

# The Physics of Cosmic Acceleration

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**Abstract** The discovery that the cosmic expansion is accelerating has been followed by an intense theoretical and experimental response in physics and astronomy. The discovery implies that our most basic notions about how gravity work are violated on cosmological distance scales. One simple fix is the introduction of a cosmological constant into the field equations for general relativity. However, the extremely small value of the cosmological constant, relative to theoretical expectations, has led theorists to explore a wide variety of alternative explanations that involve the introduction of an exotic negative-pressure fluid or a modification of general relativity. Here we briefly review the evidence for cosmic acceleration. We then survey some of the theoretical attempts to account for it, including the cosmological constant, quintessence and its variants, mass-varying neutrinos, and modifications of general relativity, such as scalar-tensor and  $f(R)$  theories and braneworld scenarios. We discuss experimental and observational tests that may allow us to distinguish between some of the theoretical ideas that have been put forward.

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## 1 Introduction

The cosmic-acceleration puzzle is among the most viscerally compelling problems in physics. Our deepest intuition about gravity—that all objects should be attracted to each other—just simply does not apply at cosmological distance scales. Rather than slowing, as Newtonian gravity predicts, the relative velocities of distant galaxies are increasing. The implication is that gravity behaves far differently than we had previously thought or that some mysterious fluid (“dark energy”) with exotic gravitational properties fills the Universe. Either way, there is new physics beyond the four fundamental forces described by the Standard Model and general relativity (GR). Cosmic acceleration thus motivates a considerable fraction of current physical-cosmology research, and it has become a major focus of particle- and string-theory efforts.

There had long been hints, stemming primarily from the disparity between the values  $\Omega_m \simeq 0.1 - 0.3$  of the nonrelativistic-mass density found by dynamical measurements and the theoretical preference for a flat Universe,  $\Omega_{\text{tot}} = 1$ , that there might be a cosmological constant. However, direct measurements with distant supernovae of a negative deceleration parameter provided the “shot heard ’round the world” [1, 2]. The case for an accelerated expansion was dramatically bolstered in 2000 with the CMB discovery of a flat Universe [3]. A combination of observations, based on galaxy surveys, the Lyman-alpha forest, baryon acoustic oscillations, but primarily the CMB, now provide constraints to cosmological parameters at a precision almost unimaginable a decade ago. The evidence for cosmic acceleration exists now at the  $\gtrsim 50\sigma$  level [4]. It can no longer be ignored.

The simplest solution involves no more than the addition of a cosmological constant  $\Lambda$  [with units of curvature, or  $(\text{length})^{-2}$ ] to Einstein’s equation. But the value required to explain cosmic acceleration is, in units where  $G = c = \hbar = 1$ , of order  $10^{-120}$ . This is not a problem in the classical world, but the quantum-theory expectation is that the cosmological constant should be of order unity, or possibly zero, should some symmetry or dynamical mechanism operate. The gravitational effects of a cosmological constant are equivalent to those of the virtual particles that continually pop in and out of existence in quantum field theory. Renormalization allows us to choose the zero point of this virtual-particle energy density, but doing so implies a cancellation of terms in the

fundamental Lagrangian to one part in  $10^{120}$ .

Dark-energy theories dodge this question. The effects of a cosmological constant in Einstein's equation can also be mimicked precisely by a homogeneous fluid of energy density  $\rho_\Lambda = \Lambda c^4/8\pi G$  and pressure  $p_\Lambda = -\rho_\Lambda$ . Dark-energy theories postulate that the vacuum itself does not gravitate (by virtue of some unspecified symmetry or dynamical mechanism), but that the Universe is filled with dark energy, an exotic negative-pressure fluid that provides the impetus for cosmic acceleration. Alternative-gravity theories investigated in this connection propose that an accelerated expansion may simply be the vacuum solution of the theory. The aim of the vast observational/experimental dark-energy effort is to determine the physics of cosmic acceleration.

While dark-energy and/or alternative-gravity theories do away with the need for a cosmological constant, those that have been developed so far require (as we will see below) the introduction of unusually tiny parameters and/or finely-tuned initial conditions. They also introduce a new question, the “coincidence problem;” i.e., why is the Universe transitioning from deceleration to acceleration so recently? None of the current models answer this question fully, although some (e.g., the tracker-field models discussed below) address it.

Theorists may debate the relative merits of various cosmic-acceleration theories—cosmological constant, dark energy, alternative gravity, anthropic arguments, etc.—but it is ultimately up to experiment to decide which is correct. The most telling empirical quantity in this regard is the (effective) dark-energy equation-of-state parameter  $w_Q \equiv p_Q/\rho_Q$ , where  $p_Q$  and  $\rho_Q$  are the dark-energy pressure and energy density, respectively. The parameter  $w_Q$  can be determined from the expansion history; i.e., by how the cosmic acceleration changes with time. If cosmic acceleration is due to a cosmological constant, then  $w_Q = -1$ , and the future expansion is de Sitter-like (i.e., exponentially expanding). In contrast, dark-energy and alternative-gravity theories predict  $w_Q \neq -1$ . Current constraints are  $w_Q \simeq -1 \pm 0.1$ . The precise value of  $w_Q$  (and its evolution with time) depends on the particular cosmic-acceleration theory. There is no consensus on how far from  $-1$  it should be, but some classification of theoretical predictions can be provided. Of course, cosmic-acceleration theories require new physics, and this new physics may also be probed experimentally in other ways, beyond just the expansion history.

This review is intended primarily to survey some of the theoretical explanations, involving both dark energy and alternative gravity, for cosmic acceleration, and secondarily to highlight the observational and experimental tests that may be pursued to test the theories. We begin with some background and a summary of the observational evidence for cosmic acceleration. We then review models that explain cosmic acceleration by the introduction of a new exotic fluid and those that work by modifying gravity. We close with a brief review of some of the observational/experimental ways forward. Refs. [5,6,7] complement this review by providing deeper analyses of observational approaches to dark energy, while others [8,9,10] provide more details about recent dynamical models for dark energy.

## 2 Background and Evidence

### 2.1 The Friedmann-Robertson-Walker cosmology

We begin by reviewing the essentials of the standard cosmological model. We refer the reader to Ch. 13 in Ref. [11] for more details.

**2.1.1 KINEMATICS** An isotropic and homogeneous expanding Universe with spatial coordinates  $x_i$  is described by the Robertson-Walker metric,<sup>1</sup>  $ds^2 = -dt^2 + a^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)]$ . The scale factor  $a(t)$  is a function of time  $t$ , where  $a(t_0) = a_0$  at the present time  $t_0$ . Cosmologists use the redshift  $z \equiv (a_0/a) - 1$  as a proxy for the age or scale factor. The redshift can be measured for distant sources; it is the fractional amount by which the wavelength of a photon has been stretched by the expansion between the time the photon is emitted and the time it is received.

The expansion rate  $H \equiv \dot{a}/a$  is a function of time, with the value  $H_0 \simeq 70 \text{ km sec}^{-1} \text{ Mpc}^{-1}$  (the Hubble constant) today, and where the dot denotes a derivative with respect to  $t$ . The deceleration parameter is then  $q \equiv -(\ddot{a}/a)/H^2 = (1+z)H'/H - 1$ , where the prime denotes a derivative with respect to  $z$ . The luminosity distance of an object of luminosity  $L$  at a redshift  $z$  is defined to be  $d_L \equiv (L/4\pi F)^{1/2}$ , where  $F$  is the energy flux received from that object. The luminosity distance is given (in a flat Universe) by

$$d_L(z) = (1+z)c \int_0^z \frac{dz'}{H(z')}. \quad (1)$$

Thus, measurement of the apparent brightness of sources of known luminosity (“standard candles”) at a variety of redshifts  $z$  can be used to determine or constrain the expansion history.

The quantity  $[H(z)(1+z)]^{-1}dz$  determines the time that evolves between redshifts  $z$  and  $z+dz$ ; it thus also determines the physical distance in this redshift interval and thus the physical volume in a given redshift interval and angular aperture. Likewise,  $a(t)$  [which can be derived from  $H(z)$ ] determines the angular sizes of “standard rods,” objects of fixed physical sizes. The angular-diameter distance is defined to be  $d_A(z) \equiv l_{\text{prop}}/\theta$ , where  $l_{\text{prop}}$  is the proper size of an object and  $\theta$  the angle it subtends on the sky. It is related to  $d_L$  through  $d_A(z) = (1+z)^{-2}d_L(z)$ . As discussed below, measurements of the volume and luminosity and angular-diameter distances can also be used to determine the expansion history.

The integral expression for  $d_L(z)$  can be Taylor expanded about  $z = 0$  to quadratic order as  $H_0 d_L(z) = cz[1 + (1/2)(1 - q_0)z + \dots]$ . The term linear in  $z$  is the well-known Hubble law. (Spatial curvature affects  $d_L(z)$  only at cubic or higher order [12].) In 1998, two groups independently used supernovae as standard candles to find better than  $3\sigma$  evidence for a negative value for  $q_0$  [1, 2], the implications of which will now be explained.

**2.1.2 DYNAMICS** The Friedmann equation,

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \sum_i \rho_i, \quad (2)$$

is the general-relativistic equation of motion for  $a(t)$  for a flat Universe filled with fluids  $i$  (e.g., nonrelativistic matter, radiation, dark energy) of energy densities  $\rho_i$ . If the fluids have pressures  $p_i$ ,

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<sup>1</sup>Given the observational evidence for negligible spatial curvature, we assume throughout a flat Universe. This simplifies considerably many of the equations. The effects of non-zero curvature are discussed in Ref. [12].

then the change  $d(\rho a^3)$  in the total energy ( $\rho = \sum_i \rho_i$ ) per comoving volume is equal to the work  $-pd(a^3)$  (where  $p = \sum_i p_i$ ) done by the fluid. This relation allows us to re-write the Friedmann equation as

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \sum_i (\rho_i + 3p_i). \quad (3)$$

A nonrelativistic source has pressure  $p = 0$  implying  $\ddot{a} < 0$ ; i.e., the relative velocities between any two galaxies should be decreasing, in agreement with our Newtonian intuition.

If we define equation-of-state parameters  $w_i \equiv p_i/\rho_i$  (e.g.,  $w_m = 0$  for matter,  $w_r = 1/3$  for radiation), then the second form, Eq. (3), of the Friedmann equation can be written  $q_0 = (1+3w_t)/2$ , where  $w_t \equiv p/\rho$  is the net equation-of-state parameter. Thus, if general relativity is correct, the observations require that the Universe has  $w_t < -1/3$ . Thus, some “dark energy,” a negative-pressure fluid, is postulated to account for cosmic acceleration.

**2.1.3 EXPANSION HISTORY** Although the original supernova measurements determined only  $q_0$ , future measurements will aim to determine the full functional dependence of  $d_L(z)$  [or equivalently,  $H(z)$ ] over the redshift range  $0 < z \lesssim \text{few}$  (the cubic correction to  $d_L(z)$  was first obtained observationally in 2004 [13]). If the Universe consists today of nonrelativistic matter (baryons and dark matter;  $w_m = 0$ ) with current energy density  $\Omega_m \equiv \rho_m/\rho_c$  (in units of the critical density  $\rho_c = 3H_0^2/8\pi G$ ) and some other exotic fluid with energy density  $\Omega_Q = 1 - \Omega_m$  and equation-of-state parameter  $w_Q$ , then  $H(z) = H_0[\Omega_m(1+z)^3 + (1 - \Omega_m)(1+z)^{3(1+w_Q)}]^{1/2}$ . In a flat Universe,  $\Omega_m + \Omega_Q = 1$ , the deceleration parameter is then  $q_0 = (1 + 3w_Q\Omega_Q)/2$ . The cosmological constant is equivalent to matter with  $w_Q = -1$ ; in this case,  $q_0 = (3/2)\Omega_m - 1$ .

Note, however, that there is no reason to expect  $w_Q$  to be constant (unless  $w_Q = -1$  precisely); it is just the simplest parametrization of a time-varying dark-energy density. In much of the current literature (including the Dark Energy Task Force Report [5]), the time evolution of  $w_Q$  is parametrized as  $w_Q = w_0 + w_a(1 - a/a_0)$ . Most generally, though,  $w_Q(z)$  may be an arbitrary function of  $z$ ; it is up to the dark-energy theory (which we have not yet specified) to predict. Fig. 1 shows the expansion history  $H(z)$ , luminosity distance  $d_L(z)$ , and deceleration parameter  $q(z)$  for four different models. The first three models are constant- $w_Q$  models with values of  $w_Q = -1.2$ ,  $-1$ , and  $-0.8$ . The fourth model is an alternative-gravity model (DGP gravity), described in Sec. 7.1 below, with variable  $w_Q$ . Measurement of  $w_Q(a)$  is the aim of observational efforts to probe the physics of cosmic acceleration.

**2.1.4 GROWTH OF STRUCTURE** So far we have assumed that the Universe is perfectly homogeneous, but this is only an approximation; the fractional density perturbation  $\delta_m(\vec{x}, t) \equiv [\rho_m(\vec{x}, t) - \bar{\rho}_m]/\bar{\rho}_m$ , where  $\bar{\rho}_m$  is the mean density, is not zero. At sufficiently early times, or when smoothed on sufficiently large scales, the fractional density perturbation is  $\delta_m \ll 1$ . In this linear regime, the density perturbation satisfies an evolution equation,

$$\ddot{\delta}_m + 2H\dot{\delta}_m - (3/2)\Omega_m H^2 \delta_m = 0. \quad (4)$$

This equation has a growing-mode solution (as a function of  $z$ )  $\delta_m(z) \propto D(z)$ , and this evolution can be determined with large-scale-structure measurements. In the standard cosmological model (i.e.,  $w_Q = -1$ ), the linear-theory growth factor  $D(z) \propto H(z)(5\Omega_m/2) \int_z^\infty (1+z)[H(z)]^{-3} dz$ . This expression is however invalid if  $w_Q \neq -1$  [14], and  $D(z)$  will in general differ for different  $w_Q$ .

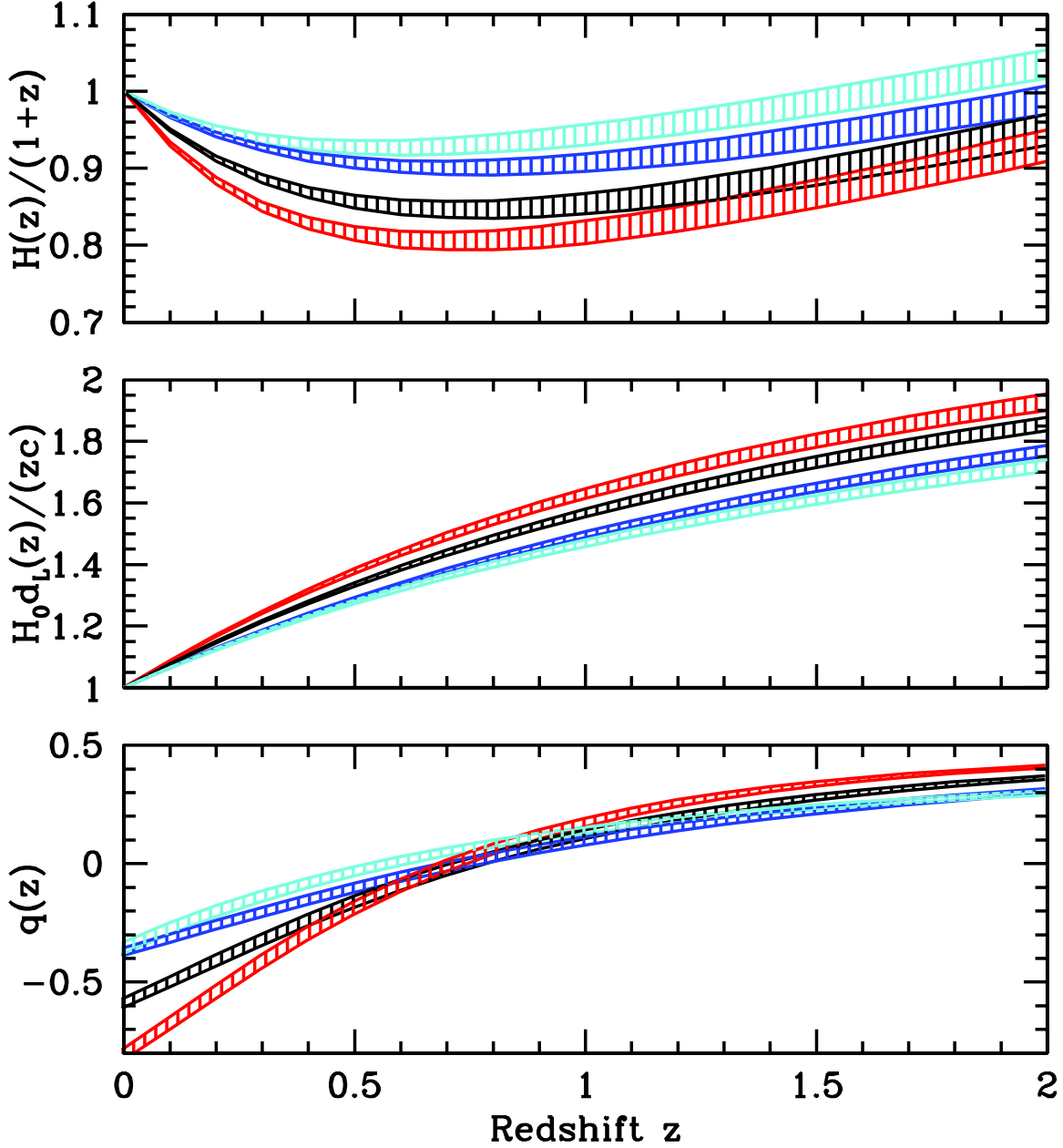


Figure 1: Examples of the expansion history  $H(z)$ , luminosity distance  $d_L(z)$ , and deceleration  $q(z)$  are shown for several different dark-energy models. The red, black, and blue curves correspond to dark-energy models with equation-of-state parameter  $w_Q = -1.2, -1, -0.8$ , respectively. The cyan curve is for a DGP alternative-gravity model (see Sec. 7.1). All models have the same matter density and assume spatial flatness. The thickness of the curves indicates the uncertainties that arise from the current uncertainty in the nonrelativistic-matter density  $\Omega_m$ .

Moreover, Eq. (4) is derived assuming that the dark energy remains perfectly homogeneous. If dark energy clusters, there may be a source for this equation (i.e., the right-hand side may be nonzero), in which case  $D(z)$  may be further affected. Alternative theories of gravity invoked to

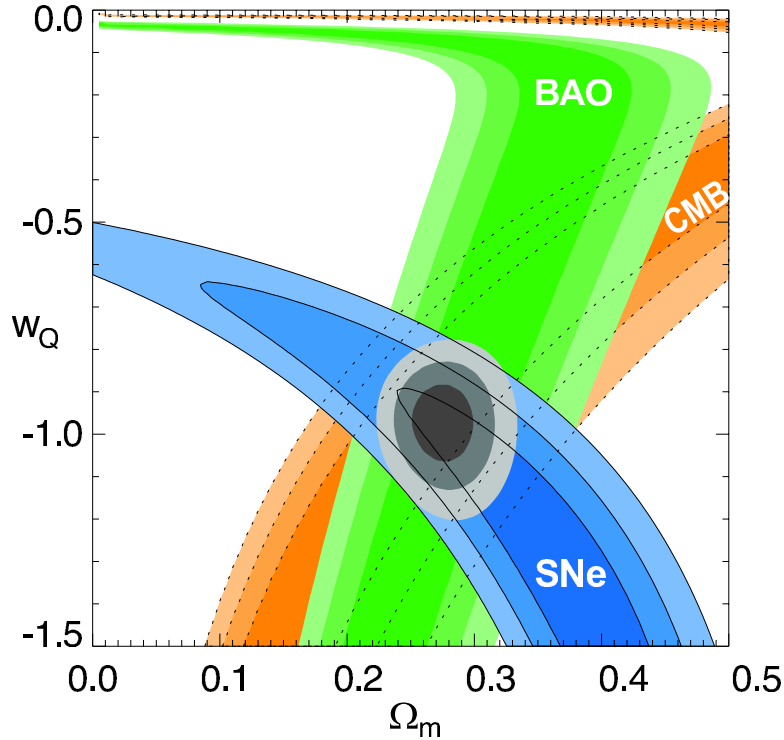


Figure 2: The 68.3%, 95.4%, and 99.7% confidence-level contours for  $w_Q$  and  $\Omega_m$  are shown, assuming a flat universe. The individual constraints from large-scale structure (using baryon acoustic oscillations), the cosmic microwave background, and the Union SN data set are shown, as well as the combined constraints. (From Ref. [15].)

explain cosmic acceleration may predict a different  $D(z)$ , even for the same expansion history.

## 2.2 The Evidence

Evidence for accelerated expansion comes from the aforementioned direct measurements of  $d_L(z)$  using Type Ia supernovae, which now suggest  $q_0 \simeq -0.7 \pm 0.1$  ( $1\sigma$  errors) [15]. However, the case for accelerated expansion is dramatically bolstered by other observations. Chief among these is the CMB measurement of a flat Universe [3], obtained by locating the first acoustic peak in the CMB power spectrum [16]; this implies a total density  $\Omega_m + \Omega_Q \simeq 1$  much greater than the matter density  $\Omega_m \simeq 0.3$  indicated by dynamical measurements. Current CMB measurements alone are now sufficiently precise that they can determine a dark-energy density  $\Omega_Q = 0.742 \pm 0.030$  (for  $w_Q = -1$  and a flat Universe) [4], a measurement that is made more precise with the addition of data from large-scale structure, the Lyman-alpha forest, baryon acoustic oscillations, the cluster abundance, and supernovae. In particular, supernova measurements provide a constraint (again, assuming  $w_Q = -1$ )  $q_0 = (\Omega_m/2) - \Omega_Q \simeq -0.7$  that is nearly orthogonal to the CMB contour  $\Omega_m + \Omega_Q \simeq 1$ , and so CMB and supernovae together provide tight limits in the  $\Omega_m$ - $\Omega_Q$  plane. The consistency of a spatially-flat Universe with dark matter and a cosmological constant with a wealth of precise data has led to the adoption of a “concordance model,” our current standard cosmological model. Current values for the parameters of this model are provided in Refs. [4, 17].



The concordance-model assumption  $w_Q = -1$  can be tested quantitatively with the data. If the Universe is flat, then the deceleration parameter is  $q_0 = (1/2)(1 + 3w_Q\Omega_Q)$ . If  $q_0 < 0$ , then  $w_Q < -(1/3)(1 - \Omega_m)^{-1}$ , or  $w_Q \lesssim -0.5$  for  $\Omega_m \lesssim 0.3$ . The observed value  $q_0 \simeq -0.7$  requires an even more negative pressure, with  $w_Q \simeq -1$ . The current constraints to the  $\Omega_m$ - $w_Q$  parameter space, assuming a constant  $w_Q$  and flat Universe, are shown in Figure 2. Very little is reliably known about the behavior of dark energy at  $z \gtrsim 1$ , except that it does not appear to have played any significant role in cosmic evolution at earlier times. Other probes of the expansion history will be discussed in Sec. 9.

### 3 Quintessence

If the history of particle physics is any guide, then it seems reasonable to speculate that the dark energy is due to a new field. For cosmology, the simplest field that can provide the missing energy between the matter density and the critical density and drive cosmic acceleration is a scalar field. Such a field in this role is sometimes referred to as “quintessence” to help distinguish it from other fields or other forms of dark energy [18].

#### 3.1 Basic Equations

**3.1.1 BACKGROUND EVOLUTION** The formal description of quintessence begins with the action,

$$S = \int d^4x \sqrt{-g} \left( \frac{R}{16\pi G} + \mathcal{L}_{SM} + \mathcal{L}_Q \right), \quad (5)$$

where  $R$  is the Ricci scalar,  $g$  the determinant of the metric. Here, the quintessence Lagrangian is  $\mathcal{L}_Q = -\frac{1}{2}(\nabla_\mu Q)(\nabla^\mu Q) - V(Q)$ , and  $\mathcal{L}_{SM}$  is the Lagrangian for Standard Model particles. The field obeys the Klein-Gordon equation,  $\square Q = V_{,Q}$ , where  $\square$  is the d’Alembertian and  $V_{,Q} \equiv \partial V / \partial Q$ , and carries stress-energy  $T_{\mu\nu} = (\nabla_\mu Q)(\nabla_\nu Q) + g_{\mu\nu} \mathcal{L}_Q$ . (We use metric signature  $(-+++)$  and adopt the curvature conventions of Ref. [19].)

The spatially-homogeneous cosmic scalar is guided by the equation of motion,  $\ddot{Q} + 3H\dot{Q} + V_{,Q} = 0$ , with energy density and pressure,

$$\rho_Q = \frac{1}{2}\dot{Q}^2 + V(Q), \quad p_Q = \frac{1}{2}\dot{Q}^2 - V(Q). \quad (6)$$

An equation-of-state parameter  $w_Q < -1/3$  is obtained when  $\dot{Q}^2 < V$ . The mechanism for obtaining  $\dot{Q}^2 \ll V$  is similar to the slow-roll mechanism in inflation (although not precisely the same given that a fraction  $\Omega_m \simeq 0.25$  of the current cosmological density is nonrelativistic matter). We illustrate here with the simple example of a potential  $V(Q) = (1/2)m^2Q^2$ . In the absence of the Hubble-friction term ( $3H\dot{Q}$ ) in the scalar-field equation of motion, the field will simply oscillate in this quadratic potential. However, if  $m \ll H$ , then the Hubble friction overdamps the oscillator. In this case,  $\ddot{Q} \ll H\dot{Q}$ ,  $V_{,Q}$ , and  $3H\dot{Q} \simeq -m^2Q$ . The field then moves little over a Hubble time, and  $\dot{Q}^2 \ll V$  is achieved. More generally, quintessence potentials are required to be very flat (have effective masses  $m_Q \equiv \sqrt{V_{,QQ}} \ll H$ ) to work.



**3.1.2 EXPANSION HISTORY AND THE QUINTESSENCE POTENTIAL** More generally, a given quintessence potential determines the expansion history and *vice versa*. For example, if quintessence has an equation-of-state parameter  $w(a)$  as a function of scale factor  $a$ , then the energy density can be reconstructed as,

$$\rho_Q(a) = \Omega_Q \rho_c \exp \left( 3 \int_a^{a_0} [1 + w(a)] d \ln a \right). \quad (7)$$

The potential and field evolution for this equation-of-state parameter can then be reconstructed from,

$$\begin{aligned} V(a) &= \frac{1}{2} [1 - w(a)] \rho(a), \\ Q(a) &= \int d\tilde{a} \frac{\sqrt{1 + w(\tilde{a})}}{\tilde{a} H(\tilde{a})} \sqrt{\rho(\tilde{a})}. \end{aligned} \quad (8)$$

The equivalence  $w(a) \leftrightarrow V(Q[a])$  is valid provided  $\dot{Q} \neq 0$ . For most quintessence models, in which the field evolves monotonically down a potential, this condition is satisfied.

**3.1.3 QUINTESSENCE PERTURBATIONS** If the quintessence field can vary in time, then it can most generally vary in space. Linearized spatial fluctuations  $\delta Q$  of the quintessence field follow the evolution equation,

$$\delta \ddot{Q} + 3H\delta \dot{Q} + \left( V_{,QQ} - \frac{1}{a^2} \nabla^2 \right) \delta Q = \dot{\delta}_m \dot{Q}, \quad (9)$$

where  $\nabla^2$  is the spatial Laplace operator in comoving coordinates, and  $\delta_m$  is the nonrelativistic-matter perturbation. Quintessence therefore responds to inhomogeneities in dark matter and baryons. Furthermore, the source term depends on  $\dot{Q}$ , so that the closer  $w_Q$  is to  $-1$ , the weaker the driving term. The nature of the response is determined by  $m_Q$  or the quintessence Compton wavelength  $\lambda_Q = m_Q^{-1}$ . In the case of constant  $w_Q$ , there is a simplification which can be written,

$$V_{,QQ} = -\frac{3}{2}(1 - w_Q) \left[ \dot{H} - \frac{3}{2}(1 + w_Q)H^2 \right]. \quad (10)$$

For a slowly varying equation-of-state parameter,  $V_{,QQ} \propto H^2$  and  $\lambda_Q \sim H^{-1}$ . From the above equations, this means that fluctuations on scales smaller than the Hubble scale dissipate with sound speed equal to the speed of light, since the coefficient of  $\nabla^2/a^2$  in Eq. (9) is unity, whence the field remains a smooth, non-clustering component. Any initial fluctuations in the quintessence field are damped out rapidly [20]. In principle, perturbations to the quintessence field serve as a source for matter perturbations—i.e., they show up as a non-zero right-hand side to Eq. (4)—and thus affects the linear-theory growth factor  $D(z)$ . However, the damping of small-scale quintessence perturbations implies that this is generically a small effect. On scales  $\gtrsim H^{-1}$ , the field is gravitationally unstable. The growth of these long-wavelength perturbations to quintessence may leave an imprint on the large-angle CMB-anisotropy pattern, which we discuss in Sec. 3.4.

## 3.2 Representative Models

Embedding scalar-field dark energy in a realistic extension of the Standard Model faces a number of challenges. A viable scenario generically requires an ultra-light scalar ( $m_Q \lesssim H \sim 10^{-42}$  GeV), with

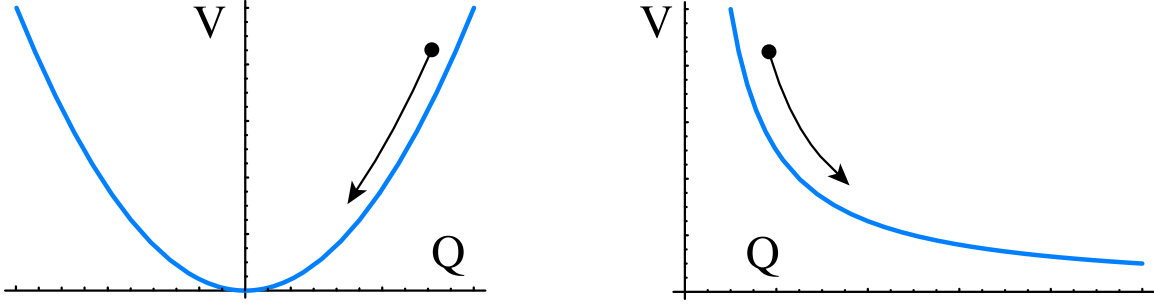


Figure 3: Two examples of potentials for the quintessence field. In the first, representative of a conventional massive scalar or PNGB, the field is relaxing towards the local minimum. In the second, representative of vacuumless potentials such as the tracker, the field is evolving towards the global minimum.

Planckian amplitude ( $Q \sim 10^{19}$  GeV) and which remains non-interacting with the Standard Model, and therefore “dark” [21, 22, 23, 24]. A second challenge comes from the coincidence problem—why is dark energy becoming dominant today? Ideally, the theory would have order-unity parameters at, say, the Planck scale, and the dark-energy density today would be insensitive to the field’s initial conditions. However, in existing models, the parameters of the potential are specially chosen so that quintessence provides acceleration today. Moreover, the scalar field generically requires finely tuned initial conditions if the field is to achieve the desired dynamics. Despite these difficulties, many models of quintessence have been proposed. Here we focus on just a few representative models.

**3.2.1 COSMIC AXION** A cosmic axion or pseudo-Nambu Goldstone boson (PNGB) is one way to have a scalar of extremely low mass and keep it dark. The first such models [25] considered a PNGB associated with a unification scale  $f$  mediating a breakdown of a global symmetry in a family of neutrinos at a scale  $\mu \sim m_\nu^2/f$ , thereby helping to explain the very light mass of the quintessence field. Models have been proposed employing string or M-theory moduli fields [26, 27], too. The resulting scalar potential,  $V = \mu^4(1 + \cos Q/f)$ , is stable against loop corrections (although not necessarily quantum-gravity effects [28, 29]), thereby protecting the mass  $\mu$ . The shift symmetry,  $Q \rightarrow Q + 2\pi f$ , disables couplings to Standard Model fields that would otherwise spoil the “darkness”. A viable scenario requires  $\mu \simeq 0.002$  eV and  $f \sim 10^{18}$  GeV [30, 31, 32]. The cosmic evolution of the field is as follows: the field has been frozen by Hubble friction through most of cosmic history; as the Hubble friction relaxes, the field begins to slowly relax towards its ground state, as illustrated in Fig. 3; in the future, the field will oscillate at the bottom of the potential, with its energy redshifting away like nonrelativistic matter. That the mass scales  $f$  and  $\mu$  are derived from the energy scale of other physics, alleviates some need to explain the coincidence problem. The initial position of  $Q$  on the potential directly determines the present-day properties of dark energy. However, the fine-tuning problem is eased because the PNGB potential is periodic; the range of starting values of  $Q$  that produces a viable scenario is a non-negligible portion of the allowed range  $Q \in [0, 2\pi f]$ .

**3.2.2 TRACKER FIELDS** Condensation of hidden-sector quark–anti-quark pairs in a supersymmetric version of QCD has been shown to give rise to a pion-like scalar field with an effective

potential  $V = M^4(Q/M_P)^{-n}$  with  $n > 0$  [33, 34]. The index  $n$  is determined by the number of fermion families and colors, and  $M$  is set by the cutoff scale. The cosmological dynamics of such a scalar are quite novel [35, 36, 37, 38]: for a broad range of initial conditions, the evolution of the field approaches, and then locks onto, a universal track with negative equation-of-state parameter such that it inevitably dominates at late times. When the scalar-field energy density is subdominant, its equation-of-state parameter is  $w_Q \approx (nw_B - 2)/(n + 2)$  where  $w_B$  is the equation-of-state parameter of the dominant or background component. Hence, the field has been rolling down the potential for most of its history, as illustrated in Fig. 3, but is now beginning to slow. As the scalar field comes to dominate, its equation-of-state parameter grows more negative, asymptoting to  $w_Q \rightarrow -1$  in the future. The universal track is uniquely determined by the mass  $M$  and index  $n$ , so that there is a one-to-one relationship between  $\Omega_Q$  and  $w_Q$  as a function of time. A viable model requires  $0 < n < 1$  and  $M \simeq 0.002$  eV. The broad insensitivity of the late-time behavior to the initial conditions is appealing—this model solves the above-mentioned fine-tuning problem. A further feature of the model is that it goes some way toward addressing the coincidence problem by allowing the dark-energy density to track the matter/radiation density over long periods of cosmological history. Still, there is no explanation as to why the acceleration is happening now, as opposed to some later time.

**3.2.3 EXPONENTIAL POTENTIAL** A scalar field with an exponential potential,  $V = M^4 e^{-\lambda Q/m_{\text{Pl}}}$ , arises in a wide-variety of extensions of Standard Model physics. In one particular case, the scalar field is the dilaton, a pion-like condensate of supersymmetric gaugino particles [39, 34]. The dynamics of this model are as follows: for  $\lambda^2 > 3(1 + w_B)$  the scalar-field energy density tracks the background fluid with  $w_Q = w_B$ ; for  $\lambda^2 < 2$  there are accelerating solutions [40]. However, the scaling solutions do not satisfactorily convert into dark energy at late times; a viable model requires  $\lambda^2 < 2$  and finely tuned initial conditions for  $Q$  and  $\dot{Q}$ . Phenomenological variations on this model have been explored, for example, in Ref. [41]. These models feature a local minimum in the exponentially decaying potential, where the field can relax and produce potential-dominated accelerating expansion.

**3.2.4 SPINTESSANCE** A scalar field with internal degrees of freedom has been considered as a dark-energy candidate; one particular example is “spintessence” [42], a complex field  $Q = Re^{i\Theta}$  spinning in a  $U(1)$ -symmetric potential  $V = V(R)$ . If the spin frequency is high enough,  $\dot{\Theta} \gg H$ , then it is rotation, rather than Hubble friction, that prevents the field from rolling immediately to its minimum. The equation-of-state parameter is  $w \approx (RV' - V)/(RV' + V)$ . Hence, a potential with shape  $RV' < V/2$  can provide  $w_Q < -1/3$ . However, the field is generically unstable to the formation of Q-balls (non-topological solitons) [42, 43], rendering this solution to the dark-energy problem unworkable. A gas of cold particles with an attractive interaction can also yield negative pressure [44], but in the relativistic regime required for cosmic acceleration, the theory winds up looking [45] like spintessence. A related idea is “oscillessence,” a single real scalar field that oscillates in a confining potential  $V(Q) \propto |Q|^n$ . This acts like a fluid with  $w_Q = (n - 2)/(n + 2)$  [46] and thus gives  $w_Q < -1/3$  for  $n < 1$ . Again, though, this model is unstable to small-scale perturbations [47].

**3.2.5 K-ESSENCE** Dark-energy models with scalar degrees of freedom with non-canonical kinetic terms in the Lagrangian display novel dynamics. K-essence defines a class of models with a

Lagrangian  $\mathcal{L}(Q, X)$  built from nonlinear functions of  $Q$  and  $X \equiv -(1/2)(\nabla_\mu Q)(\nabla^\mu Q)$ . The resulting stress-energy tensor is  $T_{\mu\nu} = \mathcal{L}_{,X}(\nabla_\mu Q)(\nabla_\nu Q) + \mathcal{L}g_{\mu\nu}$ , so that the cosmic pressure is simply  $p = \mathcal{L}$  and the energy density is  $\rho = 2Xp_{,X} - p$ . The motivation for these models is largely phenomenological, although in string-inspired models the scalar is identified with the dilaton or other moduli fields [48]. Purely kinetic k-essence, with  $\mathcal{L} = \mathcal{L}(X)$ , behaves as a barotropic fluid [49]. A k-essence counterpart to the potential-dominated tracker has  $\mathcal{L} = f(\phi)(-X + X^2)$  with  $f \propto \phi^{-n}$ . For  $0 < n < 2$ , k-essence evolves with a constant equation-of-state parameter  $w_Q = -1 + (n/2)(1 + w_B)$  until it comes to dominate the Universe, whereupon  $w_Q \rightarrow -1$  [50]. Models with multiple attractor solutions, such that the field scales with equation-of-state parameter  $w_Q = 1/3$  during the radiation era, but then runs off to a de Sitter-like solution after the onset of matter domination, have been proposed as possible solutions of the coincidence problem [51]. However, there is another aspect of k-essence that must be considered: the sound speed for the propagation of high-frequency perturbations is  $v^2 = p_{,X}/\rho_{,X}$ . The canonical scalar field has  $v^2 = 1$ . The k-essence models that predict  $v^2 < 0$  are eliminated because they are unstable to the growth of fluctuations. Density fluctuations in models with  $0 < v^2 \ll 1$  can leave a strong imprint on the CMB and large-scale structure. The apparent violation of causality in models with  $v^2 > 1$ , including models that pass from scaling in the radiation era to a present-day accelerating solution [52], suggests that additional analysis is required to understand the phenomenology of these models [53].

**3.2.6 GHOST CONDENSATE** Dark-energy scalar-field theories built from higher-order derivatives have also been studied. As an extension of k-essence, these models are motivated by string field theory or braneworld scenarios, too, and typically consist of a Lagrangian that is a nonlinear function of  $X$ ,  $\square Q$ ,  $(\nabla_\mu \nabla_\nu Q)(\nabla^\mu \nabla^\nu Q)$ , etc. In certain cases, these higher-derivative terms can stabilize theories with a leading-order kinetic term of the wrong sign (hence, a ghost). One such case, a ghost condensate [54], has equation-of-state parameter  $w_Q = -1$  but carries fluctuations with a nonlinear dispersion relation  $\omega^2 \propto k^4$ . This fluid contributes to the overall inhomogeneous density field, yet the higher-derivative terms mean that its fluctuations are sourced by higher derivatives of the local gravitational fields. Generally, the additional dynamics resulting from the higher-derivative terms allow novel behavior, such as  $w_Q \leq -1$  with a stable, but vanishing sound speed,  $v^2 \rightarrow 0$  [55]. Stable, non-relativistic fluctuations contribute like a new species of dark-matter inhomogeneities.

### 3.3 Thawing and Freezing Models

The equation-of-state parameter for dynamical dark energy is unlikely to be a constant. Using the cosmic axion and the tracker field as guides, we may identify two classes of quintessence models, thawing and freezing. Thawing models have a potential with a  $V = 0$  minimum accessible within a finite range of  $Q$ ; the field starts high up the potential, frozen by Hubble friction, with equation-of-state parameter  $w_Q = -1$ ; as the Hubble constant decays, the field begins to thaw and roll down towards  $w_Q = 0$ . Freezing models are said to be vacuumless, as the minimum is not accessible within finite range of  $Q$ , although there are no barriers; the field rolls down the potential, but decelerates so that the equation-of-state parameter evolves towards  $w_Q \rightarrow -1$ . This ignores models with nonzero local minima of the potential, but these are equivalent to a cosmological constant with massive-scalar-field excitations. The trajectories of thawing and freezing models occupy rather well-defined

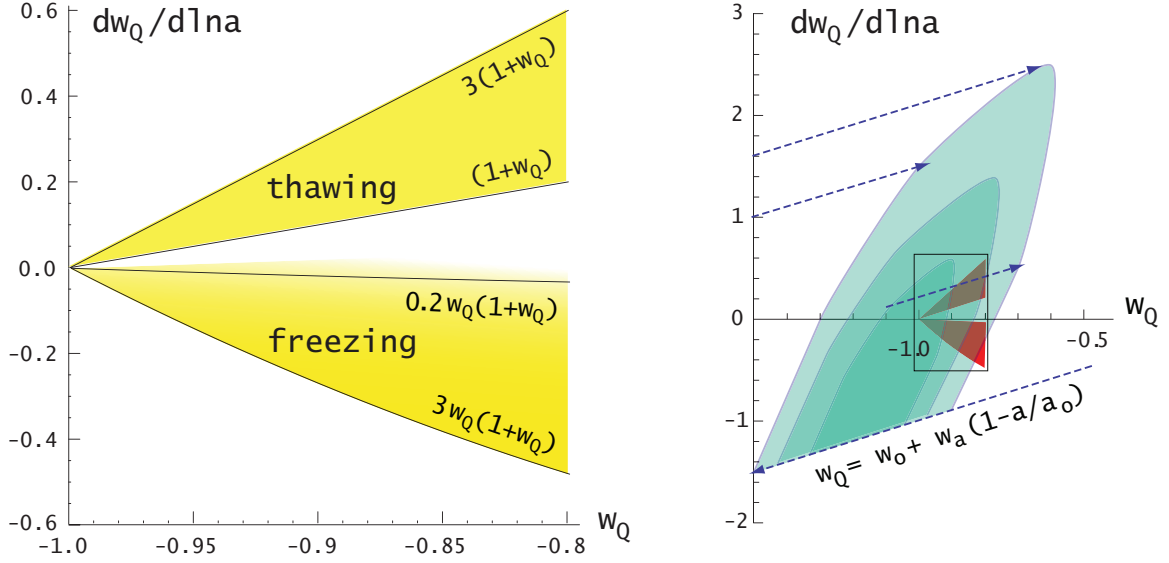


Figure 4: The  $w_Q$  vs.  $dw_Q/d\ln a$  parameter plane of dynamical dark-energy models is shown. On the left, the likely range of thawing and freezing models is shown. On the right, the current 68.3%, 95.4%, and 99.7% confidence-level constraints on the dark energy parameterization  $w_Q = w_0 + w_a(1 - a/a_0)$  [15] have been converted into the present-day values of  $w_Q$ ,  $dw_Q/d\ln a$ . The dashed lines show the direction of evolution of models located at particular points on the 99.7% confidence-level boundary.

regions of the  $w_Q$  vs.  $dw_Q/d\ln a$  parameter plane [56], which are illustrated in Fig. 4. (Plenty of models lie outside these regions [57, 58], although these tend to have metastable minima; e.g. a cosmological constant, or non-canonical kinetic terms.) These regions can be used as a guide for assessing the sensitivity of methods to test for dynamical dark energy.

### 3.4 Observables of the Models

The dark-energy observables are the energy density  $\Omega_Q$ , equation-of-state parameters  $w_0$  and  $w_a$ , and the growth factor  $D(z)$ , which is determined by the fluctuation sound speed  $v$ . The models described in this Section predict some time evolution  $a(t)$ , and thus  $w_0$  and  $w_a$ , and fluctuations that propagate at a sound speed  $v = 1$  for quintessence, or more generally  $v \geq 0$  for k-essence. The dark-energy density and equation-of-state parameter affect the expansion history  $H(z)$ . The growth rate of dark-matter and baryonic perturbations, as well as the gravitational potentials sampled by CMB photons, are sensitive to the expansion history. Fluctuations in the dark energy, dependent upon the dark-energy density, equation-of-state parameter, and sound speed, can leave an imprint on large-scale structure and the CMB. The impact of these phenomena on the CMB power spectrum are illustrated in Fig. 5. A further discussion of observational approaches will be given in Sec. 9.

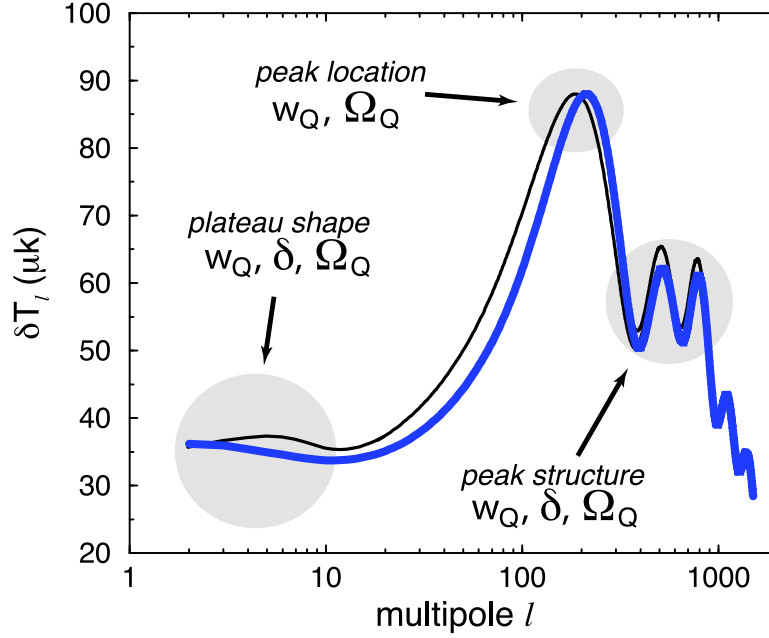


Figure 5: The effects of dynamical dark energy on the CMB temperature power spectrum are broadly illustrated. (1) Through the expansion history, the dark-energy density, noted in the figure by  $\Omega_Q$  and  $w_Q$ , influence the angular-diameter distance to last scattering, which sets the acoustic-peak multipole location. (2) Through the expansion history, dark energy influences the rate of growth of perturbations, affecting the CMB anisotropies created at late times on large angular scales. Fluctuations in the dark energy, noted by  $\delta$ , also contribute to the anisotropy pattern. (3) Dark energy can also influence the acoustic-peak structure if it has a non-negligible abundance at recombination [59].

#### 4 Mass-Varying Neutrinos

The coincidence between the mass scale  $m_\Lambda \equiv \Lambda^{1/4} \sim 10^{-3}$  eV of the cosmological constant and that of neutrino masses motivates a solution that connects cosmic acceleration to neutrino physics. This idea was pursued in Refs. [60, 61] in the idea of mass-varying neutrinos (MaVaNs). Like quintessence, the theory introduces a slowly-varying scalar field, dubbed the *acceleron*, whose value determines the neutrino mass  $m_\nu$ . The increased energy density associated with larger  $m_\nu$  affects the acceleron dynamics in such a way that the slow variation of the dark-energy density can be achieved *without* an extremely flat scalar-field potential.

More quantitatively, the energy density of the neutrino–dark-energy fluid is  $\rho_{\text{dark}} = m_\nu n_\nu + \rho_a(m_\nu)$ , where  $n_\nu$  is the neutrino number density and  $\rho_a$  the acceleron density, and the neutrinos are assumed (here, for simplicity) to be nonrelativistic. The fields of the theory are designed so that the acceleron relaxes to the value that minimizes  $\rho_{\text{dark}}$ , and the field value is thus fixed by the condition  $(\partial\rho_{\text{dark}}/\partial m_\nu) = n_\nu + (\partial\rho_a/\partial m_\nu) = 0$ . Combining this with the energy-conservation equation,  $\dot{\rho}_{\text{dark}} = -3H(\rho_{\text{dark}} + p_{\text{dark}})$ , one finds that the dark-sector equation-of-state parameter is

$$w \equiv \frac{p_{\text{dark}}}{\rho_{\text{dark}}} = -1 + \frac{m_\nu n_\nu}{m_\nu n_\nu + \rho_a}, \quad (11)$$



which gives  $w \simeq -1$ .

Specific implementations of the theory may have testable consequences for neutrino-oscillation experiments [62]. Unfortunately, however, the MaVaN idea suffers from a generic instability [63,64] to the growth of perturbations that renders it unsuitable for explaining cosmic acceleration. The dark-energy density at any given point is determined exclusively by the neutrino number density. The gradient-energy density in this model is too small to prevent the growth of spatial fluctuations. Thus, the sound speed is  $c_s^2 = w < 0$ , giving rise to a dynamical instability to the rapid growth of perturbations to the MaVaN energy density. A similar instability arises generically in other models that similarly attempt to couple dark matter and dark energy [65,66].

## 5 Phantom Energy

The simplest dark-energy models (single-field models with canonical kinetic terms) have  $w_Q \geq -1$ . However, current data are consistent with  $w_Q < -1$ ; for example, a recent analysis finds  $-1.14 < w_Q < -0.88$  (95% CL) [17]. It is thus interesting to ask, what if the dark energy is phantom energy [67]? i.e., what if it has an equation-of-state parameter  $w_Q < -1$ ? Generally, dark energy with  $w_Q < -1$  fits to observational data with a slightly lower energy density  $\Omega_Q$  than dark energy with  $w_Q > -1$ . There are important differences, though, stemming from the fact that  $w_Q < -1$  implies a violation of the null energy condition. This means the energy density grows, rather than decays, with time. In a phantom-dominated Universe, the scale factor and expansion rate diverge in finite time, ripping apart everything—galaxies, stars, atoms—before the Universe terminates in a “big rip” singularity [68,69]. For example, assuming the equation-of-state parameter maintains the constant value  $w_Q = -1.1$ , the end would arrive in  $\sim 100$  Gyrs. Spacetime diagrams classifying the fate of the Universe with different types of dark energy were presented in Ref. [70].

Theoretical models of phantom dark energy require exotic physics, such as a scalar field with negative kinetic energy or higher-derivative terms [67,71]. A quantum field with negative kinetic energy is unstable; even if “dark,” then gravitational interactions, unless cut off at a sufficiently low energy scale [71,72], can catalyze a catastrophe. Curved-space quantum-field-theory models of phantom energy are discussed in Refs. [73,74]. Quantum effects may strengthen a big rip or a sudden singularity (a singularity in which the scale factor remains finite but its derivatives diverge) [75] when the spacetime-curvature radius shrinks to Planckian radius [76,77]. There are other mechanisms that could masquerade as phantom energy, such as novel photon [78] or dark-matter interactions [79], as well as new gravitational phenomena [73]. Whereas a canonical or k-essence scalar cannot cross the  $w_Q = -1$  barrier [80,81,82], such evolution may be achieved in the presence of higher-derivative terms [83].

## 6 Scalar-Tensor and f(R) Theories

An alternative approach to cosmic acceleration is to change gravity. With quintessence, we assume that general relativity is correct, but that the Universe contains some exotic new substance that drives cosmic acceleration; i.e., the left-hand side ( $G_{\mu\nu}$ ) of Einstein’s equation remains unaltered, but we introduce a new source  $T_{\mu\nu}$  for the right-hand side. Here we alter general relativity; i.e., replace the left-hand side of Einstein’s equation, or change gravity even further.



## 6.1 Scalar-tensor theories

We begin by reviewing scalar-tensor theories, perhaps the most widely studied class of alternative-gravity theories. A wide array of experimental tests of such theories have been investigated in detail [19]. Scalar-tensor theories appear as low-energy limits of string theories, and other alternative-gravity theories, such as  $f(R)$  theories (to be discussed below), can be recast as scalar-tensor theories. They can be understood heuristically as models of gravity with a variable Newton's constant.

**6.1.1 THE ACTION AND FIELD EQUATIONS** In scalar-tensor theories, the Einstein-Hilbert action  $S_{\text{EH}} = (16\pi G)^{-1} \int d^4x \sqrt{-g} R$  for gravity is replaced by an action (see, e.g., Ref. [84]),

$$S = \int d^4x \sqrt{-g} \left[ b(\lambda) R - \frac{1}{2} h(\lambda) g^{\mu\nu} (\partial_\mu \lambda) (\partial_\nu \lambda) - U(\lambda) + \mathcal{L}_M(g_{\mu\nu}, \psi_i) \right], \quad (12)$$

where  $\lambda(\vec{x}, t)$  is the eponymous scalar field;  $\mathcal{L}(g_{\mu\nu}, \psi_i)$ , the matter Lagrangian, is a function of the metric and matter fields  $\psi_i$ ; and  $b(\lambda)$ ,  $h(\lambda)$ , and  $U(\lambda)$  are functions that determine the form of the scalar-tensor theory. The presence of a spatially varying field  $b(\lambda)$  multiplying the curvature  $R$  in Eq. (12) implies that scalar-tensor theories are theories of gravity with a Newton's constant that depends on  $b(\lambda)$ . The other terms in the action are then kinetic- and potential-energy terms for the new field. Although Eq. (12) suggests that three functions [ $b(\lambda)$ ,  $h(\lambda)$ , and  $U(\lambda)$ ] are required to specify the theory, we can redefine  $b$  to be the new field and then derive new functions  $U(b)$  and  $h(b)$ .

Variation of the action with respect to the metric leads to the equation of motion (the generalization of Einstein's equation),

$$G_{\mu\nu} = b^{-1}(\lambda) \left[ \frac{1}{2} T_{\mu\nu}^{(M)} + \frac{1}{2} T_{\mu\nu}^{(\lambda)} + \nabla_\mu \nabla_\nu b - g_{\mu\nu} \square b \right], \quad (13)$$

where  $G_{\mu\nu}$  is the Einstein tensor,  $T_{\mu\nu}^{(M)}$  the stress tensor for matter, and

$$T_{\mu\nu}^{(\lambda)} = h(\lambda) (\nabla_\mu \lambda) (\nabla_\nu \lambda) - g_{\mu\nu} \left[ \frac{1}{2} h(\lambda) g^{\rho\sigma} (\nabla_\rho \lambda) (\nabla_\sigma \lambda) + U(\lambda) \right], \quad (14)$$

is the stress tensor for the scalar field. There is also an equation of motion,

$$h \square \lambda + \frac{1}{2} h' g^{\mu\nu} (\nabla_\mu \lambda) (\nabla_\nu \lambda) - U' + b' R = 0, \quad (15)$$

for the scalar field, where  $' \equiv d/d\lambda$ .

**6.1.2 FRIEDMANN EQUATIONS** The equation of motion for the scale factor  $a(t)$  in a spatially-flat Robertson-Walker Universe is

$$H^2 \equiv \left( \frac{\dot{a}}{a} \right)^2 = \frac{\rho}{6b} + \frac{h\dot{\lambda}^2}{6b} - H \frac{\dot{b}}{b} + \frac{U}{6b}, \quad (16)$$

with scalar-field equation of motion,

$$\ddot{\lambda} + 3H\dot{\lambda} = 3\frac{b'}{h} \left( \dot{H} + 2H^2 \right) - \frac{h'\dot{\lambda}^2}{2h} - \frac{1}{2} \frac{U'}{h}. \quad (17)$$

Several things are clear from these equations. First, there is considerable freedom in the choice of the functions  $b(\lambda)$ ,  $h(\lambda)$ , and  $U(\lambda)$ , and so it is difficult to make general statements about the validity of scalar-tensor theories. Secondly, specification of these functions does not alone determine the phenomenology; the initial conditions for the new scalar degree of freedom must also be specified.

Although the detailed Friedmann and scalar-field equations are different, there are explanations for cosmic acceleration in these theories analogous to those in ordinary quintessence theories. For example, if  $U(\lambda)$  is sufficiently shallow, there may be solutions to the equations of motion in which  $\lambda$  is displaced from the minimum of  $U(\lambda)$ , and rolls slowly. In this case, the time derivatives in the equations of motion will become negligible; the Friedmann equation becomes approximately  $H^2 \simeq U/(6b) \simeq \text{constant}$ ; and a roughly de Sitter expansion ensues. Given the additional terms in the Friedmann equation and scalar-field equation of motion that depend on derivatives of  $b$  and  $h$ , the details may differ, and a wider range of behaviors may be possible. However, the form of the left-hand side of Eq. (17) implies that the rolling of the scalar field generically slows with time; a general-relativistic cosmological behavior is consequently an attractor in many scalar-tensor theories [85].

**6.1.3 BRANS-DICKE THEORY AND SOLAR SYSTEM TESTS** The Brans-Dicke theory is defined by  $b(\lambda) = \lambda/(16\pi G)$ ,  $h(\lambda) = \omega/(8\pi\lambda G)$ , and  $U(\lambda) = 0$ , where the Brans-Dicke parameter  $\omega$  is a constant. Solution of the field equations in the Solar System gives rise to a parameterized-post-Newtonian (PPN) parameter  $\gamma = (\omega + 1)/(\omega + 2)$  for this theory. This parameter is measured in time-delay experiments in the Solar System to be  $\gamma = 1 + (2.1 \pm 2.3) \times 10^{-5}$  [86], leading to a bound  $\omega \gtrsim 5 \times 10^4$ . The generalization of this Solar System constraint to scalar-tensor theories with other choices of  $b(\lambda)$ ,  $h(\lambda)$ , and  $U(\lambda)$  depends on the specifics of those functions. Roughly speaking, though, the bound applies as long as the curvature at the minimum of  $U(\lambda)$  is sufficiently shallow so that the motion of  $\lambda$  within the Solar System is not restricted; this will be quantified more precisely for  $f(R)$  theories below.

## 6.2 $f(R)$ Theories

**6.2.1 THE ACTION AND FIELD EQUATIONS** One class of alternative-gravity theories that have received considerable attention in recent years are  $f(R)$  theories, in which the Einstein-Hilbert action is replaced by an action [87],

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} f(R) + S_{\text{matter}}, \quad (18)$$

where  $f(R)$  is a function whose form defines the theory. Such actions, which generalize the Einstein-Hilbert action, may arise as low-energy limits of string theory. Note that the  $G$  in this action is not necessarily the Newton's constant measured in terrestrial experiments.

The field equations are obtained by varying the action with respect to  $g_{\mu\nu}$ ; the result is

$$f'(R)R_{\mu\nu} - \frac{1}{2}f g_{\mu\nu} - \nabla_\mu \nabla_\nu f'(R) + \square f'(R)g_{\mu\nu} = 8\pi G T_{\mu\nu}. \quad (19)$$

Taking the trace and setting  $T_{\mu\nu} = 0$ , we find a constant-curvature vacuum solution (i.e., a de Sitter spacetime) with scalar curvature  $R_0$ , with  $f'(R_0)R_0 = 2f(R_0)$ .

**6.2.2 EXAMPLE:  $1/R$  GRAVITY** In  $1/R$  gravity [88], we choose  $f(R) = R - \mu^4/R$ , with  $\mu$  a constant; this theory has a self-accelerating vacuum solution with  $R = 12H^2 = \sqrt{3}\mu^2$ . The field equation for this theory is

$$8\pi GT_{\mu\nu} = \left(1 + \frac{\mu^4}{R^2}\right) R_{\mu\nu} - \frac{1}{2} \left(1 - \frac{\mu^4}{R^2}\right) R g_{\mu\nu} + \mu^4 (g_{\mu\nu} \square - \nabla_{(\mu} \nabla_{\nu)}) R^{-2}. \quad (20)$$

Some intuition about the model can be obtained from the trace,

$$\square \frac{\mu^4}{R^2} - \frac{R}{3} + \frac{\mu^4}{R} = \frac{8\pi GT}{3}, \quad (21)$$

where  $T = g^{\mu\nu} T_{\mu\nu}$ . For an effectively pressureless source (e.g., the Sun),  $T = -\rho$ , where  $\rho$  is the mass density. This should be compared with the general-relativistic Einstein-equation trace,  $R = 8\pi G\rho$ . First of all, note that (for constant  $R$ ) the new equation is quadratic, rather than linear, in  $R$ ; this suggests that there may be two different constant-curvature solutions for the same  $\rho$ . Given that  $\mu^2 \sim H \ll G\rho$  in the Solar System, it is tempting to assume an approximate “GR-like” solution  $R \simeq 8\pi G\rho$ . However, this solution is violently unstable to small-wavelength perturbations [89] (and it produces the wrong spacetime outside the Sun). The other solution, which has  $R \simeq \mu^2$  everywhere throughout the Solar System (and which is very different from  $R \sim G\rho$ ), is stable. However, this solution predicts a PPN parameter  $\gamma = 1/2$ , which disagrees with experimental constraints [90, 91].

The other significant (and perhaps more important) difference between  $1/R$  gravity and general relativity arises from the term  $\square(\mu^4/R^2)$  in Eq. (21). In general relativity, the trace equation,  $R = 8\pi G\rho$ , is a constraint equation that determines  $R$  uniquely. However, in  $1/R$  gravity, the scalar curvature  $R$  becomes a dynamical variable; i.e., there is a new scalar degree of freedom.

The original  $1/R$  gravity theory is just one example of an  $f(R)$  theory. Given the flexibility allowed in the choice of  $f(R)$ , it is dangerous to draw general conclusions about  $f(R)$  theories from  $1/R$  gravity. Still, there are several lessons: (1) if the  $f(R)$  theory is to explain cosmic acceleration, there are likely to be mass parameters comparable to  $H$  in the theory. (2) There is a scalar degree of freedom, dormant in general relativity, that comes to life in  $f(R)$  theories. (3) Solar System constraints to the theory may be severe. (4) There may be more than one solution, for the same source, to the field equations. (5) One of the solutions may be unstable to small-wavelength perturbations.

**6.2.3 THE EQUIVALENCE BETWEEN  $f(R)$  AND SCALAR-TENSOR THEORIES** In the general case, the physics of  $f(R)$  theories can be understood by noting that they are equivalent to scalar-tensor theories [90]. Consider the following action for gravity with a scalar field  $\lambda$ :

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} [f(\lambda) + f'(\lambda)(R - \lambda)] + S_{\text{matter}}. \quad (22)$$

The  $\lambda$  equation of motion gives  $\lambda = R$  if  $f''(\lambda) \neq 0$ , demonstrating the equivalence with Eq. (18). Eq. (22) is thus equivalent to the scalar-tensor action, Eq. (12), if we identify  $b(\lambda) = f'(\lambda)$ ,  $U(\lambda) = -f(\lambda) + \lambda f'(\lambda)$ , and  $h(\lambda) = 0$ . In other words,  $f(R)$  theories are equivalent to scalar-tensor theories with vanishing kinetic term. The absence of a kinetic term seems to suggest that the scalar degree of freedom remains dormant, but if we change to an Einstein-frame metric  $g_{\mu\nu}^E = b'(\lambda)g_{\mu\nu}$  and

canonical scalar field  $\varphi$  through  $f'(\lambda) = \exp\left(\sqrt{16\pi G/3}\varphi\right)$ , then the Jordan-Brans-Dicke-frame (JBD-frame) action [Eq. (22)] becomes, in the Einstein frame,

$$S = \int d^4x \sqrt{-g_E} \left[ \frac{1}{16\pi G} R_E - \frac{1}{2} g_E^{\mu\nu} (\partial_\mu \varphi)(\partial_\nu \varphi) - V(\varphi) \right], \quad (23)$$

with

$$V(\varphi) = \frac{\lambda(\varphi)f'(\lambda(\varphi)) - f(\lambda(\varphi))}{16\pi G[f'(\lambda(\varphi))]^2}. \quad (24)$$

In this frame, the propagating scalar degree of freedom is apparent.

In the Einstein frame, scalar-tensor theories look like general relativity with a canonical scalar field. The difference, though, is that the Einstein-frame metric  $g_E$  is *not* the metric whose geodesics determine particle orbits; it is the JBD-frame metric  $g$ . Thus, scalar-tensor theories in the Einstein frame resemble general relativity with an extra, non-geodesic, force on the particle. These may also be generalized by “chameleon theories” [92], where the scalar coupling to matter may differ for different matter fields. Viewed in the JBD frame, scalar-tensor theories are those in which there is a new propagating scalar degree of freedom, in addition to the usual two propagating tensorial degrees of freedom.

**6.2.4 FRIEDMANN EQUATIONS** The Friedmann equations for  $f(R)$  theory are now obtained from Eq. (16),

$$H^2 = \frac{1}{6} \frac{\rho}{f'(\lambda)} - H \frac{d}{dt} \ln f'(\lambda) + \frac{1}{6} \frac{\lambda f'(\lambda) - f(\lambda)}{f'(\lambda)}, \quad (25)$$

while the scalar-field equation of motion, Eq. (17), now provides the constraint  $\lambda = 12 H^2 + 6 \dot{H}$ . The self-accelerating solution,  $H^2 = \lambda/12$ , can be obtained by setting  $\rho = 0$  and time derivatives equal to zero in these equations. We further see that it is the scalar-field potential,  $U(\lambda) = \lambda f'(\lambda) - f(\lambda)$ , in the last term of Eq. (25) that is driving the accelerated expansion.

**6.2.5 SOLAR SYSTEM CONSTRAINTS** The absence of a kinetic term for  $\lambda$  implies a Brans-Dicke parameter  $\omega = 0$  and thus a PPN parameter  $\gamma = 1/2$  for  $f(R)$  gravity, generalizing the result for  $1/R$  gravity. However, as discussed above, this Solar System constraint only applies if the function  $U(\lambda)$  in the scalar-tensor theory is sufficiently close to flat that the scalar field can move freely in the Solar System. This is true if the following four conditions are satisfied [93]: (i)  $f(R)$  is analytic at  $R = R_0$ , where  $R_0$  is the background value of  $R$ ; (ii)  $f''(R_0) \neq 0$ ; and (iii)  $|f'(R_0)/f''(R_0)| r_{\text{SS}}^2 \ll 1$ , where  $r_{\text{SS}} \sim \text{AU}$  is the distance scale over which the Solar System tests are carried out; and (iv)  $|f'(R_0)/f''(R_0)| \ll R_0(r_\odot/GM_\odot)$ . If these conditions are violated, then the linear-theory analysis that concludes that  $\gamma = 1/2$  breaks down. In this case, a fully nonlinear analysis is required to determine  $\gamma$ .

Theories can be constructed, by violating condition (iv) above, that exhibit a “chameleon mechanism,” whereby the nonlinear solution satisfies Solar System constraints [94, 95, 96]. These theories require the effective mass to be large in the Solar System, and small in intergalactic space. The GR-like solution  $R \simeq 8\pi G\rho$  inside the Sun matches onto the GR-like solution with  $\rho \simeq \rho_{\text{MW}}$  (with  $\rho_{\text{MW}}$  the mass density in the Milky Way) outside the Sun, but within the Milky Way. That solution then transitions to the cosmological solution in intergalactic space. Functional forms for  $f(R)$  that allow such behavior require several small parameters. Ref. [97] provides a classification of such models. For example, an  $f(R)$  that resembles a broken power law, with  $f(R) \propto R^n$  (with  $n > 0$ )

as  $R \rightarrow 0$  and  $f(R) \propto a + b/R^n$  (with  $a$  and  $b$  appropriately chosen constants), may work [94, 96]. In these theories, the scalar-field dynamics on cosmological scales become very “stiff;” i.e., the phenomenology of these theories is almost indistinguishable from those in which there is simply a cosmological constant [98].

These models also imply a tail-wags-the-dog effect whereby a change in the ambient density surrounding the Solar System, from ISM densities to IGM densities, can change the results of PPN tests by 5 orders of magnitude. In some  $f(R)$  theories, particularly those with a chameleon mechanism, the usual  $1/r^2$  force law of gravity is modified. This seemingly trivial change may have profound implications for almost every area of astrophysics, from Solar System scales to the dynamics of galaxy clusters, few of which have yet to be thought through carefully.

**6.2.6 PALATINI FORMALISM** In the usual formulation (the “metric formalism”) of general relativity, the Einstein-Hilbert action is varied with respect to the metric  $g_{\mu\nu}$  to obtain Einstein’s equations. However, an alternative approach, the “Palatini formalism,” is to vary the action with respect to the connection  $\Gamma_{\mu\nu}^\rho$  as well as the metric. If applied to the Einstein-Hilbert action, this approach results in the same gravitational field equations, and it also yields the standard relation between the metric and the connection. However, for a more general  $f(R)$  action, the Palatini formalism gives rise to a different theory. Solutions to cosmic acceleration may also be obtained with the Palatini formalism [99], possibly without violating Solar System constraints. However, the Christoffel symbol is now evaluated using a different metric,  $\tilde{g}_{\mu\nu} = f'(R)g_{\mu\nu}$ , whereas particle trajectories still follow geodesics of  $g_{\mu\nu}$ . Moreover,  $R$  is obtained from the algebraic relation  $Rf'(R) - 2f(R) = 8\pi GT$  between the Ricci scalar and the trace of the stress-energy tensor. The gravitational implications depend sensitively on the source stress tensor. At the quantum level, these theories generally result in new matter couplings that may have even more dire empirical consequences [100] in the form of violations of the equivalence principle [101].

## 7 Braneworld Gravity and Related Ideas

The alternative-gravity theories discussed above introduce a new scalar degree of freedom. Another possibility is to modify gravity by changing the dimensionality of space. We now discuss such “braneworld” scenarios, as well as braneworld-inspired ideas. In braneworld scenarios, our  $(3+1)$ -d world is a subspace of a higher-dimensional spacetime. Unlike earlier extra-dimensional models (e.g., Kaluza-Klein theories), Standard Model fields may be restricted to lie on our brane, while gravitational fields may propagate in the extra dimensions (the “bulk”) as well.

### 7.1 DGP Gravity

**7.1.1 THE ACTION** DGP (for Dvali-Gabadadze-Porrati) gravity [102, 103] postulates a  $(4+1)$ -dimensional Universe in which the bulk of the five-dimensional spacetime is Minkowski space with an embedded  $(3+1)$ -dimensional brane (our Universe) on which matter fields live. The gravitational action is

$$S_{(5)} = \int d^5x \sqrt{-g} \frac{R}{16\pi G^{(5)}} + \int d^4x \sqrt{-g^{(4)}} \left[ \frac{R^{(4)}}{16\pi G} + \mathcal{L}_{SM} \right], \quad (26)$$

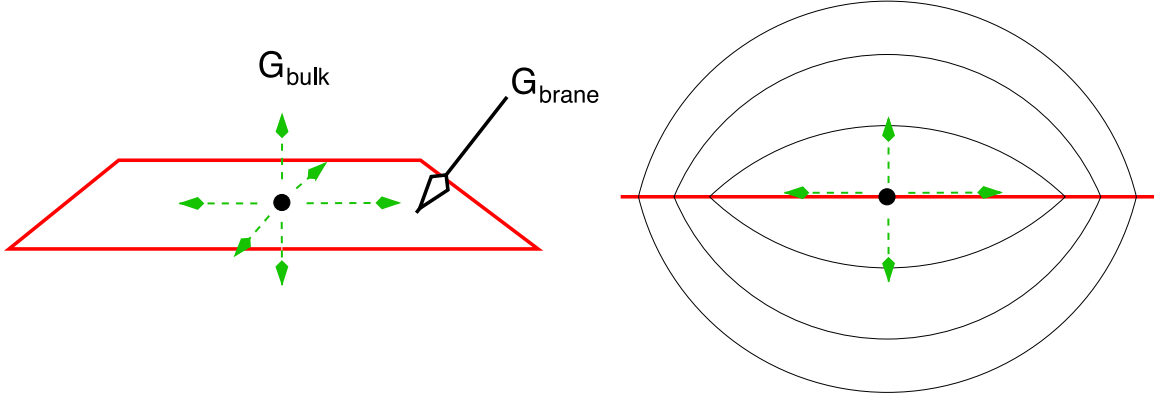


Figure 6: An illustration of the DGP mechanism. Added to our  $(3+1)$ -d spacetime (the “brane”) is an additional spatial dimension, the “bulk.” The presence of stress-energy on the brane provides an energy cost for the propagation of gravitons with wavelengths  $< r_0$  into the bulk, thus making the gravitational force law  $\propto r^{-2}$  at distances  $r < r_0$  on the brane, but  $\propto r^{-3}$  at distances  $r > r_0$ . On the right are illustrated approximate equipotential curves for the gravitational field. (From Ref. [104].)

where  $G^{(5)}$  is the 5-d gravitational constant (note that its dimensions are different than those of  $G$ ), and  $g$  ( $R$ ) the 5-d metric determinant (Ricci scalar), and  $g^{(4)}$  ( $R^{(4)}$ ) the induced metric determinant (Ricci scalar) on the brane.

**7.1.2 HEURISTIC PICTURE** Before proceeding with the cosmological solution, it is instructive to consider DGP gravity in the weak-field limit. If we take  $g_{AB} = \eta_{AB} + h_{AB}$ , with  $|h_{AB}| \ll 1$ , then the linearized field equations tell us that the 4-d metric components  $h_{\mu\nu}$ , wherein resides the nonrelativistic potential, have Fourier ( $p^\mu$ ) components,

$$h_{\mu\nu}(p) = \frac{8\pi G}{p^2 + 2(G/G^{(5)})p} \left[ T_{\mu\nu}(p) - \frac{1}{3} \eta_{\mu\nu} T^\alpha_\alpha(p) \right], \quad (27)$$

for a stress-energy source  $T_{\mu\nu}$  on the brane. This suggests a crossover distance,  $r_0 = (1/2)(G^{(5)}/G)$ . For Fourier modes  $p \gg r_0^{-1}$ ,  $h_{\mu\nu}(p) \propto p^{-2}$ , implying the usual static gravitational potential  $V(r) \propto r^{-1}$  for  $r \ll r_0$ . But for Fourier modes  $p \ll r_0^{-1}$ ,  $h_{\mu\nu}(p) \propto p^{-1}$ , implying  $V(r) \propto r^{-2}$  at larger distances; i.e., gravity is weaker at distances  $r \gtrsim r_0$ .

The static gravitational potential in DGP gravity differs from that in fundamental theories with small extra dimensions. If there is an extra dimension curled up into a size  $R_5 \sim \text{mm}$ , and the graviton is free to propagate equally in our three spatial dimensions and this extra small dimension, then the gravitational force law steepens to  $r^{-3}$  at distances  $\lesssim \text{mm}$ . In DGP gravity, however, the extra dimension is large, not small, and there is an energy cost for the propagation of gravitons with wavelengths  $\lesssim r_0$  into the bulk. At  $r \lesssim r_0$ , the gravitons are thus confined to the brane, and we have ordinary gravity; for  $r \gtrsim r_0$ , the gravitons can escape into the bulk and the force law is that for a 5-d spacetime, as shown in Fig. 6.

**7.1.3 COSMOLOGICAL SOLUTION** The action can be varied to obtain the field equations. The brane is then assumed to be filled with a homogeneous fluid of pressure  $p$  and energy density  $\rho$  [102, 105]. The cosmological metric takes the form (assume a flat Universe for simplicity),  $ds^2 =$



$N^2(t, \xi)dt^2 - A^2(t, \xi)d\vec{x}^2 - B^2(t, \xi)d\xi^2$ , where  $\xi$  is the coordinate for the fifth dimension. The (generalized) Einstein equations yield equations of motion for the metric variables  $N(t, \xi)$ ,  $A(t, \xi)$ , and  $B(t, \xi)$ . The usual scale factor for our Universe is then  $a(t) = A(t, \xi = 0)$ , and it satisfies an equation of motion (the DGP Friedmann equation)

$$H^2 \pm \frac{H}{r_0} = \frac{8\pi G}{3}\rho. \quad (28)$$

There are two solutions for the expansion [cf., the discussion of  $f(R)$  models above]. If we take the minus sign in Eq. (28), then at early times, when  $H \gg r_0^{-1}$ , we recover the usual Friedmann equation. But when  $H$  decreases, the new term kicks in, and  $H \rightarrow r_0^{-1}$  at late times; i.e., the Universe asymptotes at late times to a de Sitter phase. (The plus sign in Eq. (28) results in an eternally decelerating Universe.)

**7.1.4 SOLAR SYSTEM TESTS** Unlike quintessence models, which retain general relativity, DGP gravity is an alternative-gravity theory, and it makes predictions for modified gravitational physics, beyond a modified expansion rate, and in particular for a modified spacetime in the Solar System. On the face of it, DGP gravity resembles at Solar System distance scales a theory with a gravitational scalar degree of freedom. This can be seen from the tensor structure,  $T_{\mu\nu} - (1/3)\eta_{\mu\nu}T_\mu^\mu$ , that acts as the source for the linearized gravitational field in Eq. (27). This tensor structure resembles that in an  $\omega = 0$  scalar-tensor theory and in massive gravity (the extra scalar being the longitudinal mode of the graviton), but differs from the structure  $T_{\mu\nu} - (1/2)\eta_{\mu\nu}T_\mu^\mu$  in general relativity. The extra scalar degree of freedom in DGP gravity may be understood as a fluctuation in the brane surface. The difference means that a relativistic particle (e.g., a photon) is affected differently by the same source, leading to a PPN parameter  $\gamma = 1/2$ , again in disagreement with measurements. This is a DGP equivalent of the van Dam-Veltman-Zakharov discontinuity [106, 107] that appears in massive gravity.

However, Eq. (27) provides only the  $(3+1)$ -d components of the field. The approximations that lead to this linearized equation involve a highly nonlinear metric perturbation in the bulk, even when the source is weak, calling the derivation of Eq. (27) into question. A proper treatment involves a perturbative expansion not only in  $h_{\mu\nu}$ , but also in  $r/r_*$ , where  $r_* = (r_g r_0^2)^{1/3}$ , and  $r_g = 2GM/c^2$  is the Schwarzschild radius [108]. The field equations for the spherically symmetric spacetime can then be solved perturbatively in three different distance regimes, with the following results: (1) the spacetime looks like that in general relativity, with fractional corrections  $O((r/r_*)^{3/2})$ , at small distances,  $r \ll r_*$ ; (2) it looks like that in an  $\omega = 0$  scalar-tensor theory (i.e., the static potential is still  $\propto r^{-1}$ , but light deflection is described by  $\gamma = 1/2$ ) at distances  $r_* \ll r \ll r_0$ ; and (3) it then falls off more steeply, as  $r^{-2}$ , at distances  $r \gg r_0$ .

For example,  $r_* \simeq 150$  pc for the spacetime around the Sun. Thus, Solar System tests of gravity occur deep within the GR-like regime, and DGP gravity is thus consistent with these tests. Still, the spacetime is not *precisely* Schwarzschild; there are corrections  $O((r/r_*)^{3/2})$ . These corrections may be tested by future experiments [109], although the  $r^{-3/2}$  dependence of the correction means that the theory *cannot* be parametrized with the usual PPN formalism.

Light-deflection experiments in the Solar System are unlikely to be constraining, as the fractional correction to the general-relativistic value for the deflection angle will be  $\sim (r/r_*)^{3/2} \sim 10^{-11}$ , while the smallest value probed is  $\sim 10^{-4}$ . However, measurements of perihelion advances may



be more promising. DGP gravity leads to a correction,  $\sim 5 \mu\text{as}/\text{year}$ , to the perihelion advance of a planetary orbit [109]. Unlike the general-relativistic perihelion-advance rate, which decreases for larger- $r$  orbits, the DGP correction is  $r$  independent and can thus be distinguished from a general-relativistic correction (or from those that occur in the usual PPN expansion). Moreover, Solar System tests at large distances may be equally, or more, effective at testing DGP gravity as those at short distances. Thus, improved lunar laser ranging experiments may be sensitive to DGP gravity [110], as might BepiColombo and MESSENGER, European Space Agency and NASA satellites, respectively, to Mercury. However, probes of the outer Solar System, like Cassini, might also probe DGP gravity.

**7.1.5 EXPANSION HISTORY** By re-arranging Eq. (28), the expansion history can be written,  $H(z) = (H_0/2) \left[ 1 - \Omega_m + \sqrt{(1 - \Omega_m)^2 + 4\Omega_m(1+z)^3} \right]$ . At  $z \gg 1$ , this approaches the standard form,  $H(z) \simeq \sqrt{\Omega_m} H_0 (1+z)^{3/2}$ , and  $w_Q \rightarrow -1$  in the distance future,  $z \rightarrow -1$ . The deceleration parameter for this model is  $q_0 = 3\Omega_m(1 + \Omega_m)^{-1} - 1$ , and thus there is a relation between  $q_0$  and  $\Omega_m$ . A value  $\Omega_m = 0.274$  implies  $q_0 = -0.355$ , which is only marginally consistent with current data. A better fit to observations can be obtained by adding a cosmological constant or curvature [111], or in models based on other manifestations of braneworlds.

Fig. 1 shows the expansion history, luminosity distance, and deceleration for the DGP model. If the expansion history can be measured with sufficient precision to distinguish this functional form from, e.g., a constant- $w_Q$  model, then this may provide an avenue toward testing the model.

**7.1.6 GROWTH OF STRUCTURE** The distance scales relevant for large-scale structure generally occur at  $r \gtrsim r_*$ , where the behavior of DGP gravity differs from that of general relativity. The growth of linear density perturbations can be described in DGP gravity in terms of an effective Newton's constant,  $G_{\text{eff}} = G(1 + 3/\beta)$  [112], with  $\beta = 1 - 2r_0 H \left[ 1 + \dot{H}/(3H^2) \right]$ . The effects of this altered gravitational constant can be taken into account approximately by changing the last term in Eq. (4); the factor  $\Omega_m H^2$  that appears there arises from the Friedmann equation  $\Omega_m H^2 = 8\pi G\rho/3$ . The change in the linear-theory growth factor  $D(z)$  can be appreciable in these models; it is a  $\sim 30\%$  correction at  $z = 0$ . This contrasts dramatically with quintessence models, which do not generally affect  $D(z)$  significantly.

## 7.2 Related ideas

There have been other attempts to modify gravity to account for cosmic acceleration that are inspired by DGP gravity or the massive-gravity theories it resembles.

**7.2.1 DEGRAVITATION** The idea of degravitation [113, 114] is to replace Einstein's equation,  $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ , by  $[1 + F(L^2 \square)] G_{\mu\nu} = 8\pi G T_{\mu\nu}$ , where  $F(x)$  is a monotonically decreasing filter function with the limits  $F(x) \rightarrow 0$  for  $x \rightarrow \infty$ , and  $F(x) \gg 1$  for  $x \rightarrow 0$ . Here,  $L$  is a distance scale (presumably  $\sim H_0^{-1}$ ) at which the force of gravity weakens. Thus, Newton's constant acts as a high-pass filter; long-wavelength modes of the stress-energy tensor do not source the gravitational field. An analogous modification of electrodynamics is precisely equivalent to electrodynamics with a massive photon. Likewise, the structure of degravitation shares some similarities with massive gravity, although the mapping is not precise.

**7.2.2 THE FAT GRAVITON** The idea here [115] is to postulate that virtual gravitons with invariant masses  $\gtrsim \text{meV}$  just simply do not propagate; the cosmological constant due to zero-point fluctuations conveyed by gravity is then that observed. Such models can be constrained by considering cosmological gravitational-lensing systems [116], as the angular deflection of photons in such systems implies momentum transfers (presumably carried by virtual gravitons) larger than this energy scale.

**7.2.3 MODIFIED FRIEDMANN EQUATIONS** Braneworld scenarios that generalize the DGP theory by allowing for a wider range of dynamics in the bulk can produce an effective expansion law  $H^2 \propto \rho^n$  on the brane [117], and this has motivated phenomenological models of dark energy. One such example is the “cardassian” model whereby  $H^2 = (8\pi G\rho/3) + B\rho^n$  [118]; supernova and CMB distances suggest  $n \lesssim 0.4$ . An alternative parameterization of the effects of extra dimensions proposes  $H^2 + (1 - \Omega_M)H_0^2(H/H_0)^\alpha = 8\pi G\rho/3$  [119]. During the matter era, the equation-of-state parameter of the inferred dark energy is  $w_{\text{eff}} = -1 + \alpha/2$  until  $z \sim 1$ , and asymptotes to  $w_{\text{eff}} \rightarrow -1$  in the future. Rough arguments suggest that  $\alpha \lesssim 1$  is necessary for consistency with observations.

**7.2.4 A PHENOMENOLOGICAL APPROACH** In Refs. [120, 121, 122] the existence of a new gravitational theory is posited that changes the amount of spacetime curvature produced per unit mass. The Friedmann equation is modified so that the matter-dominated expansion becomes progressively more de Sitter-like, mimicking the evolution under dynamical dark energy with equation-of-state parameter  $w_Q \simeq -1$ . Metric perturbations likewise respond differently to inhomogeneities in the matter and radiation, leading to a characteristic “gravitational slip” whereby the potential  $\psi$  appearing in the geodesic equation,  $\ddot{\vec{x}} = -\vec{\nabla}\psi$  differs from the potential  $\phi$  in the Poisson equation,  $\nabla^2\phi = 4\pi G\delta\rho$ . Scalar-tensor and  $f(R)$  theories, braneworld scenarios, and DGP gravity, as well as massive gravity, all predict  $\phi \neq \psi$  in the presence of non-relativistic matter, in contrast to general relativity. This suggests a parameterized post-Friedmann description of modified gravity, whereby a new parameter,  $\varpi \equiv \psi/\phi - 1$  characterizes the degree of departure from general relativity, in analogy to the post-Newtonian parameter  $\gamma$ . The imposed time- and scale-dependence of  $\varpi$ , along with two further assumptions—conservation of the radiation and matter stress-energy tensor, and the absence of new gravitational effects mimicking a “dark fluid” momentum flux or velocity relative to the cosmic rest frame—are sufficient to complete the description of linearized metric perturbations. A  $\varpi \neq 0$  affects the rate of growth of perturbations, the integrated Sachs-Wolfe effect, and the weak-gravitational-lensing deflection angle. Hence, observations of the CMB and large-scale structure may be used to test for the consistency of general relativity on cosmological scales.

### 7.3 Comments

There are a number of theoretical questions that must be addressed if braneworld scenarios are to explain cosmic acceleration. The simplest DGP model is only marginally consistent with the observed cosmic acceleration; either some new exotic fluid or a more complicated implementation of the braneworld must be introduced to improve the agreement with data. Braneworld scenarios introduce new small parameters, and they do not solve the coincidence problem. Moreover, it has been questioned whether the local perturbative solutions for the spherically symmetric DGP spacetime can be sewn together into a single global solution [123]. There are also questions about the stability of the self-accelerating phase to the growth of small-scale fluctuations [124, 125]. Still,

braneworld scenarios and related ideas are worth further theoretical attention as they connect cosmology to novel ideas from string and supergravity theories and provide a range of phenomenological consequences, beyond simply the alteration of the expansion rate for which they were introduced to explain.

## 8 The Landscape Scenario

We have concentrated on theories of cosmic acceleration based on the introduction of new fields or modifications of gravity, both intended as alternatives to the simple postulate of a cosmological constant. But cosmic acceleration may just be due to a cosmological constant. If so, then the physics of cosmic acceleration is just the physics of the cosmological constant. We have refrained from discussing theories of the cosmological constant (for reviews, see Refs. [126,127]), but we make an exception for the recently developed landscape scenario [128].

Like quintessence, the landscape scenario allows for a range of possible values for the vacuum energy. Unlike quintessence, these possibilities are arranged in a “discretuum,” rather than a continuum, of values. The spacing between these values is comparable to the observed value of the cosmological constant. Additional arguments, which we do not discuss here, can be provided to explain why we live in one the lowest of these.

To understand the idea, recall that the electromagnetic field  $F_{\mu\nu}$  is a two-form (an antisymmetric rank-two tensor) sourced by a charge  $e$  that follows some worldline. In 1+1 dimensions (or equivalently, between two parallel plates), the electric field and its energy density are constant, and quantization of the electron charge  $e$  implies that both the field and energy density are quantized, the latter taking on values  $\rho \propto n^2 e^2$ , where  $n$  is an integer.

Similarly, a four-form field  $F_{\mu\nu\rho\sigma}$  in 3+1 spatial dimensions is sourced by coupling to an “electrically” charged membrane (a 3-brane), and in string theory, there are also analogs of magnetic charges (5-branes). Quantization conditions, analogous to the Dirac quantization condition in electromagnetism, then require that the field, and the associated energy density, take on discrete values,  $\rho = (1/2)n^2 q^2 m_{\text{Pl}}^4$  [129, 130, 128].

Suppose now that there is a “bare” cosmological constant  $\lambda = O(m_{\text{Pl}}^2)$  which, for the sake of argument, may be negative. Then the effective cosmological constant  $\Lambda$  can take on values  $\Lambda = \lambda + 4\pi n^2 q^2 m_{\text{Pl}}^2$ . There are thus an infinite range of possible values of  $\Lambda$ . The requirement that there be one that is  $\Lambda \lesssim 10^{-120} m_{\text{Pl}}^2$  requires  $q \lesssim 10^{-120} \lambda^{1/2} m_{\text{Pl}}^{-1}$ ; i.e., there is still a fine-tuning problem. Put another way, if  $q \sim 1$ , then the closest that  $n^2 q^2$  will get to  $-\lambda/m_{\text{Pl}}^2$  is  $\sim 1$ , or in other words, the density of states is constant in  $n$ .

However, in string theory, there may be a large number  $J$  of four-form fields; e.g., a typical value may be  $J \simeq 100 - 500$ . If so, then the cosmological constant takes on values  $\Lambda = \lambda + 4\pi \sum_i n_i^2 q_i^2 m_{\text{Pl}}^2$ . Taking all  $q_i = q$ , for the sake of argument, each combination  $\{n_1, n_2, \dots, n_J\}$  describes a different vacuum with a contribution  $\lambda_n \equiv 4\pi q^2 m_{\text{Pl}}^2 \sum_i n_i^2$  to the vacuum-energy density. The number of states with  $n^2 \equiv \sum_i n_i^2$  in the range  $n^2 \rightarrow n^2 + dn^2$  is  $(dN/dn^2)dn^2$ , where  $(dN/dn^2) = (2\pi)^{J/2} n^{J-2} [2\Gamma(J/2)]^{-1}$ , the density of states, is proportional to the area of an  $J$ -sphere of radius  $n$ . The typical spacing between states is thus  $4\pi q^2 m_{\text{Pl}}^2 \Delta(n^2)$  where  $\Delta(n^2) = (dN/dn^2)^{-1}$ . If we assume  $\lambda_n \simeq m_{\text{Pl}}^2$ , then  $n^2 \simeq (4\pi q^2)^{-1}$ . Taking  $4\pi q^2 \simeq 0.01$ , we find  $\Delta(n^2) \simeq 10^{-120}$  for  $J \simeq 200$ . Thus, the presence of many four-form fields allows for far more closely spaced levels in the

cosmological-constant discretuum, and thus explains how a value  $10^{-120} m_{\text{Pl}}^2$  may arise in string theory.

## 9 The Observational Way Forward

### 9.1 The expansion history

The evidence for dark energy or modified gravity comes from measurements that probe the expansion history of the Universe, and extensions of these measurements provide perhaps the most promising avenues for further empirical inquiry. Current data show that the cosmic expansion is accelerating and constrain the dark-energy density to within a few percent. If we assume the equation-of-state parameter  $w_Q$  is constant, then it is constrained to be within 12% of  $-1$  (at the 95% C.L.) [17].

The question now is whether cosmic acceleration is due just to a cosmological constant, or whether there is something more interesting going on. Thus, a number of avenues are being pursued to measure  $w_Q$  more precisely to see whether it can be shown to be different from  $-1$ . These probes have recently been reviewed thoroughly by the Dark Energy Task Force (DETF) [5] and in Refs. [6, 7], so we simply summarize them here. In principle, the expansion history can be determined with a variety of cosmological observations (e.g., quasar-lensing statistics, cluster properties, the Lyman-alpha forest, Alcock-Paczynski test, direct measurements of the age of the Universe, etc.). However, the DETF focused upon supernovae, galaxy-cluster abundances, baryon acoustic oscillations, and weak gravitational lensing, reflecting a rough consensus in the community that these four currently provide the most promising avenues. We caution, however, that there may still be room for new ideas. Either way, it is generally agreed that given systematic errors inherent in any particular technique, several complementary methods will be required to provide cross-checks.

**9.1.1 SUPERNOVAE** Supernovae have played a crucial role in establishing cosmic acceleration, and they are likely to provide even more precise constraints on the expansion history in the future. The supernovae used here are Type Ia supernovae, explosions powered by the thermonuclear detonation of a white dwarf when its mass exceeds the Chandrasekhar limit. These explosions can be distinguished from those produced by other mechanisms (e.g., Type II supernovae, powered by iron-core collapse in supergiants) from the details of their spectra and light curves. The fact that the star ignites very rapidly after exceeding the Chandrasekhar limit implies that Type Ia supernovae should be good standard candles. Thus, their observed brightness provides the luminosity distance  $d_L(z)$ . Measurements support this simple notion, and details of the spectra and light curves can be used to correct for relatively small changes in the supernova luminosities.

Supernova searches will be particularly valuable if they can reach redshifts  $z \sim 1$ , where the effects of different  $w_Q$  values become most dramatic (see Fig. 1. Progress with supernovae will require improved systematic-error reduction, better theoretical understanding of supernovae and evolution effects, and greater statistics. Both ground-based and space-based supernova searches can be used to determine the expansion history. However, for redshifts  $z \gtrsim 1$ , the principal optical supernova emission (as well as the characteristic silicon absorption feature) gets shifted to the infrared which is obscured by the atmosphere, and this provides (much of) the case for a space-based observatory.

**9.1.2 BARYON ACOUSTIC OSCILLATIONS** In recent years, baryon acoustic oscillations (BAO) have become increasingly attractive as a possibility for determining the expansion history. The acoustic oscillations seen in the CMB power spectrum are due to oscillations in the photon-baryon fluid at the surface of last scatter. The dark matter is decoupled and does not participate in these oscillations. However, since baryons contribute a non-negligible fraction of the nonrelativistic-matter density, oscillations in the baryon-photon fluid get imprinted as small oscillations in the matter power spectrum at late times [131,132]. These oscillations have now been detected in galaxy surveys [133]. The physical wavenumber at which these oscillations occur is well understood from linear perturbation theory, and so they provide a standard ruler. Thus, baryon acoustic oscillations measure the angular-diameter distance  $d_A(z) = (1+z)^{-2}d_L(z)$ . Measurement of clustering along the line of sight may also provide information on the expansion history  $H(z)$ . Issues with BAO include nonlinear evolution of the acoustic peaks in the matter power spectrum and systematic and astrophysical effects [134] that could mimic features in the power spectrum.

**9.1.3 CLUSTER ABUNDANCES** Galaxy clusters are the largest gravitationally bound objects in the Universe. The spatial density of clusters in the Universe can be determined from models of structure formation. The observed number of clusters depends on the spatial density as well as on the volume per unit solid angle on the sky and per unit redshift interval [135]. This volume depends on the quantity  $[H(z)(1+z)]^{-1}$ , and so clusters measure the expansion history  $H(z)$ .

The theories predict the cluster abundance as a function of the cluster mass. The trick, then, is to obtain the cluster mass from the cluster observables—namely, the luminosity and temperature of the x-ray emission, the Sunyaev-Zeldovich effect [136], cluster dynamics, and/or the effects of weak gravitational lensing by the cluster on background galaxies. There is now a large industry that amalgamates theory, simulations, and multiwavelength cluster observations in an effort to develop a reliable cluster-mass indicator.

**9.1.4 WEAK LENSING** Weak gravitational lensing by large-scale density fluctuations along the line of sight to distant galaxies can distort the images of those galaxies [137]. Large-distance correlations in the mass thereby induce long-distance correlations in the observed ellipticities of the distant galaxies. Measurements of these ellipticity correlations can thus be used to determine the power spectrum of the mass as a function of angular wavenumber. If the power spectrum is already known, e.g., from the CMB, as a function of the physical wavenumber, then the observed amplitude determines the physical wavenumber corresponding to an angular wavenumber. Thus, weak lensing measures the angular-diameter distance  $d_A(z)$ . Weak lensing probes the gravitational potential, and thus the total mass, unlike galaxy surveys, which use luminous galaxies to trace the mass distribution. The challenge with weak lensing is to understand the subtle experimental effects that might mimic weak-lensing-induced ellipticity correlations. There may also be intrinsic alignments of the galaxies [138] that might look like a weak-lensing signal.

**9.1.5 OTHER PROBES OF THE EXPANSION HISTORY** There may be other ways to measure the expansion history. If the ages of stellar populations can be obtained from their spectra at a variety of redshifts, then the expansion rate  $dz/dt$  may be obtained directly [139]. There may be other luminous standard candles; for example, the gravitational-wave signal from supermassive-black-hole binaries [140] may give a new method to determine luminosity distance if a suitable measure of redshift can be obtained from an optical counterpart. It has also been recently suggested that by

comparing the biases and redshift-space distortions for two different galaxy populations, constraints to  $D(z)$  and  $H(z)$  may be obtained [141] in a way that is limited ultimately by the number of galaxies, rather than the number of Fourier modes in the density field.

## 9.2 Growth of structure

The rate of growth of density inhomogeneities [i.e., the linear-theory growth factor  $D(z)$ ] depends on the cosmic expansion rate. Moreover, different theories that predict the same background cosmic evolution may lead to different rates of perturbation growth. For example, DGP theories are expected to have a significant effect on  $D(z)$ , and we have discussed above a phenomenological approach (parametrized-post-Friedmann) to the growth of perturbations in alternative-gravity theories. Of the four avenues discussed above, clusters, BAO, and weak lensing may also provide measurements of  $D(z)$ , in addition to measurements of  $H(z)$ .

## 9.3 Lorentz violation and other tests

The new physics, gravitational or otherwise, implied by cosmic acceleration may have other observable/experimental consequences, apart simply from its effect on cosmic expansion. For example, we have discussed Solar System tests of alternative-gravity theories for cosmic acceleration and the differing effects of various models on the growth of large-scale structure.

Tests of Lorentz violation provide another avenue. The rest frame of the CMB provides us with a preferred frame in the Universe. Since a cosmological constant has the same density in every inertial frame, it can manifest no effects of Lorentz violation. If, however,  $w \neq -1$ , either due to dark energy or modified gravity, and if that new physics is somehow coupled non-gravitationally to ordinary matter, then the preferred cosmological frame may show up in tests of Lorentz violation. Typically, however, we expect these violations to be extremely small by laboratory standards. First of all, dark-energy fields must be exceedingly weakly coupled to Standard Model particles if they are to remain dark. Moreover, the timescale for evolution of these fields is the Hubble time, far longer than laboratory timescales.

Cosmological observations might allow for the experimental timescale to be comparable to the Hubble time. For example, Ref. [21] pointed out that if quintessence couples to the pseudoscalar of electromagnetism, there will be a uniform rotation of the linear polarization of photons propagating over cosmological distances. This could be probed by looking for a mean misalignment between the linear polarization of cosmological radio sources with the position angles of their images. It can also be tested by looking for the parity-violating polarization correlations it produces in the CMB polarization [142].

In addition to these and laboratory tests of Lorentz violation, preferred-frame effects in gravitational physics may also arise if the quintessence field couples in some nontrivial way [143]. Eotvos-like experiments may also be used to search for couplings of ordinary matter to the quintessence field. If cosmic acceleration is due to a scalar-tensor theory, then the variable Newton's constant implied by the theory may suggest that other fundamental constants vary with time [144].

It is easy to speculate how various dark-energy theories may give rise to Lorentz violation, preferred-frame effects, or variation of fundamental constants. But in the absence of any clear front-runner theories, it is much more difficult to say which, if any of these, will be more constraining.



## 10 Conclusions

Cosmic acceleration provides an intriguing puzzle. Occam's razor suggests that the phenomenon may be explained simply by a cosmological constant. This may be an acceptable phenomenological explanation, but it would be more satisfying to have a physical explanation for the observed value of  $\Lambda$ . The unexpectedly small value inferred for  $\Lambda$  leads us to suspect that instead the apparent cosmological constant may be the false-vacuum energy associated with the displacement of some field from its minimum and/or that there may be new gravitational physics beyond Einstein's general relativity. Plenty of interesting ideas for dark energy and alternative gravity have been conjectured, but there is no clear front runner. The models are all toys, awaiting any new, corroborating or contraindicating evidence.

Some have argued that no new physics is required, that nonlinear behavior in general relativity may exhibit subtleties that allow for an accelerated expansion. For example, Ref. [145] proposed that superhorizon perturbations may induce accelerated expansion in our observable Hubble patch. This idea has been disproved [146,147,148], but it has not yet been disproven that sub-horizon nonlinearities may explain the observations. Alternatively, it has been suggested that the luminosity-distance-redshift data can be explained if we reside at the center of a Gigaparsec-scale void in an otherwise Einstein-de Sitter Universe. But such a radially-inhomogeneous, anti-Copernican scenario conflicts with other observations [149,150]. Any future proposals that attempt to dispense with new physics will have to explain the vast catalog of phenomena already explained by the standard cosmological model.

New theories of gravitation can in principle work. However, it has proved to be more difficult than may have originally been anticipated to alter gravity to explain cosmic acceleration without violating Solar System constraints. The scalar-tensor or  $f(R)$  theories that do succeed seem contrived and/or manifest themselves in a way that is virtually indistinguishable from a cosmological constant. Braneworld scenarios introduce the possibility of interesting gravitational physics in the Solar System and in large-scale structure, but the simplest models must be ornamented with additional ingredients to work. Generally, alternative-gravity theories that alter the long-range  $1/r^2$  force law may have profound implications for a variety of astrophysical systems, few of which have yet been explored carefully.

The simplest paradigm, quintessence, does not suffer from instabilities, and can be viewed as an effective theory for more complicated models. Quintessence models do require small parameters and/or finely-tuned initial conditions, and they do not address the coincidence problem. Still, the resemblance of some quintessence fields to both fundamental or composite scalars appearing in existing models of physics beyond the Standard Model give the hope that new particle discoveries, at the Large Hadron Collider or beyond, may provide the clues to connect this dark energy field to the world of luminous matter.

The next step for cosmological studies should be to determine if  $w_Q$  departs significantly from  $-1$ . If it does, then the step beyond that will be to measure its time evolution  $w_a$ . The  $w_0$ - $w_a$  measurement may then tell us something qualitative about dark-energy dynamics (e.g., thawing or freezing potentials). If so, we can then go further from there.



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## Appendix A: Acronyms

BAO: Baryon Acoustic Oscillations  
 CMB: Cosmic Microwave Background  
 DETF: Dark Energy Task Force  
 DGP: Dvali-Gabadadze-Porrati  
 GR: General Relativity  
 JBD: Jordan-Brans-Dicke  
 MaVaN: Mass Varying Neutrino  
 PNGB: Pseudo-Nambu Goldstone Boson  
 PPN: Parametrized Post-Newtonian

## Appendix B: Key Terms

*Dark Energy:* A negative-pressure fluid comprising  $\sim 75\%$  of the cosmic energy budget, postulated to account for the accelerated cosmic expansion.

*Quintessence:* A dynamical dark energy. Literally, the fifth element in the cosmic energy budget, beyond radiation, baryons, neutrinos, and dark matter.

*Equation-of-State Parameter:* The ratio of the homogeneous pressure to the energy density, and denoted  $w = p/\rho$ .

*Braneworld:* Scenario in which Standard Model fields are confined to a membrane in a higher-dimensional spacetime, but gravity propagates everywhere.

*Landscape scenario:* The idea that string theory predicts a huge number of false vacua with different but closely spaced vacuum-energy densities.

## Appendix C: Highlighted References

Refs. [1] & [2]: These two papers report the first direct evidence, from supernova measurements of the luminosity-distance–redshift relation, for an accelerated cosmic expansion.

Refs. [5] & [6]: These two articles provide the most up-to-date and detailed reviews of observational probes of the expansion history.

Ref. [10]: This is a comprehensive and detailed review of dynamical models of dark energy.

Ref. [21]: This article provides a cogent explanation of the difficulties in building a dark cosmic scalar field in a realistic model of particle physics.

Ref. [104]: This article provides a clear and detailed recent review of DGP gravity and cosmology.

Ref. [126]: This is a classic review on the cosmological constant.

## Appendix D: Summary Points

1. The cosmic expansion is observed to be accelerating.
2. The physical mechanism responsible for the cosmic acceleration is unknown. Interpreting the observational and experimental evidence in the context of Einstein’s general relativity, the causative agent appears to be an exotic fluid, referred to as “dark energy,” with negative pressure.
3. A cosmological constant is equivalent to such a fluid with a constant energy density. However, the value of this energy density, in units where  $G = c = \hbar = 1$  is  $10^{-120}$ , and there is no good explanation for the smallness of this value.
4. Quintessence postulates that the dark energy is due to the negative pressure associated with the displacement of some new scalar field from the minimum of its potential. Such theories generally predict an equation-of-state parameter for dark energy of  $w_Q \neq -1$ , as opposed to the cosmological constant, which has  $w_Q = -1$ .
5. Other explanations for cosmic acceleration propose that a new gravitational theory supplants Einstein’s general relativity on cosmological scales. However, new theories are tightly constrained by precision tests of gravitation within the Solar System.
6. In the absence of a clear front-runner theory, most efforts are directed towards refining measurements of the cosmic expansion history to determine more precisely the value of  $w_Q$ .
7. A combination of cosmological observations are expected to gain the most traction towards understanding the physics of cosmic acceleration. Most attention has been focused on four techniques: supernovae, baryon acoustic oscillations, cluster abundances, and weak lensing.

## Appendix E: Problems with the Cosmological Constant

A cosmological constant with  $\Lambda = 3\Omega_\Lambda H_0^2/c^2$  provides a phenomenological description of dark energy; it implies that the vacuum “weighs” something; i.e., that the vacuum gravitates. However, there is no physical understanding for why empty space would act as a source for the gravitational field. The particle-physics vacuum contributes an effective cosmological constant, but with an energy density many orders of magnitude larger than is observed. This gross mismatch between theory and observation—noted both by Pauli [152] and Zeldovich [151]—is one of the deepest physical enigmas of our time. In quantum field theory, renormalization allows us to reset the energy density of the vacuum to zero, and for many years it was generally assumed that some mechanism made this cancellation precise and stable. However, the discovery of cosmic acceleration suggests that the cosmological constant is (in the absence of quintessence or some alternative-gravity explanation for cosmic acceleration) small, but non-zero, and this has now changed the character of the cosmological-constant problem. If the observational trend continues to favor dark energy with

$w_Q$  consistent with  $-1$ , the challenge will be to explain why the cosmological constant is so small, yet non-zero.

## Appendix F: Extra resources

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## Appendix G: Future Issues

Will future results from the Large Hadron Collider have any impact on dark-energy theory? Could the discovery of supersymmetry, a nonstandard Higgs, or large extra dimensions change the way we think about dark energy?

Will string theory make a robust prediction for the cosmological constant? or perhaps otherwise explain the physics of cosmic acceleration?

Can an elegant and consistent modification to general relativity be found to explain cosmic acceleration while still satisfying Solar System constraints?

Will there be NASA and ESA satellite missions to study dark energy within 5-10 years?

How much will ground-based observations and experiments refine our knowledge of the physics of cosmic acceleration?

Will new connections between other probes of new physics (e.g., dark-matter searches, gravitational waves, probes of gravity on submillimeter scales? Lorentz-invariance violation) and dark energy be found?

The relevant future observations include measurements of the cosmic expansion history with greater accuracy and studies of the growth of large-scale structure. More work must be done to determine the best avenue forward will be in this regard.

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