Adventures in Friedmann Cosmology: An Educationally Detailed Expansion of the Cosmological Friedmann Equations

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Abstract

The general relativistic cosmological Friedmann equations that describe how the scale factor of the universe evolves are expanded explicitly to include energy species not usually seen. The evolution of a universe as indicated by the Friedmann equations when dominated by a single, isotropic, stable, static, perfect-fluid energy species are discussed and compared. These energy species include phantom energy ($w < -1$), cosmological constant ($w = -1$), domain walls ($w = -2/3$), cosmic strings ($w = -1/3$), normal matter ($w = 0$), radiation and relativistic matter ($w = 1/3$), and a previously little-discussed specie of energy called "ultralight" ($w > 1/3$). A very brief history and possible futures of Friedmann universes when dominated by stable energy species are discussed.

I. THE FRIEDMANN EQUATION OF ENERGY, EXPANDED

The Friedmann equation of energy for a uniform cosmology evolving under general relativity is typically written in the form[1]

$$H^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2}, \tag{1}$$

where H is the Hubble parameter, G is the gravitational constant, ρc^2 is energy density, c is the speed of light, R is a scale factor of the universe, and k is a unitless constant related to the curvature of the universe. Note that $H = \dot{R}/R = \dot{a}/a$, where a is the dimensionless scale factor of the universe such that $a = R/R_o$ and R_o is the scale factor of the universe at some canonical time t_o. An example of R_o is the average distance between galaxies. A general derivation of both the Friedmann equation of energy and the Friedmann equation of acceleration (discussed below) can be found in numerous sources and is not given here.[1].

Average density ρ is typically broken down into the known energy species: matter, radiation, and a cosmological constant. Then

$$\rho = \rho_0 + \frac{\rho_3}{a^3} + \frac{\rho_4}{a^4}. \tag{2}$$

Here, ρ varies as a, and hence time t, as the universe evolves. Classically, $\rho_0 c^2$ is associated with the cosmological constant (sometimes labeled Λ), $\rho_3 c^2$ is the present value of matter energy density, and $\rho_4 c^2$ is the present value of the radiation energy density.

It is instructive, however, to break up the energy density further into even hypothesized energy species. Staying with integer powers of a, then

$$\rho = \sum_{n=-\infty}^{\infty} \rho_n \; a^{-n}. \tag{3}$$

Here, all of the ρ_n values on the right remain fixed to their values at the scale factor: $a = 1$. It will be assumed here that only positive values of ρ_n and a can exist. Define a critical density $\rho_c = 3H^2/(8\pi G)$ and designate $\Omega = \rho/\rho_c$. Dividing each of these by ρ_c at $a = 1$ yields

$$\Omega = \sum_{n=-\infty}^{\infty} \Omega_n \; a^{-n}. \tag{4}$$

As above, Ω varies with a, and hence time t, whereas all of the Ω_n values are fixed at the time when $a = 1$.

Considering only the epoch of $a = 1$ yields

$$\Omega_{\rm total} = \sum_{n=-\infty}^{\infty} \Omega_n. \tag{5}$$

It is instructive to rewrite this sum using popular labels such that

$$\begin{aligned} \Omega_{\rm total} = & \; \Omega_{\rm phantom\ energy} + \Omega_{\rm cosmological\ constant} + \Omega_{\rm domain\ walls} \\ + & \; \Omega_{\rm cosmic\ strings} + \Omega_{\rm matter} + \Omega_{\rm radiation} + \Omega_{\rm ultralight}. \end{aligned} \tag{6}$$

These labels will be explained in detail in the following sections.

The last "curvature" term in Eq. (1) can be resolved into more familiar quantities. Toward this end, divide Eq. (1) by H^2. Therefore, Eq. (1) can be rewritten as

$$1 = \Omega - \frac{kc^2}{H^2 R^2}. \tag{7}$$

Given these inputs, a more generalized Friedmann equation of energy can be written in a dimensionless form that explicitly incorporates all possible stable, static, "integer", isotropic energy species. Eq. (1) is now divided by the square of the Hubble parameter when $a = 1$, dubbed H_o^2, so that

$$(H/H_o)^2 = \sum_{n=-\infty}^{\infty} \Omega_n \; a^{-n} + (1 - \Omega_{\rm total}) \; a^{-2}. \tag{8}$$

Writing out these terms with familiar labels yields

$$\begin{aligned} (H/H_o)^2 \sim & \; \textstyle\sum_{n=-\infty}^{-1} \Omega_{\rm phantom\ energy} \; a^{-n} \\ + & \; \Omega_{\rm cosmological\ constant} \; a^0 + \Omega_{\rm domain\ walls} \; a^{-1} \\ + & \; \Omega_{\rm cosmic\ strings} \; a^{-2} + (1 - \Omega_{\rm total}) \; a^{-2} \\ + & \; \Omega_{\rm matter} \; a^{-3} + \Omega_{\rm radiation} \; a^{-4} \\ + & \; \textstyle\sum_{n=5}^{\infty} \Omega_{\rm ultralight} \; a^{-n}. \end{aligned} \tag{9}$$

The equation can be written to highlight *only* present day observables by further substituting $1/a = R_o/R = (1 + z)/(1 + z_o) = (1 + z)$, where z is redshift and z_o is the redshift at the epoch where $a = 1$.

II. ATTRIBUTES OF THE FRIEDMANN EQUATIONS

A. The Gravitational Horizon

Over what scale lengths do the Friedmann equations operate? The maximum scale length is referred to as the "gravitational horizon." Energy outside of an object's gravitational horizon will have no gravitational affect on that object.[8] There is little observational evidence that limits the maximum size of the current gravitational horizon of our local universe. If the gravitational horizon operates like a light cone, however, it would be expanding at the speed of light.[9] If so, the gravitational horizon of a point in space is exactly the particle horizon for that point, encompassing the volume of space from which light or any relativistic particle could have come to that point since the universe started.[10] Then, in a sense, we can only be gravitationally affected by energy species that we can see.

Inside the gravitational horizon, only anisotropic distributions of energy can cause local energy species to gravitate toward any particular direction. Isotropic energy distributions that surround any particular point, no matter how distant, will not accelerate matter at that point with respect to the rest frame of the universe, as stated by Birkhoff's theorem. Similarly, the expansion rate of the universe inside any given sphere surrounded only by isotropic energy is determined by applying the Friedmann equations to energy distributed *inside* the sphere.[11]

B. The Rest Frame With Respect to the Universe

In cosmology, a "rest frame with respect to the universe" or "cosmic rest frame" is frequently cited. This might seem counter-intuitive, since special relativity has no preferred inertial frames. Every place in the universe has a definable "rest frame," however, delineated by the energy in the universe itself. The universe rest frame, at any point, is the one frame where the average velocity of energy species in the universe is found to be zero. In this frame, for example, the cosmic microwave background (CMB) photons will appear the same in all directions: isotropic. It is usually assumed that all species of energy are at rest, on the average, with respect to all other species of energy. Note that were any specie of energy to dominate the universe, the frame where this energy specie is at rest would determine the rest frame for that universe.

Our Earth is not so privileged so as to be at rest with respect to the CMB frame. Analyses of the dipole of the microwave background seen from Earth shows that our Milky Way Galaxy is moving at about 600 km sec^{-1} with respect to this rest frame.[12]

C. Energy Conservation

What do the Friedmann equations mean in terms of energy conservation over any finite (called "non-local") scale? Very little, actually. The Friedmann equations only define how the energy that is in the universe drives the expansion of the universe. As indicated above, the energy density of any stable energy specie n goes as a^{-n}, decreasing as the universe expands for positive n. Since this energy dilution occurs *everywhere in the universe*, there is no place for the energy to go. Within the confines of the Friedmann equations, measured with respect to the rest frames of the universe, this energy is lost.

One might find solace by demanding that single components of energy remain constant and conserve energy individually as the universe evolves. As will be seen in the following sections, examples where this works include normal matter, cosmic strings, and domain walls. The mass of a single electron, for example, does not change as the universe expands. In these cases, the loss of energy density is still unexplained but it can at least be completely attributed to the geometric expansion of the universe.

However, at least one type of known energy has components that themselves change energy as the universe expands: radiation. Individual pieces of radiation, photons for example, become redshifted and individually lose energy as the universe expands.

More generally, the speed relative to the cosmic rest frame of any freely moving component of any energy specie will decay as the universe expands. This kinetic energy drop related to the reduced speed does not go anywhere – it just disappears from the universe and from the Friedmann equations. In general, a relativistic energy form has an integer specie number n one integer higher than the same type of energy specie at rest. Radiation is just one example of this.

To better understand the kinetic energy loss, start by considering a particle moving at a relativistic speed $\gamma = 1/\sqrt{1 - v^2/c^2}$ with respect to cosmic rest frame. Now consider that this particle was detected at speed v by an observer at redshift z. What will be this particle's speed when it arrives locally, at $z = 0$? The quantity $\gamma v a$ is constant for anything moving with speed v with respect to the cosmic rest frame.[34] Written in terms of redshift, $\gamma va = \gamma v(1+z)^{-1}$. In general, γva is proportional to the momentum of the particle, but for relativistic particles, $v \sim c$ so that the energy of the particle is just c times the momentum. Therefore, for relativistic particles, the kinetic energy of any particle decays as $(1+z)$, just like radiation. As an example, it is straightforward to see that a particle detected with a highly relativistic velocity of $\gamma = 100$ at a redshift of $z = 1$ will be detected to have a $\gamma \sim 50$ locally.

It is interesting to estimate the loss rate of kinetic energy by all non-relativistic $n \sim 3$ matter in the present expanding universe. For non-relativistic matter, $\gamma \sim 1$ so that $\gamma va \sim va$ is approximately constant. Taking the time derivative of this invariant gives $adv/dt + vda/dt = 0$. Therefore $dv/dt = -(v/a)da/dt = -vH$, where it is remembered that $H = (1/a)(da/dt)$.

Now for non-relativistic particles, the kinetic energy density $\rho_K c^2 \sim (1/2)\rho v^2$, where here $\rho = \rho_3 a^{-3}$ stands for the particle density. Therefore the change in the particle kinetic energy density over time is $d\rho_K c^2/dt = (1/2)v^2 d\rho/dt + \rho v dv/dt$. Note that here $d\rho/dt = d(\rho_3 a^{-3})/dt = -3\rho_3 a^{-4} da/dt = -3\rho_3 a^{-3} H$. Combining these relations gives the change in kinetic energy over time as $d\rho_K c^2/dt = (1/2)v^2(-3\rho_3 a^{-3} H) - \rho_3 a^{-3} v^2 H$. Note that the first term gives the amount of kinetic power lost by the diluting mass density, while the second term gives the amount of kinetic power lost to particles slowing. When $a = 1$ at the present epoch, these terms combine to give $d\rho_K c^2/dt = (-5/2)v^2 \rho_3 H_o$.

Applying this to the present universe, it will be assumed that all matter in the universe, including dark matter, has an effective speed relative to the cosmic rest frame of $v = 600$ km sec^{-1}, that matter makes up 30 % of the critical energy density, and that $H \sim H_o \sim 70$ km sec^{-1} Mpc^{-1} today.[3,12] Therefore $\rho_c \sim 3H_o^2/(8\pi G) \sim 10^{-17}$ kg km^{-3} would be critical density today. Given these values, the average kinetic energy density disappearing from matter in the universe in one second, measured today, is about 6 x 10^{-33} kg meter^{-1} sec^{-3}. Given that the energy of a single hydrogen atom is about $m_P c^2 \sim 1.5$ x 10^{-10} kg meter^2 sec^{-2}, this approximately corresponds to the loss of one hydrogen atom over the volume of the Earth for every thirty seconds that pass.

At any one time in the universe, all of the energy is typically constrained to add up to Ω_{total}. When $\Omega_{\text{total}} = 1$, this in itself is a form of energy conservation, saying that the local energy, no matter what form it takes, is just enough to cause spacetime to be flat. A value of $\Omega_{\rm total} = 1$ is indicated by many modern astronomical observations.[2] When $\Omega_{\rm total}$ is constrained to be exactly unity, it cannot be moved away from unity within the bounds of standard Friedmann cosmology, even if all of its constituent Ω_n values vary wildly with z.

In Newtonian gravity, the energy of a rising free body moves from "kinetic" to "potential" energy. Even there, potential energy, although it might be readily converted back to kinetic energy, is a mathematical construct used to balance out energy conservation equations. There is no equivalent potential energy for Friedmann cosmology. In general, energy is not conserved over large distances and times in general relativity.[4]

Mathematically, the general issue of conservation of energy is subtle in general relativity. Let's start with the time component of the $T^{\alpha\beta}$, assuming that energy is conserved

$$\nabla_\mu T^{\alpha\beta} = 0. \tag{10}$$

Since we've taken only the time component of $T^{\alpha\beta}$, Eq.(10) represents only the conservation of energy. The remaining components would yield the momentum conservation along the three spatial directions. Expanding Eq.(10) using the rules of covariant differentiation we obtain

$$\frac{\partial T^{\alpha\beta}}{\partial x^\mu} + \Gamma^\alpha_{\mu\lambda} T^{\alpha\beta} - \Gamma^\beta_{\mu\lambda} T^{\alpha\lambda} = 0. \tag{11}$$

If we were to consider the version of Eq.(11) at a local level, then the Christoffel connections would vanish, leaving only partial derivatives with respect to time. Noether's theorem states that energy is the time translation invariant.[1] A physical quantity, according to the rules of mechanics, is conserved only if the coordinates that it generates are missing from the Lagrangian. Since energy is the generator of infinitesimal time translations, the appearance of time in the above Lagrangian indicates that energy is not conserved over finite times and finite distance scales.

Nevertheless, energy is considered to be conserved on infinitesimal scales (called "locally") in general relativity, on very nearby scales where curvature changes are not important. A mathematical statement of this is that $T^{\alpha\beta}_{;\nu} = 0$, where T is the stress energy tensor that accounts for all the local energy, and the semicolon indicates a type of time derivative.

D. Perfect Fluids and Gravitational Pressure

The Friedmann equations typically quantify the cosmological evolution of perfect fluids. Perfect fluids are characterized by only two variables: energy density (ρ) and pressure (P). These two variables can be isolated in an effective modified Poisson equation for gravity from general relativity in the weak field limit that includes pressure such that[1]

$$\bigtriangledown^2\phi = 4\pi G(\rho + 3P/c^2), \tag{12}$$

where ϕ is the Newtonian potential in the limit of weak field gravity. The gravitational influence of pressure has no Newtonian analogue. In general, the equation of state of any perfect fluid is given by $w = P/(\rho c^2)$ so that $\rho \propto a^{-3(1+w)}$.[5] Therefore, when energy is conserved locally for a perfect fluid, $n = 3w + 3$ and so $w = n/3 - 1$. The terms in the Friedmann equations with $w < 0$ ($n < 3$) indeed have repulsive ("negative") gravitational pressure. Locally conserved perfect fluids will be assumed by default for the rest of this paper.

For a universe dominated by a single specie of perfect fluid energy where w is unchanging, $c_s^2 = \partial P/\partial\rho = w$, where c_s is the sound speed in the perfect fluid. When $w < 0$ ($n < 3$), the sound speed becomes imaginary, causing instabilities. However, instabilities in these cosmological "fluids" may be only on small scales and may not affect the large scale nature of the universe. Therefore, it is common to assume that all perfect fluid energy species act homogeneously on cosmological scales.

Another phenomenological problem occurs for perfect fluids with $w < 1$ ($n < 0$) and $w > 1$ ($n > 6$). Energy species with this property have a formal sound speed greater than the speed of light, possibly making them unphysical.[6]

Perfect fluids are not the only hypothesized types of energy species. One simple alternative is called a "Chaplygin gas" which has $P = -A/\rho$ where A is a positive constant.[7] Given a positive ρ, a Chaplygin gas would also have a positive sound speed. This type of fluid will not be considered further here.

E. The "Other" Friedmann Equation: The Friedmann Equation of Acceleration

The above Friedmann equation of energy is actually only the time-like part of a larger four-vector with three spatial components. Each of the three space-like components is assumed identical here because all energy forms are equivalently considered to be spatially isotropic. The single remaining unique space-like component is referred to as the Friedmann equation of acceleration. Both Friedmann equations are derived from the same set of assumptions. The Friedmann equation of acceleration is less cited and perhaps more difficult to deal with because it is a second order differential equation. The second order, however, indicates the accelerative nature of this equation. The Friedmann equation of acceleration is typically written in the form[1]

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P/c^2). \tag{13}$$

Like the Friedmann equation of energy, the Friedmann equation of acceleration can also be written in a dimensionless form that explicitly incorporates all integer n energy species. To do this, however, one must assume an equation of state that connects pressure to density. The most commonly used equation of state is the above discussed perfect fluid relation where energy is locally conserved. Written in terms of w, an expanded Friedmann equation of acceleration becomes

$$\frac{\ddot{a}}{a} = -\left(\frac{H_o^2}{2}\right)\left(\frac{8\pi G\rho}{H_o^2}\right)(1+3w). \tag{14}$$

As with the Friedmann equation of energy, define a critical density $\rho_c = 3H^2/(8\pi G)$ and designate $\Omega = \rho/\rho_c$. The equation is also written in terms of the present epoch where $a = 1$, $H = H_o$, $n = 3w + 3$, and Ω_n's are fixed at their $a = 1$ values. Then, using Eq. (4),

$$\frac{\ddot{a}}{a} = H_o^2 \sum_{n=-\infty}^{\infty} \left(1 - \frac{n}{2}\right)\ \Omega_n\ a^{-n}. \tag{15}$$

Note that this Friedmann equation of acceleration can also be obtained by taking the time derivative of the Friedmann equation of energy. In this sense, when P and ρ are related by a perfect fluid equation of state and local energy is conserved, the two Friedmann equations are not independent.

An expanded Friedmann equation of acceleration written with popular labels reads

$$\begin{aligned} \frac{\ddot{a}}{H_o^2 a} \sim &\quad \textstyle\sum_{n=-\infty}^{-1}(1-\frac{n}{2})\Omega_{\text{phantom energy}}\ a^{-n} \\ + &\quad \Omega_{\text{cosmological constant}}\ a^0 + (\tfrac{1}{2})\ \Omega_{\text{domain walls}}\ a^{-1} \\ - &\quad (\tfrac{1}{2})\ \Omega_{\text{matter}}\ a^{-3} - \Omega_{\text{radiation}}\ a^{-4} \\ + &\quad \textstyle\sum_{n=5}^{\infty}(1-\frac{n}{2})\ \Omega_{\text{ultralight}}\ a^{-n}. \end{aligned} \tag{16}$$

Note that both the $n = 2$ cosmic string term and the $n = 2$ curvature term, tracked by the $(1 - \Omega_{\rm total})$ term in Eq. (9), are absent as they do not alter the magnitude of acceleration of the universe.

F. Minimally Simplified Friedmann Equations

It is insightful to rewrite the Friedmann equations only in terms of how the scale factor a changes with time when $\Omega_{\rm total} = 1$ and one energy specie n (or $w = n/3-1$) dominates. Then, the Friedmann equation of energy, specifically Eq. (8), can be combined with $H = (\dot{a}/a)$ to yield

$$\dot{a} = H_o\ a^{1-n/2}, \tag{17}$$

and

$$\ddot{a} = H_o^2(1 - n/2)\ a^{1-n}. \tag{18}$$

In terms of w we have

$$\dot{a} = H_o\ a^{-\frac{3}{2}w-\frac{1}{2}}, \tag{19}$$

and

$$\ddot{a} = H_o^2 \left(-\frac{3}{2}w - \frac{1}{2}\right)\ a^{-3w-2}. \tag{20}$$

G. Minimally Simplified Friedmann Solutions

It is insightful to solve the Friedmann equations for $a(t)$ when $\Omega_{\rm total} = 1$ and one energy specie n (or $w = n/3 - 1$) dominates. Solutions are straightforward. The solutions can be generalized when $w > -1$ $(n > 0)$. The Friedmann equation of energy, Eq. (17) can then be written

$$a^{(n-2)/2}\ da = H_o\ dt. \tag{21}$$

Integrating gives

$$\left(\frac{2}{n}\right) a^{n/2} = H_o t + k, \tag{22}$$

where k is an integration constant. For $n > 0$, demanding that $a = 0$ when $t = 0$ yields $k = 0$. Therefore,

$$a = \left(\frac{n}{2}\right)^{2/n}\ (H_o t)^{2/n}. \tag{23}$$

Written in terms of w yields

$$a = \left(\frac{3}{2}w + \frac{3}{2}\right)^{\frac{2}{3w+3}} (H_o t)^{\frac{2}{3w+3}}. \tag{24}$$

Solutions to Friedmann's equations for universe when multiple energy species are simultaneously important are given in Lake.[14]

III. ENERGY SPECIES IN THE FRIEDMANN EQUATIONS

Energy can take any number of different forms – it may never be known how many. Energy is frequently referred to by the form it takes. Species of energy can be classified by how they gravitationally affect the universe in the Friedmann equations. Stable, "integer" energy species change their energy density, as measured relative to the cosmic rest frames, only as an integer power of universe scale factor: $\rho \propto a^{-n}$. Static matter and isotropic radiation are both examples of stable, integer energy species. The number n of an energy specie can be interpreted as a count of the number of effective spatial dimensions in which a unit of an energy specie is confined. Energy species with $w < 0$ ($n < 3$) are typically referred to as "dark energy." A list of all known and hypothesized perfect fluid energy species is given in Table I and will be discussed below. How these stable energy species interact are extrapolated from Vilenkin.[15] and summarized in Table II.

A. Phantom Energy: $w < -1$

Energy species in the Friedmann equations with both negative w and n are called "phantom energy." Phantom energy is considered unconfined in any spatial direction and always pervades the entire universe. Any positive $\Omega_{\text{phantom energy}}$ that multiplies any term with $w < -1$ ($n < 0$) increases in magnitude as the universe expands. If no other phantom energy terms exist, and if the given phantom energy does not undergo a phase transition, then it will eventually grow from any miniscule density to dominate the energy and expansion dynamics of the universe.[16] One reason that phantom energy is controversial is because its formal sound speed, $c_s = c\sqrt{w}$, has a magnitude greater than the speed of light.

To see how the universe evolves with time in $\Omega_{\text{total}} = 1$ cosmologies dominated by phantom energy, one cannot use the single-term general solution Eq. (23) because it applies only to

$w > -1$ ($n > 0$). It is straightforward to integrate Eq. (17), however, which becomes, for $w < -1$,

$$\left(\frac{2}{n}\right) a^{n/2} - \left(\frac{2}{n}\right) = H_o t. \tag{25}$$

Note that t is measured from the time that the $w < -1$ ($n < 0$) phantom energy term begins to dominate the expansion of the universe. Then

$$a = \left[\frac{1}{1 - (-n/2) H_o t}\right]^{2/(-n)}, \tag{26}$$

when written in terms of w becomes

$$a = \left[\frac{1}{1 - \frac{-3w-3}{2} H_o t}\right]^{\frac{2}{-3w-3}}. \tag{27}$$

The universe scale factor a goes to infinity in a finite amount of time, when the denominator approaches zero! That is the time of the "Big Rip".[16] For very negative values of $3w$ and n, the phantom energy rips apart the universe quite quickly after dominating the expansion rate, while for values of $3w$ near -1 (n near zero), the Big Rip will occur far into the future.

B. Volume Energy: Cosmological Constant: $w = -1$

The fundamental type of energy that evolves as $w = -1$ ($n = 0$) in the Friedmann equations has energy distributed uniformly in the volume of space and is known as the "cosmological constant" or Λ. This specie of energy is unconfined in any spatial direction. Note that *not* just any volume of energy would evolve as $w = -1$ energy. Cosmological constant energy must remain isotropic with respect to all rest frames in the universe, even as the universe expands. One *cannot* make $w = -1$ cosmological constant energy by simply spreading $w = 0$ ($n = 3$) particles uniformly. The reason is that these particles would either expand with the universe and so change their average energy density, or move increasingly rapidly with respect to the expanding universe, with the possible exception of a single point.

Energy species with $w = -1$ have been hypothesized as vital components of the universe as far back as Einstein, whose original intent was to create a repulsive gravity component that kept a static universe of $w = 0$ matter particles from collapsing. Since then, a cosmological constant has been suggested a number of times to account for possible anomalies in cosmological data.[18] Often such re-introductions were met with skepticism. Modern supernova data, however, indicate that standard candle supernovae appear less bright than expected with only $w = 0$ matter dominating the universe. These data are well fit with a significant high-density $w = -1$ cosmological constant term.[19,20] Additionally, modern analyses of cosmic microwave background data indicate a flat $\Omega_{\rm total} = 1$ universe which has only a $w = 0$ component of about 30 percent, consistent with $w = -1$ making up the rest.[2] Both of these measurements are also consistent with modern galaxy clustering estimations of w and have led to a modern "concordance model" where an $w = -1$ component makes up about 70 percent of the energy of the universe.[21]

Cosmological constant $w = -1$ energy has gravitationally repulsive pressure. Although the energy density of $w = -1$ matter is positive, and is gravitationally attractive, Eq. (13) shows that the repulsive pressure is three times more forceful. This repulsive pressure effectively accelerates the expansion of the universe. A summary of how $w = -1$ energy interacts with other energy species is given in Table II.

Suppose "cosmological constant" $w = -1$ dark energy could somehow be completely contained inside non-interacting boxes of constant size – would these boxes still cosmologically evolve as $w = -1$? The answer is "no." As the universe expands, each box would still contain the same amount of $w = -1$ energy, but the energy density between the boxes would be zero. As the boxed got further apart, the number density of boxes would decrease like $w = 0$. Therefore, all of the boxes together would evolve as $w = 0$.

In a similar vein, were a small box containing cosmological constant energy dropped near the Earth, it would fall just as if the box were filled with any small amount of $w = 0$ particles. That is one realization of the Equivalence principle.

It is not possible for pervasive $w = -1$ energy to move with respect to the universe.[22] Conversely, it is also not possible, for example, for particles of $w = 0$ matter to move with respect to pervasive $w = -1$ energy. Therefore, for example, motion of $w = 0$ matter in a universe with $w = -1$ energy will yield no drag or accelerative aberration force.

The Friedmann equation of energy for the case where a $w = -1$ $(n = 0)$ energy specie solely dominates an $\Omega_{\rm total} = 1$ universe is simply solved analytically. The solution is *not* given by Eq. (24) which applies only when $w > -1$. The solution is easily found by directly integrating Eq. (19) which yields

$$\ln a = H_o t + \ln a_o, \tag{28}$$

where t can be considered the time since $w = -1$ energy began to dominate the expansion, when $a = a_o$. The solution can then be written

$$a = a_o \, e^{H_o t}. \tag{29}$$

This is a classic solution where a cosmological constant drives an exponentially accelerating universe. The universe is said to be in a "de-Sitter" phase, and the Hubble parameter H is a constant H_o. Such a phase is hypothesized to have dominated the early universe in a phase called "inflation."[23] Our universe appears to be slowly re-entering into another such phase now.

C. Sheet Energy: Domain Walls: $w = -2/3$

A fundamental type of energy that evolves as $w = -2/3$ ($n = 1$) in the Friedmann equations has energy confined to sheets. For $w = -2/3$, the sheet density distribution must be isotropic in all cosmic rest frames and at all times of an evolving universe. The term "sheet" is used to mean that the thickness of the contained energy in one dimension is small compared to the gravitational horizon size. Note that *not* just any sheet of energy would dilute as $w = -2/3$ energy. A sheet of energy that acts as $w = -2/3$ is by definition not moving in the direction perpendicular to its area, with respect to the rest frames of surrounding universe, at every point along its entire surface, at all times. The sheet also retains its surface energy density as the universe expands. This would not be possible, for example, for a sheet of $w = 0$ ($n = 3$) particles. Such a sheet that started in the rest frame of the universe at all points on the sheet would quickly dilute in average surface energy density if it expanded with the universe. If the sheet of $w = 0$ particles did not expand with the universe, only a single point in the sheet would be able to remain in the rest frame of the universe – points on the sheet far from this center would quickly be seen to moving relativistically, relative to local matter, as the universe expanded.

Therefore, $w = -2/3$ sheet energy is intrinsically different than a sheet of uniformly spread $w = 0$ particles, and any other energy specie w. Such a $w = -2/3$ energy specie is called a (cosmological) *domain wall*.[24] A domain wall might arise as a topological defect that occurred during a phase transition in the early universe. Note that since the surface energy density in the $w = -2/3$ sheet remains constant as the universe expands – the sheet does not "stretch" to become thinner.

Were domain walls to form before the last epoch of inflation, they would likely have been diluted by inflation to be too sparse to be cosmologically observable or important today. No credible claim for the detection of a cosmological domain wall has ever been made. Nevertheless, attempts have been made to describe the expansion history of our universe with a network of domain walls.[25,27]

The general concept of domain walls is found in other branches of physics. A general physics definition of (non-cosmological) domain wall is a region between two volumes that formed slightly differently. A common example of a non-cosmological domain wall can be seen (and heard!) in a glass of ice when water is poured into it. Many times, an ice-cube will "crack" and show internal sheets that are defects in the pure ice. These cracks have likely formed among (several) domain walls in the ice (W. Cantrell 2006, private communication). Ice domain walls are not expected to have negative gravitational pressure.

Note that if a person were to go up to a flat, static, cosmological domain wall, that person would feel gravitational repulsion from it.[15] Although the positive energy density in a domain wall is gravitationally attractive, the stronger negative pressure will result in a net gravitational repulsion, as indicated in Table II.

Domain walls would act differently, in a cosmological sense, if they were curved.[15,26] First assume that it is possible that domain walls could be curved into balls.[15] This situation is similar to containing any other specie of energy in a $w = 0$ box. As the universe expands, each "domain ball" would still contain the same amount of energy, but the energy between the balls would be zero. The number density of balls would therefore decrease like $w = 0$ particles as the universe expanded. Therefore, all of the balls together would evolve in the Friedmann equations as $w = 0$.

Were a small box containing a domain ball dropped near the Earth, it would fall just as if the box were filled with any small amount of $w = 0$ particles.[15] That is one aspect of the Equivalence principle.

An intermediate case between domain walls and domain balls occurs when a domain wall obtains curvature on the scale of the gravitational horizon size. This sheet curvature will create an energy specie between domain walls and domain balls, with an effective $-2/3 < w < 0$ and so $1 < n < 3$.

A domain wall moving perpendicular to its plane will increase w so that $-2/3 < w < -1/3$. The increase in w derives from the same physics that causes photons to lose energy – the wall continually moves into frames where it has less speed relative to the cosmic standards of rest. When moving relativistically, the w of a domain wall would actually approach $w \sim -1/3$ ($n = 2$).

Were a domain wall energy sheet rolled up into a static, straight, hollow tube, so that radius of the tube is small compared to the gravitational horizon but the length of the tube is long compared to the gravitational horizon, this resulting "domain tube" would act like cosmological $w = -1/3$ "string" matter, as described below.

When domain walls dominate the Friedmann equations, then $H^2 = \Omega_1 a^1$ so that $(\dot{a}/a)^2 \sim a$. Solving for a as a function of time it is straightforward to see that $a \sim t^2$.

D. Line Energy: Cosmic Strings: $w = -1/3$

A fundamental type of energy that evolves as $w = -1/3$ ($n = 2$) in the Friedmann equations has energy confined to lines. The term "line" is used to mean that the confinement radius of the energy in two dimensions is small compared to the gravitational horizon size. Note that *not* just any line of energy would dilute as $w = -1/3$ energy. For $w = -1/3$, the line density distribution must be isotropic in all cosmic rest frames and at all times of an evolving universe. A line of energy that acts as $w = -1/3$ is by definition not moving in the direction perpendicular to its length, with respect to the rest frames of the universe, at every point along its length, at all times. The line also retains its surface energy density as the universe expands. This would not be possible, for example, for a line of $w = 0$ ($n = 3$) particles. Such a particle line that started in the rest frame of the universe at all points in the line would quickly dilute in energy density as $w = 0$ if it expanded with the universe. If the $w = 0$ line did not expand with the universe, only a single point in the line would be able to remain in the rest frame of the universe – points on the line far from this "center" would quickly be seen to moving relativistically, relative to local matter, as the universe expanded.

Therefore, a $w = -1/3$ line energy is intrinsically different than a line of uniformly spread $w = 0$ particles, and any other energy specie w. Such a $w = -1/3$ energy specie is called a *cosmic string*.[26] A cosmic string might arise as a topological defect that occurred during a phase transition in the early universe.[24] Note that since the linear energy density in the $w = -1/3$ string remains constant as the universe expands – the string does not "stretch" to become thinner.

Were cosmic strings to form before the last epoch of inflation, they would likely have been diluted by inflation to be too sparse to be cosmologically observable or important today. No enduring claim for the detection of a cosmic string has yet been made.

The general concept of topological defect strings is not foreign to other branches of physics. A general physics definition of a topological string is a region between two areas that formed slightly differently. A common example of non-cosmological topological string can be seen on a pond when it freezes to ice. Many times, sheets of surface ice meet along wandering boundaries that can be considered strings.

Note that if a person were to go up to a straight, static, cosmic string, that person would feel no gravitational attraction toward it or away from it.[15] This is indicated in Table II. Although the positive energy density in a cosmic string is gravitationally attractive, the negative pressure is gravitationally repulsive at the same magnitude, so that the combined result yields no local gravitational effect.

How would cosmic strings act, cosmologically, if they were curved into closed loops? This situation is similar to containing any other specie of energy in a box. As the universe expands, each cosmic string loop would still contain the same amount of energy, but the energy between the loops would be zero. The number density of loops would therefore decrease like $w = 0$ matter particles. Therefore, all of the loops together would evolve as $w = 0$ particles as the universe expanded.

In a similar vein, were a small box completely containing a cosmic string loop dropped near the Earth, it would fall just as if the box were filled with any small amount of $w = 0$ particle energy. That is one aspect of the Equivalence principle.

A string with significant curvature with a scale less than the gravitational horizon will act like an energy specie intermediate between no-curvature, long cosmic strings, and high-curvature, short string loops. The curved string would have an effective equation of state parameter $-1/3 < w < 0$ and hence an effective number of confined dimensions $2 < n < 3$.

A cosmic string moving perpendicular to its length will increase w so that $-1/3 < w < 0$. The increase in w derives from the same physics that causes photons to lose energy – the string continually moves into frames where it has less speed relative to cosmic standards of rest. When moving relativistically, the w of a cosmic string would actually approach $w \sim 0$ ($n = 3$).

When cosmic strings dominate the Friedmann equations, then $H^2 = \Omega_2 a^{-2}$ so that $(\dot{a}/a) \sim a^{-1}$. Solving for a as a function of time it is straightforward to see that $a \sim t$.

E. Point Energy: Matter: $w = 0$

A fundamental type of energy that evolves as $w = 0$ ($n = 3$) in the Friedmann equations has energy confined to a point. The term "point" is used to mean that the confinement radius of the energy in all three spatial dimensions is small compared to the gravitational horizon size. Examples of $w = 0$ particles are numerous and include the reader, baryonic matter, dark matter, massive fundamental particles, and topological defects like magnetic monopoles.[28,29] Conglomerates of $w = 0$ particles such as heavy nuclei, molecules, and black holes also act cosmologically as $w = 0$. These particles may be held together by some fundamental force, such as the strong nuclear force, electromagnetism or even gravity itself.

As indicated in other sections, $w = 0$ components can even be made from containing other energy species. Volume energy $w = -1$ ($n = 0$) could act like $w = 0$ energy were it somehow able to be confined to finite three-dimensional boxes. Domain wall $w = -2/3$ ($n = 1$) energy could act like $w = 0$ point energy where the domain wall curled up into a compact object. Cosmic string $w = -1/3$ energy could act like $w = 0$ point energy were the string curled up into a compact object. Even $w = 1/3$ radiation could be confined to a small mirrored box and act as $w = 0$ energy.

Note that if one were to go up to a static $w = 0$ energy component, one would feel the classic gravitational attraction to it that Newton discovered over 300 years ago: $F = GMm/r^2$. Static energy species with $w = 0$ exhibit no gravitational pressure.

A particle moving in any spatial direction will increase w so that $0 < w < 1/3$. When moving relativistically, the w of a particle would actually approach $w \sim 1/3$ ($n = 4$) and be considered a form of radiation.

It is interesting to wonder what the w of the average baryonic particle today in the universe. Consider that the average speed of matter in our universe relative to the cosmic rest frame is v. Now the momentum p of a particle moving with speed v is $p = m_o v/\sqrt{1 - v^2/c^2} =$

$m_o v\gamma$. Here m_o is the rest mass of a particle with rest energy $E_o = m_o c^2$. Now w is the ratio between isotropic pressure and density. Averaged over all three spatial dimensions, $P = pc^3/3$ so that $w = P/(\rho c^2) = pc/(3E_o) = m_o v c\gamma/(3m_o c^2) = (v/3c)\gamma$. Given our Galaxy's measured present speed of 600 km sec^{-1} with respect to the cosmic rest frame is typical of all particles in the universe, then $w \sim 6.7 \times 10^{-7}$. Written in term of n, use $\Delta n = 3\Delta w = 2 \times 10^{-6}$, so that, for the average particle in the present universe, $n \sim 3.000002$.

When particles dominate the Friedmann equations, then $H^2 = \Omega_3 a^{-3}$ so that $(\dot{a}/a) \sim a^{-3/2}$. Solving for a as a function of time it is straightforward to see that $a \sim t^{2/3}$.

F. Radiation: $w = 1/3$

A fundamental type of energy that evolves as $w = 1/3$ ($n = 4$) in the Friedmann equations has energy confined not only in all three spatial dimensions but moving relativistically with respect to its local rest frame in the universe. This familiar energy specie is known as "radiation." Possibly the most common example is electromagnetic radiation, but other commonly discussed examples include gravitational radiation and neutrinos.

Hypothetically, any $w = 0$ particle could be accelerated to near the theoretical speed limit dubbed "speed of light" and subsequently act as effective $w = 1/3$ radiation in the cosmological sense of the Friedmann equations. Additionally, $w = 1/3$ radiation components can be made from any energy specie that can be spatially confined in three dimensions and boosted to a relativistic speed.

Does any radiation have $w = 1/3$ exactly? The authors take the didactic view in this section that all radiations have some amount of particle rest mass, even if it is minuscule, has no aggregate cosmological significance, and even no reasonable possibility of being measured. Therefore, just as particle energy typically has some speed and so a w slightly greater than zero, radiation energy might have some rest mass and so a w slightly smaller than 1/3. The present rest mass limits on radiation include the photon[30] at 1.2×10^{-54} kg, the neutrino[31] at about 1.8×10^{-42} kg, and gravitational radiation[33] at 4.5×10^{-69} kg.

This gravitational pressure of $w = 1/3$ radiation is positive. Because of gravitational pressure, radiation gravitates twice as strongly as $w = 0$ matter energy with the same average energy density. This caused radiation dominated epochs of our early universe to expand more slowly than subsequent matter and cosmological constant epochs.

Suppose all radiation could be completely contained inside static non-interacting boxes – would it still cosmologically evolve as $w = 1/3$ radiation? The answer is "no." As the universe expands, each box would still contain the same amount of radiation energy as measured inside each box. There is no impetus for the trapped radiation to lose energy as the universe expands. The energy density between the boxes would remain zero, however. Therefore, the number density of boxes would decrease, as the universe expands, as the number of spatial dimensions: $n = 3$. All of the boxes together would therefore evolve as $w = 0$ particle energy.

Were a small box containing radiation dropped near the Earth, it would fall just as if the box were filled with $w = 0$ energy.[32] That is one aspect of the Equivalence principle.

When radiation dominates the Friedmann equations, then $H^2 = \Omega_4 a^{-4}$ so that $(\dot{a}/a) \sim a^{-2}$. Solving for a as a function of time it is straightforward to see that $a \sim t^{1/2}$.

G. Ultralight: $w > 1/3$

Are there any other types of energy in the universe? The only remaining realm for stable energy species have $w > 1/3$ ($n > 4$). This specific possibly was discussed and limited recently, where such energy species were referred to as *ultralight*.[35] The term ultralight energy contrasts with dark energy as being beyond light in terms of attractive gravitational pressure. In analogy, ultralight is to light what ultraviolet light is to violet light.

The idea of energy evolving with effective $w > 1/3$ has been hypothesized before in the context of time-varying scalar fields.[36] An epoch with $w = 1$ ($n = 6$) might exist were a scalar field to slow the expansion rate of the universe as it exited inflation.[37,38] A contracting universe dominated by a scalar field with effective $w > 1$ would create homogeneity similar to an expanding, inflating universe.[39] The stable ultralight species discussed in this section, however, are not scalar fields.

Species of ultralight with $w > 1$ might be unphysical because their formal sound speed, $c_s = c\sqrt{w}$, is greater than the speed of light. Only ultralight species with $1/3 < w < 1$ ($4 < n < 6$) would have a sound speed less than c.

A particularly mundane specie of ultralight energy has $w = 2/3$ ($n = 5$). All of the known stable energy species and all of the energy species described above have integer powers of $3w$ and n. In fact, the next in the linear integer progression after $n = 4$ radiation is $n = 5$

ultralight. The $w = 2/3$ ultralight would dilute cosmologically as a^5, also an integer power of the universe scale factor a. Furthermore, $w = 2/3$ ultralight can be a stable perfect fluid as its sound speed c_s is neither imaginary nor greater than c. Therefore, the $w = 2/3$ ultralight energy specie will be highlighted here as particularly interesting.

Why would any energy form act with a $w > 1/3$? Although there are clear dimensional paths to understanding most other integer $3w$ species, there is no such clear path for understanding ultralight. Possibly, ultralight might respond, at least partly, to other spatial dimensions, such as those spatial dimensions hypothesized in string theory[40] or Randall-Sundstrum cosmologies.[41] If so, such energy species might only act as lower w energy species as these dimensions became cosmologically unimportant. Alternatively, ultralight might somehow incorporate multiple sensitivities to the time dimension.

The gravitational pressure of $w > 1/3$ ultralight is attractive, so that $w > 1/3$ ultralight cosmologically gravitates more strongly than any other specie of energy.

Were a small box containing and confining some ultralight energy dropped near the Earth, it would fall just as if the box were filled with any small amount of $w = 0$ particle energy. That is one aspect of the Equivalence principle.

When $w = 2/3$ ($n = 5$) ultralight dominates the Friedmann equations, then $H^2 = \Omega_5 a^{-5}$ so that $(\dot{a}/a) \sim a^{-5/2}$. Therefore, as the universe expands, ultralight dilutes even faster than light. Solving for a as a function of time it is straightforward to see that $a \sim t^{2/5}$.

Ultralight is not considered a candidate to make a significant contribution to the energy budget of the universe today, although ultralight might have affected the universe in the distant past. The reason derives from the high attractive gravitational pressure of ultralight would slow the expansion of the universe and extend the duration of epochs when ultralight dominated. The strongest present limit on the $w = 2/3$ ultralight energy specie comes from primordial nucleosynthesis results.[35] Were nucleosynthesis one of these epochs, the longer duration would create a higher fraction of heavy elements than detected today. Given that $\Omega_{\rm radiation} \sim 2 \text{ x } 10^{-5}$ in today's universe,[1], $\Omega_{\rm ultralight} < 10^{-11}$ today.[35]

H. More Complicated Forms of Energy

Besides energy species that change energy density only as a simple power-law of universe scale factor a, other forms of energy might exist. One set of possibilities includes unstable topological defects of higher order than monopoles such as textures.[45,46] Another set of possibilities involves energetic scalar fields such as the hypothesized field that fueled inflation, vector fields such as the electromagnetic field, or even tensor fields, like the general relativistic formulation of gravity. A simple such field is a quantum scalar field typically designated ϕ. Such fields may respond to a potential energy function $V(\phi)$ that may be driven on its own time scale and not be simply related to the universe scale factor. When invoked to explain dark energy observations in cosmology, a scalar field may be called *quintessence*.[47,48]

The Friedmann equation of energy dominated by such a scalar field ϕ related to potential V is written as[47]

$$H^2 = (1/2)\dot{\phi}^2 + (1/2)\bigtriangledown\phi^2 + V(\phi). \tag{30}$$

These fields typically define an equation of state such that

$$w = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}. \tag{31}$$

Although V's exist that make $3w$ an integer, such cases are exceptional. Note from this equation that any evolving scalar field where the potential $V(\phi) << \frac{1}{2}\dot{\phi}^2$ for any phase will effectively have $w = 1$ during that phase.[36] Alternatively, when $V(\phi) >> \frac{1}{2}\dot{\phi}^2$, then $w = -1$ for any finite potential V. The free parameters allowed by these cosmological fields give these fields the potential of being significantly more complicated than the "stable" fields considered here in other sections. They will not considered further here.

IV. CHANGES BETWEEN ENERGY SPECIES

A. Slow Changes

Even stable energy species do not always stay in the same form. Changes between stable energy species that occur on a time scale comparable to the total expansion time of the universe can be referred to as "slow" changes. One clear example of this has been discussed frequently here: the continuous cosmological slowing of energy moving relative to the cosmic rest frame as the universe expands. Given a monotonically expanding universe, even radiation will eventually decelerate to appear nearly at rest with respect to the cosmic rest frames so that all $w \sim 1/3$ radiation will eventually become $w \sim 0$ matter.

It is interesting to wonder how long will it be before even today's $w \sim 1/3$ photons morph into $w \sim 0$ particles nearly at rest with respect to the expanding universe. This strongly depends on the unknown rest mass of the photon and the possibly unknowable expansion trajectory of the future universe. Nevertheless, a grossly approximate lower mass limit can be estimated assuming the present expansion rate applies indefinitely into the future and assuming that photons have the rest mass right at the present experimental limit.

For photons, $m_o \sim 10^{-54}$ kg is a recent upper limit of the rest mass.[30] This corresponds to a rest energy of $E_o = m_o c^2$ of about 10^{-65} kg meters^{-2} sec^{-1}. Let's assume that all photons have just this rest mass. The total energy of a visible photon is about $E = 10^{-18}$ kg meters^{-2} sec^{-1}. Remember that $E = E_o \gamma$, where $\gamma = 1/\sqrt{1 - v^2/c^2}$. Therefore, $\gamma = E/E_o \sim 10^{47}$. Expanding γ and solving for v shows that these photons are going about $(1 - 10^{-94})c$, where c is the fastest possible speed.

Recall again the cosmological relation for matter moving at speed v relative to the cosmic rest frame: $v\gamma = v_a \gamma_a a$, where the right side quantities are measured at in the cosmic rest frame when the universe has expansion factor a, and the left side quantities are measured at the cosmic rest frame when $a = 1$. The relative expansion factor needed to bring photons to near their rest mass is therefore about $a \sim \gamma_a \sim 10^{47}$. Specifically, this will bring photons down to a speed where $(v/c)\gamma \sim 1$, so that $v = c/\sqrt{2}$.

Given a "de Sitter" cosmology where $w = -1$ energy dominates and $H = H_o$ remains constant into the future, the corresponding time this would take would be given by Eq. (29) so that $a/a_o = a/1 = e^{H_o t}$. Therefore, $t = (\ln a)/H_o \sim 108/H_o$ or about 108 "Hubble times." Given that $1/H_o \sim 1/(70 \text{ km sec}^{-1} \text{ Mpc}^{-1}) \sim 1.4 \text{ x } 10^{10}$ years, photons may become non-relativistic about 1.5 trillion years in the future.

Other methods exist where one energy species can slowly morph into another. One example of this is an energy specie that slowly becomes confined as the universe evolves. Let's take cosmic strings as examples. Were $w = -1/3$ static cosmic strings to be slowly increasing their curvature scale length relative to scale length of the gravitational horizon, these strings might morph into loops wholly contained within the gravitational horizon, and so act like $w = 0$ matter particles so far as the Friedmann equations are concerned. The same logic holds for domain walls. Conversely, the gravitational horizon itself might expand from where the curvature of strings and walls were effectively negligible, to where strings and walls are effectively curved, yielding the same effect.

B. Fast Changes

Stable energy may undergo a rapid change of species. Possibly the most familiar transitions between energy species occurs for when energy shifts between $w = 0$ ($n = 3$) matter and $w = 1/3$ ($n = 4$) radiation. Examples include atomic, molecular, collisional, or nuclear transitions between $w = 0$ matter particles that result in the emission of $w = 1/3$ radiations such as photons. Conversely, $w = 1/3$ radiations such as photons could themselves collide and create $w = 0$ matter. When considering a rapid transition between energy forms in general, though, the magnitude of energy *densities* are equivalent and do not depend the magnitude or sign of the gravitational pressure P.

Energy species can also change into each other rapidly during a phase transition in the universe. Once such rapid phase transition occurred at the end of inflation, when dark energy, presumably $w = -1$ energy, rapidly changed into (mostly) $w = 1/3$ radiation energy.

A leading term in the fast transition of $w = 0$ ($n = 3$) matter to $w = 1/3$ radiation is fusion in the central regions of stars. The power density would then be $d\rho_*/dt = n_* L_*$, where n refers to number density, L refers to absolute luminosity, and the subscript * designates stars. Assume there are $n_* \sim 10^{-69}$ galaxies meter^{-3} x 10^{11} stars galaxy^{-1}, and that each star produces on average $L_* = 4$ x 10^{26} kg meter^2 sec^{-3}. The total average power converted would then be on the order of $n_* L_* \sim 4$ x 10^{-32} kg meter^2 sec^{-3}. Given that the energy of a single hydrogen atom is about $m_P c^2 \sim 1.5$ x 10^{-10} kg meter^2 sec^{-2}, this approximately corresponds to a conversion of one hydrogen atom to radiation over the volume of the Earth for every four seconds that pass.

V. THE PAST AND FUTURE ACCORDING TO THE FRIEDMANN EQUATIONS

The Friedmann equations may be wrong or incomplete. Other classes of solutions are occasionally considered. For example, one popular alternative to the classical Friedmann energy equation has $H^2 = A\rho + B\rho^b$, where $b < 2/3$ is a time dependent number. A universe so described is labeled Cardassian.[50] For Friedmann cosmologies, $b = 0$.

Nevertheless, no exceptions to the Friedmann equations have been found on cosmological scales. So far as humanity's present ability to measure and understand, the Friedmann equations have been able to explain the overall evolution of our universe. Therefore, the history of the universe and its future, as extrapolated from the Friedmann equations, is given below.

A. A Really Short History of the Universe in Terms of Dominating Energy Species

The flatness of the universe and the homogeneity of the microwave background do not tell us details about the past history of the universe. If these attributes are caused by the expansion history of the universe, they tell us that the universe must have spent a significant amount of time in a $w < -1/3$ ($n < 2$) mode in order to create the flatness and the homogeneity seen today.

Using the above framework, however, a particularly brief summary of the history of the universe is possible that highlights primarily which stable energy specie dominated the Friedmann equations. A summary of the this section is given in Table III. So far as modern cosmological data discerns, the furthest back that the universe can be traced is to the Planck time, an epoch where all the known forces have the same strength, and where $w = 1/3$ ($n = 4$) radiation dominated. The Planck time occurred at about $t_P \sim 10^{-43}$ sec after an extrapolated time where the density would be formally infinite, called the "Big Bang." At the Planck time and for all times afterwards, the universe expanded.[51]

The general lack for the present need of any $(1-\Omega_{\text{total}})$ "curvature" term in the Friedmann equations indicates that the universe entered into at least one "inflationary epoch" where some sort of dark energy with $w < -1/3$ dominated in the past. When any energy specie that has $w < -1/3$ dominates, Ω_{total} for the universe moves toward unity. It is commonly assumed, though, and will be assumed here, that this dark energy had $w = -1$ ($n = 0$). If inflation was related to the grand unification energy scale of forces, it likely started at about a time near 10^{-35} seconds after the Big Bang.[1]

At the end of inflation, the $w = -1$ energy decayed into a mixture of $w = -1$, $w = 0$, and $w = 1/3$ energy species, with the $w = 1/3$ relativistic energy specie dominating the energy density and determining the expansion rate. Starting at a time of about 10^{-32} seconds after the Big Bang, this was the start of the "radiation epoch."

As the universe further expanded, $w = 1/3$ energy density diluted relative to the $w = 0$ matter energy density and the still diminutive $w = -1$ energy density. In time, the $w = 1/3$

radiation density diluted below the $w = 0$ ($n = 3$) matter energy density. At a time about 70,000 years after the Big Bang, this was the start of the "matter epoch" of the universe.

As the universe continued to expand, the $w = 1/3$ radiation energy density further diluted to cosmological insignificance, as it remains today. Today, the $w = 0$ matter energy density has now diluted so much that the remaining constant $w = -1$ dark energy density is again beginning to dominate the total energy density of the universe, and hence the expansion rate of the universe.

B. Big Rip, Big Freeze, or Big Crunch: Possible End States for the Universe

Assuming the universe remains dominated by stable energy species, there are only three possible end states. All of them are extrapolated from the Friedmann equations. The first is called a Big Rip where a gravitationally repulsive phantom energy – that pervades everywhere – grows to infinite magnitude and so rips everything apart. The next possibility is a Big Freeze, where the universe expands forever, everything slows down and cools off, and eventually the universe is a sea of expanding static, stable energy units. The last possibility is a Big Crunch. Here the universe expands for only a finite time and then re-collapses in a heat bath that compresses everything together toward infinite density. Which of these possible fates awaits our universe is discussed below and summarized in Table IV.

If the dominant energy specie somehow changes into other stable energy species, other futures for our universe are possible. These include oscillating universes, where the universe re-expands again after a Big Crunch,[39] and a cascading universe where a small phantom energy will grow to dominate the universe until it decays into daughter products that contain yet another phantom energy.[17]

C. The Future of a Flat, Minimal Universe

An instructive, minimal case occurs for a "flat" universe, when $\Omega = 1$ in one energy specie. Here the Friedmann equations of energy and acceleration take on very simple forms, namely those shown in Eq. (17) and Eq. (18) and their solutions given in the above specie-specific solutions. Inspection of these minimal Friedmann equations shows with great clarity how the universe evolves when dominated by different energy species. The end states are summarized in Table IV.

Starting at the lowest w, when a $w < -1$ (and hence $n < 0$) phantom energy specie dominates, the universe expands, the magnitude of the speed of expansion continually increases, and the magnitude of the acceleration of the expansion increases faster than exponentially. This universe will end in a finite time in a Bip Rip.

When a $w = -1$ ($n = 0$) cosmological constant energy specie dominates the universe, the magnitude of the speed of the expansion continually increases exponentially, and the magnitude of the acceleration of the expansion continually increases exponentially. This universe will end in an infinite time in a Big Freeze.

When a $w = -2/3$ ($n = 1$) domain wall energy specie dominates, the magnitude of the speed of expansion continually increases, but the magnitude of the acceleration of the universe remains a positive constant. This universe will also end in an infinite time in a Big Freeze.

When a $w = -1/3$ ($n = 2$) cosmic string energy specie dominates, the universe "coasts" at constant expansion speed but zero acceleration. This universe will also end in an infinite time in a Big Freeze.

When a $w = 0$ ($n = 3$) particle energy specie dominates the universe, the magnitude of the expansion speed continually decreases to zero, as does the magnitude of the acceleration of the expansion. The universe will asymptotically approach a Big Freeze. Finally, the same analyses holds true of a $w = 1/3$ ($n = 4$) radiation dominated universe, as well as any ultralight $w > 1/3$, ($n > 4$) universe.

D. The Future of a Curved, Minimal Universe

We next look at universes also dominated by a single energy specie but where $\Omega \neq 1$, so that a curvature term is present. According to the Friedmann equation of energy, the future of the expansion speed of the universe might then become a competition between the term in Eq. (21) that describes the evolution of this energy specie, and the trailing "curvature" term. Curvature will not affect the magnitude of the acceleration of the expansion. Table IV summarizes these possibilities.

Let's first look at energy species that dilute slower than a^{-2}. These have $w < -1/3$ and $n < 2$. These energy species will dominate the curvature term, which goes as a^{-2}. In these universes, it does not matter what Ω_{total} and the curvature is at any time – the universe will always expand and qualitatively act as the $\Omega = 1$ universe described above, ending either in a Big Rip or a Big Freeze.

In the case of a universe dominated by cosmic strings, there is a tie between the energy term and the curvature term – both go as a^{-2}. Qualitatively, however, this universe will evolve as a $\Omega = 1$ universe with no curvature, as described above, ending in a Big Freeze.

In the case of a universe dominated by matter, radiation, or ultralight, a finite curvature term will eventually grow to dominate any of these terms as the universe expands. If $(1 - \Omega_{\text{total}})$ is positive, then $\Omega_{\text{total}} < 1$, and the universe will expand forever and end in a Big Freeze. This is called an "open universe."

If $(1 - \Omega_{\text{total}})$ is negative, then $\Omega_{\text{total}} > 1$, and eventually the expansion rate $\dot{a}$ will fall to zero. At that time a maximum expansion scale factor of the universe can be found. Past this time, these universes will collapse into a Big Crunch.

It is also useful to understand that any universe that spends a significant amount of time being dominated by a $w < -1/3$ $(n < 2)$ energy phase drives Ω_{total} toward unity. Alternatively, a universe that spends significant time being dominated by a $w > -1/3$ phase drives Ω_{total} away from unity.

If present observations continue to support the concordance cosmology where 70 percent of the energy in the universe is in a form where $w = -1$, and if the energy species in the universe today are stable, then the universe should continue to expand and the average w of the universe will continue to approach -1. The fate of the universe will then be a Big Freeze. It is possible that the $w = -1$ energy will evolve as a field and/or decay into energy species with higher w. If so, the ultimate future of the universe is really unknown.

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w	n	Dimensional Type	Name
<-1	<0	unknown	Phantom energy
-1	0	volume	Cosmological constant
-2/3	1	sheet	Domain wall
-1/3	2	line	Cosmic string
0	3	point	Matter
1/3	4	relativistic point	Radiation
>1/3	>4	unknown	Ultralight

TABLE I: Energy species are listed that may drive the expansion rate of the universe as described by the Friedmann equations. The entries include known and hypothesized energy species that are considered to be stable perfect fluids over cosmological scales.

Energy Specie	Phantom Energy	Cosm. Constant	Domain Walls	Cosm. Strings	Particle Matter	Radiation Energy	Ultralight Energy
Phantom	Repel	Repel	Repel	Repel	Repel	Repel	-
Cosm. Const.	Repel	Repel	Repel	Repel	Repel	Repel	-
Domain Walls	Repel	Repel	Repel	Repel	Repel	Neutral	Attract
Cosm. String	Repel	Repel	Repel	Neutral	Neutral	Attract	Attract
Matter	Repel	Repel	Repel	Neutral	Attract	Attract	Attract
Radiation	Repel	Repel	Neutral	Attract	Attract	Attract	Attract
Ultralight	-	-	Attract	Attract	Attract	Attract	Attract

TABLE II: Given two stable energy species, how do they act toward each other? Behavior extrapolated from Vilenkin.[15]

Name	Time (Approximate)	Dominant Energy	w
Planck Time	10^{-43} sec	Radiation	1/3
Inflation	10^{-35} sec	Cosm. Const.	-1
Radiation Epoch	10^{-32} sec	Radiation	1/3
Matter Epoch	70,000 years	Matter	0
Modern Epoch	13.7 billion years	Cosm. Const.	-1

TABLE III: A brief history of the universe in terms of the stable energy specie that dominated the gravitational expansion in the Friedmann equations.

Dominating Energy Specie	Equation of State	Universe Density	Universe Fate
Phantom energy	$w < -1$	$\Omega_{\text{total}} = \text{any}$	Big Rip
Cosm. const., domain walls, or cosmic strings	$-1 \leq w \leq -1/3$	$\Omega_{\text{total}} = \text{any}$	Big Freeze
Matter, radiation, or ultralight	$w > -1/3$	$\Omega_{\text{total}} \leq 1$	Big Freeze
Matter, radiation, or ultralight	$w > -1/3$	$\Omega_{\text{total}} > 1$	Big Crunch

TABLE IV: The future of a universe dominated by a single energy specie. Flat and curved universes are considered.