimental results.

Search for WIMPs

Accelerator searches

Extensive unsuccessful searches for the particles involved in supersymmetric models have been performed at particle accelerators throughout the world. Thus substantial regions of prime neutralino dark matter parameter space have already been eliminated. This does not yet mean that low-energy supersymmetry is unlikely to exist, since only a small portion of the allowed mass range under 1 TeV has been explored. Since supersymmetry predicts a Higgs boson with mass under about 120 GeV, such a discovery would be very important, especially if the Higgs boson showed non-standard properties indicative of supersymmetry. It is correct to think of the particle physics search for supersymmetry as a powerful search for the dark matter.

Direct detection of WIMPs

A satisfying solution to the dark matter problem would be the detection of WIMPs from our Galactic halo as they move past and through the Earth. This would also allow measurement of the local density of dark matter and establish beyond doubt that the dark matter is non-baryonic cold dark matter. There are several ways to do this, and currently two methods are being aggressively pursued.

The most exciting result would be direct detection of the WIMP particles in the laboratory. Since we roughly know the speed ($\sim 220 \text{ km s}^{-1}$) and the density ($\rho \sim 0.3$ proton masses cm^{-3}), we can say that for a WIMP of mass of order 10–100 GeV, roughly 100 000 dark matter particles per second pass through every square centimeter of the Earth. However, if WIMPs exist, they are very weakly interacting particles, so it is quite rare that one of them will interact at all; most of them pass right through the Earth unimpeded. In addition, if a WIMP does elastically scatter off a nucleus, the deposited energy is usually in the keV to 100 keV range, too small to be noticed

except by exquisitely sensitive equipment. These difficulties, however, have not stopped many groups throughout the world from developing devices capable of detecting WIMPs. The detection rates turn out to be within and just beyond the reach of current experimental efforts.

The basic idea is to detect the small energy deposited when a WIMP scatters off a nucleus in some well-instrumented piece of material. When a WIMP scatters off a nucleus in a crystal, the nucleus recoils, causing dislocation in the crystal structure, vibrations of the crystal lattice (i.e. phonons or heat) and also ionization. These signals can be detected. For example, some experiments use kilogram size germanium and silicon crystals and attempt to detect the ionization and phonon signals. Other groups use sodium iodide crystals and look for the scintillation light caused by the ionized electrons or search for crystal dislocations in samples of billion year old mica. Another possibility is to record the recoil of an ionized atom in a gas using drift chamber techniques.

The main difficulties in these experiments come from the fact that the WIMP events are rare and that there are many backgrounds that deposit similar amounts of energy on much more frequent time-scales. Thus the experiments operate deep underground, where ionizing COSMIC RAYS are less frequent, and typically operate their detectors at extremely cold temperatures to keep thermal excitations low. Also many types of shielding, as well as redundant detection methods, are now becoming standard. Even so, these are difficult experiments and tiny amounts of radioactivity in the detector or shielding can swamp the expected signal. With effort, a background rate of under one event per kilogram of detector per day can be achieved. The expected signal is highly dependent on the supersymmetry model, but typically is in the range from 10^{-5} – 10 events $\text{kg}^{-1} \text{day}^{-1}$. The events can be separated from the background in two ways. In some detectors the background (non-WIMP) interactions can be recognized and simply ignored. In the larger detectors this is not possible, so they use the fact that the WIMP event rate is predicted to be larger in June than in December. This annual modulation in event rate is caused by the Earth's orbit either being aligned with the Sun's motion in the Galaxy (in June) or anti-aligned (in December).

The current generation of detectors have detection thresholds of around 1 event $\text{kg}^{-1} \text{day}^{-1}$, with hopes that within the next few years signals as small as 10^2 events $\text{kg}^{-1} \text{day}^{-1}$ will be detectable. Thus there is a reasonable chance that dark matter neutralinos will be detected by this type of direct detection within the next few years. It is also clear, however, that there are many values of the supersymmetry parameters that predict detection rates of below the 10^{-2} events $\text{kg}^{-1} \text{day}^{-1}$ threshold, and so would not be detectable in the near future by these methods.

Indirect detection of WIMPs

A great deal of theoretical and experimental effort has gone into another potential technique for WIMP detection. The idea is that if the halo is made of WIMPs, then these WIMPs will have been passing through the Earth and Sun for several billion years. Since WIMPs will occasionally elastically scatter off nuclei in the Sun or Earth, they will occasionally lose enough energy, or change their direction of motion enough, to become gravitationally captured by the Sun or Earth. The orbits of such captured WIMPs will repeatedly intersect the Sun (or Earth) resulting in the eventual settling of the WIMPs into the core. As the number density increases over time, the self-annihilation rate will increase. Since ordinary neutrinos can result from WIMP self-annihilation, one predicts a stream of neutrinos coming from the core of the Sun or Earth. Neutrinos easily escape the Solar core and detectors on Earth capable of detecting neutrinos coming from Sun or Earth have operated for some time. The energy of such neutrinos is roughly $1/2$ to $1/3$ of the WIMP mass, so these neutrinos are much higher energy than the MeV scale solar neutrinos from nuclear reactions that have already been detected. The higher energy of these WIMP annihilation neutrinos makes them easier to detect than ordinary solar neutrinos and somewhat compensates for their much fewer numbers. It also makes them impossible to confuse with ordinary solar neutrinos. Thus the presence of a source of high-energy neutrinos emanating from the centers of the Sun and Earth would be taken as evidence for WIMP dark matter.

While the above chain of reasoning may seem long, it appears to be robust, and several experimental groups are in the process of designing and building detectors capable of seeing such a neutrino signal. For this signal, it is not the mass of the detector that is relevant, but the surface area. Neutrinos from the core of the Sun or Earth produce muons in the atmosphere and rock around the detectors, and it is primarily these muons that the detectors watch for. Muons are also copiously created by cosmic rays entering the Earth's atmosphere, so there is a substantial background of 'downward' traveling muons. These detectors, then, are located deep underground, where the rock shields many of the background muons, and they also focus on 'upward' traveling muons, that are much more likely to have been created by neutrinos that have traveled through Earth and interacted in the rock just below the detector. Thus surprisingly, the best way to see high-energy neutrinos from the Sun is to go deep underground at night (when the Sun is 'under' the Earth).

The new generation of detectors are designed to have very large surface areas. A comparison of direct and indirect detection methods indicates that for a typical neutralino a kilogram of direct detector germanium has about the same sensitivity as 10^4 – 10^6 m^2 of indirect

detector. The new generation of detectors should have areas in this range and should be able to start to probe realistic supersymmetry models, but again viable supersymmetric models exist which predict rates too small for these detectors to measure, and a definitive test of the WIMP hypothesis does not seem possible in the near future.

MACHOs and microlensing

An exciting development in the dark matter story is the detection of MACHOs by three separate groups, MACHO, EROS and OGLE. All three groups monitored millions of stars in the LARGE MAGELLANIC CLOUD (LMC), in the SMALL MAGELLANIC CLOUD (SMC) or in the GALACTIC BULGE, for signs of gravitational microlensing, and all three groups have found it. It has now become clear that these objects constitute some new component of the Milky Way, but whether they constitute a substantial part of the dark matter or new stellar components of the Galaxy or LMC is not clear.

MICROLENSING is a powerful new tool for discovering and characterizing populations of dark objects in our Galaxy, and the current experiments may have the capability to give a definitive answer to the question of whether the dark matter in our Galaxy is baryonic. The reason is that the microlensing searches are probably sensitive to any objects in the range $10^{-8}M_{\odot} < m < 10^3M_{\odot}$, just the range in which such objects are theoretically allowed to exist. Objects made purely of H and He with masses less than $\sim(10^{-9}-10^{-7}M_{\odot})$ are expected to evaporate owing to the microwave background in less than present age of the universe, while dark matter objects with masses greater than $\sim 10^3M_{\odot}$ would have disrupted known GLOBULAR CLUSTERS. So unlike the searches for WIMP dark matter, which if they detect nothing will remain inconclusive, if the MACHO searches find nothing, we would at least know what the dark matter is not. However, the MACHO searches have found something, as we describe below.

Microlensing is also described in the article on GRAVITATIONAL LENSING. The idea of microlensing rests on Einstein's observation that, if a massive object lies directly on the line of sight to a much more distant star, the light from the star will be lensed and form a ring around the lens. The ring is called the 'Einstein ring' and it sets the scale for all the microlensing searches. In the lens plane, the radius of the ring is given by

$$r_E = 610R_{\odot} \left(\frac{m}{M_{\odot}} \frac{L}{\text{kpc}} x(1-x) \right)^{1/2}$$

where R_{\odot} and M_{\odot} are the solar radius and mass, m is the MACHO mass, L is the distance to the star being monitored and x is the distance to the MACHO divided by L . The formation of a ring is very unlikely, but even with

imperfect alignment two images result and a large magnification can occur.

Since the MACHO, Earth and source star are all in relative motion, the star appears to brighten, reaches a peak brightness, and then fades back to its usual magnitude. Thus the signature for a microlensing event is a time-symmetric brightening of a star occurring as a MACHO passes close to the line of sight. When a microlensing event is detected, one fits the lightcurve and extracts the peak magnification A_{max} , the time of the peak, t_0 , and event duration \hat{t} . The primary physical information comes from \hat{t} , which depends on the MACHO velocity, the MACHO mass, the source distance, and the lens distance. The source distance can be determined since it is visible, but unfortunately, one cannot determine the other three physical parameters from \hat{t} . However, statistically, one can use information about the halo density and velocity distribution, along with the distribution of measured event durations to gain information about the MACHO masses. Using a standard model of the dark halo, MACHOs of Jupiter mass ($10^{-3}M_{\odot}$) typically cause events lasting 3 days, while brown dwarf mass MACHOs ($0.1M_{\odot}$) cause events lasting about a month.

Assuming a halo made entirely of MACHOs, the probability of any MACHO crossing in front of a star is about 5×10^{-7} . Thus many millions of stars must be monitored in order to see a handful of microlensing events. In addition, if one wants to see microlensing from objects in the dark halo, the monitored stars must be far enough away so that there is a lot of halo material between us and the stars. Therefore, the best stars to monitor are those in the LMC and SMC at distances of 50 kpc and 60 kpc respectively, stars in the galactic bulge at 8 kpc and stars in nearby galaxies such as M31 at 750 kpc.

Microlensing experiments

There are several experimental groups that have undertaken the search for microlensing and have returned results. All together about a dozen events have been detected towards the LMC, a couple towards the SMC, and more than 400 towards the galactic bulge. For detection of dark matter MACHOs, it is primarily the LMC events that are relevant. All survey collaborations operate a medium-size telescope and monitor millions of stars nightly. Since the fields are crowded, each CCD frame contains hundreds of thousands of stellar images, the brightness of each which must be determined by computerized photometry. These stellar brightnesses are arranged sequentially in a lightcurve, each of which is finally searched for microlensing-like bumps. All together many terabytes of data have been analyzed by the survey experiments.

Most of the monitored stars are constant brightness as one expects, but about one-half of 1% are variable.

These are mostly identified as variable stars of known types. Several signatures of microlensing, including the unique shape of a microlensing lightcurve are used to pick out microlensing events from this background of variable stars. For example, the MACHO collaboration analyzed about 9.5 million lightcurves from its 2 yr LMC data set and found six to eight microlensing events.

Experimental results

In order to interpret microlensing events, the efficiency with which an experiment can detect microlensing is needed. Bad weather, bad seeing, crowded stars, etc cause microlensing events to be missed by the experiment. The MACHO collaboration finds an efficiency of around 30% for microlensing events that last 50 days. For events lasting less than a few days and events lasting longer than 6 months the efficiency is very low.

Using the efficiency and a model of the dark halo, the number of microlensing events one expects to see if the halo consisted entirely of MACHOs is found to be about a dozen for the MACHO collaboration 2 yr data set. This quantity is related to the lensing 'optical depth', the roughly 5×10^{-7} probability that at any time a source star is lensed (if the halo is made entirely of MACHOs).

Using the eight observed MACHO collaboration LMC events, the observed optical depth is $\sim (2.5 \pm 1 \times 10^{-7})$, roughly half the value if the halo consisted entirely of MACHOs. A careful likelihood analysis of these events gives, for a standard dark halo model, a most likely MACHO halo fraction of 0.5 ± 0.3 and a most likely mass of the MACHOs of around $0.5 M_{\odot}$. This result depends on the assumption that the events are due to lenses in the Galactic halo and on the model of the galactic halo used.

Another analysis based on noticing that none of the eight detected events had durations of less than 20 days can rule out low-mass MACHOs. This is because $\dot{t} \propto m^{1/2}$, and no short-duration events have been observed toward the LMC. This analysis gives the strongest constraints to date on the baryonic content of the dark halo. The EROS and MACHO collaboration limits show that objects in the mass range from $10^{-7} M_{\odot}$ to $10^{-3} M_{\odot}$ cannot make up the entire dark halo. Objects in the range from $3.5 \times 10^{-7} M_{\odot}$ to $4.5 \times 10^{-5} M_{\odot}$ make up less than 10% of the dark halo. Thus we now know that the dark matter is not mostly objects of Earth mass, or Jupiter mass, or any combination thereof. The only compact baryonic dark matter candidates left are objects in the brown dwarf and higher mass range. This result is independent of any assumptions about the observed microlensing events, but does depend on the model of the dark halo.

In 2002, an international team of astronomers observed a dark matter object directly for the first time. Images and spectra of a MACHO microlens were taken by the NASA/ESA HUBBLE SPACE TELESCOPE (HST) and the European Southern Observatory's Very Large Telescope

(VLT). This result is a strong confirmation that gravitational microlensing is the cause of the events detected by teams searching for MACHO dark matter.

In addition, Christopher Kochanek of the Harvard-Smithsonian Centre for Astrophysics in Cambridge, Massachusetts and Neal Dalal of the University of California, San Diego have used radio telescopes and gravitational lensing to search for cold dark matter. They have studied seven galaxies, each magnified by four nearer ones. Because each lensing galaxy is in a slightly different position, the researchers got four different images of each of the seven distant galaxies. The four images should have been identical. But each is actually slightly different. The difference was enough to have been caused by the kind of clumps of dark matter around lensing galaxies that mathematical models predict.

Interpretation of results: dark matter or not?

The naive interpretation of the microlensing results is that between 20% and 80% of the dark matter in the Milky Way has been identified. However, the result that the mass of the objects is above the brown dwarf limit of $0.1 M_{\odot}$ is surprising. Main sequence stars with masses above $0.1 M_{\odot}$ would have been seen and therefore cannot be the dark matter. Several interpretations are possible.

First, perhaps the MACHOs are white dwarf stars or neutron stars. These are dark remnants of an earlier generation of stars, but it is problematic to have enough of these around to be the dark matter and not have detected the other byproducts of such an early stellar population. Second, perhaps the model of the Galactic halo used is incorrect, and the masses of the MACHOs are actually safely below the brown dwarf limit. However, most reasonable halo models investigated do not have this property. Next, perhaps MACHOs are primordial black holes, or other exotic objects not currently known. This is possible, but quite surprising. Most importantly, perhaps the microlensing events are not due to halo lenses, and therefore are not telling us about the dark matter. In a typical microlensing event, the distance of the MACHO is not determined, so it is not known where the lens population is located. The estimate of the amount of MACHO dark matter relies on an assumed distribution of lens material, and therefore on the model of the Galaxy and LMC.

It has been suggested that the lenses could be faint stars in the LMC itself, or in some small undiscovered dwarf galaxy between the Sun and the LMC. These possibilities are being vigorously pursued, but strong arguments have been given against both possibilities. Thus the outcome is very unclear at the moment.

If one could measure the distance to the MACHO, that would be enough to distinguish between the above possibilities. Thus a 'microlensing parallax' satellite has been proposed to measure these distances. Other ways to

measure the distance to lenses, and therefore determine whether or not they are part of the dark halo, include ground-based lensing parallax, binary lens caustic crossing, and the finite source star effect. Unfortunately, events where these effects can be measured are rare, so luck, or new larger surveys and better follow-up, is needed to resolve this issue of what the MACHOs are.

The next generation of microlensing surveys and follow-up efforts are underway or being planned and should determine some lens distances, as well as gather more events. Thus, while the question of baryonic dark matter remains open, the next few years should bring an answer.

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