KSUPT-07/1 June 2007

Resource Letter: BE-1: The Beginning and Evolution of the Universe

Bharat Ratra[1] and Michael S. Vogeley[2]

[1]*Department of Physics, Kansas State University, 116 Cardwell Hall, Manhattan, KS 66506*
ratra@phys.ksu.edu

[2]*Department of Physics, Drexel University, 3141 Chestnut Street, Philadelphia, PA 19104*
vogeley@drexel.edu

This Resource Letter provides a guide to the literature on the evolution of the Universe, from an early inflation epoch to the complex hierarchy of structure seen today. References are listed for the following topics: general relativity; observations of the expanding Universe; the hot early Universe and nucleosynthesis; theory and observations of the cosmic microwave background; Big Bang cosmology; inflation; dark matter and dark energy; theory of structure formation; the cold dark matter model; galaxy formation; cosmological simulations; observations of galaxies, clusters, and quasars; statistical measures of large-scale structure; measurement of cosmological parameters; and some open questions in cosmology.

I. INTRODUCTION

It is the current opinion of many physicists that the Universe is well-described by what Fred Hoyle termed a Big Bang Model, in which the Universe expanded from a denser hotter childhood to its current adolescence, with present energy budget dominated by dark energy and less so by dark matter, neither of which have been detected in the laboratory, with the stuff biological systems, planets, stars, and all visible matter are made of (called baryonic matter by cosmologists) being a very small tracer on this dark sea, with electromagnetic radiation being an even less significant contributor. Galaxies and groups and clusters of galaxies are locally distributed inhomogeneously in space but on large enough scales and in a statistical sense the distribution approaches isotropy. This is supported by other electromagnetic distributions such as the X-ray and cosmic microwave backgrounds which are close to isotropic. As one looks out further into space, as a consequence of the finite speed of light, one sees objects as they were at earlier times, and there is clear observational evidence for temporal evolution in the distribution of various objects such as galaxies.

At earlier times the Universe was hotter and denser, at some stage so hot that atoms could not exist. Nuclear physics reactions between protons, neutrons, etc., in the cooling expanding Universe resulted in the (nucleo)synthesis of the lighter elements (nuclei) such as D, ^{4}He, and ^{7}Li, with abundances in good accord with what is observed, and with the photons left over forming a residual cosmic microwave background (CMB) also in good agreement with what is observed.

Given initial inhomogeneities in the mass distribution at an earlier time, processing of these by the expansion of the Universe, gravitational instability, pressure gradients, and microphysical processes, gives rise to observed anisotropies in the CMB and the current large-scale distribution of nonrelativistic matter; the situation on smaller spatial scales, where galaxies form, is murkier. Observations indicate that the needed initial inhomogeneities are most likely of the special form known as scale invariant, and that the simplest best-fitting Big Bang Model has flat spatial geometry. These facts could be the consequence of a simple inflationary infancy of the Universe, a very early period of extremely rapid expansion, which stretched zero-point quantum-mechanical fluctuations to larger length scales and transmuted them into the needed classical inhomogeneities in the mass-energy distribution. At the end of the inflationary expansion all radiation and matter is generated as the Universe moves into the usual Big Bang Model epoch. Inflation has roots in models of very high energy physics. Because of electromagnetic charge screening, gravity is the dominant large scale force, and general relativity is the best theory of gravity.

Our Resource Letter attempts to elaborate on this picture. Given the Tantalus principle of cosmology (and most of astrophysics), that one can see but not "touch" — which makes this a unique field of physics — there have been many false starts along wrong trails and even much confusion and many missed opportunities along what most now feel is the right track. Given space constraints we cannot do justice to what are now felt to be false starts, nor will we discuss more than one or two examples of confusion and missed opportunities. We attempt here to simply describe what is now thought to be a reasonable standard model of cosmology and trace the development of what are now felt to be the important threads in this tapestry; time will tell whether our use of "reasonable standard" is more than just youthful arrogance (or possibly middle-aged complacence).

This Resource Letter is divided into two main parts. The first part, Secs. II–VI, lists relevant publications. The second part, Secs. VII—XIII, is a summary of the current standard model of cosmology, with emphasis in parts on some historical roots. More precisely, the first part comprises of lists of: periodicals (Sec. II); more technical books (Sec. III); historical and biographical references (Sec. IV); less technical books and journal articles (Sec. V); and internet resources (Sec. VI). The second part summarizes and lists historically significant and more modern papers as well as review articles for: the foundations of the Big Bang Model (Sec. VII, which summarizes research in the half century from Einstein's foundational paper on modern cosmology until the late 1960's discovery of the CMB radiation), as well as some loose ends; inflation, which provides a widely felt to be reasonable explanation of the Big Bang (Sec. VIII); dark energy and dark matter (Sec. IX), the two (as yet not directly detected) main components of the energy budget of the present Universe; the growth of structure in the Universe (Sec. X); observations of large-scale structure in the Universe (Sec. XI); estimates of cosmological parameters (Sec. XII); and a discussion of what are now thought to be relevant open questions and directions in which the field appears to be moving (Sec. XIII).

While already possibly too long, this Resource Letter would have been even longer if we had covered other subjects now under discussion. Consequently, we have excluded a number of topics, including quantum cosmology, the multiverse scenario, string gas cosmology, braneworld and higher dimensional scenarios, and other modifications of the Einstein action for gravity. (We note that one motivation for modifying

Einstein's action is to attempt to do away with the construct of dark matter and/or dark energy. While it is probably too early to tell whether this can get rid of dark energy, it seems unlikely that this is a viable way of getting around the idea of dark matter.)

For original papers written in languages other than English, we cite only an English translation, unless this does not exist. We only cite books that are in English. For books that have been reprinted we cite only the most recent printing we are aware of.

II. PERIODICALS

Articles of relevance are found in periodicals that service a number of fields of physics, including astrophysics, cosmology, general relativity, and elementary particle physics. Periodicals are available on the web now, usually through subscription.

During the last few decades the most relevant research journals have been:

Astronomical Journal
Astronomy and Astrophysics
Astronomy and Astrophysics Letters
Astrophysical Journal
Astrophysical Journal Letters
Monthly Notices of the Royal Astronomical Society
Physics Letters B
Physical Review D
Physical Review Letters

During the last few decades relevant research articles have also appeared in:

Astroparticle Physics
Astronomy and Astrophysics Supplement
*Astronomy Letters (*originally *Soviet Astronomy Letters)*
*Astronomy Reports (*originally *Soviet Astronomy)*
Astrophysical Journal Supplement
Astrophysics and Space Science
Classical and Quantum Gravity
General Relativity and Gravitation
Gravitation and Cosmology
International Journal of Modern Physics A
International Journal of Modern Physics D
JETP Letters
Journal of Cosmology and Astroparticle Physics (electronic)
*Journal of Experimental and Theoretical Physics (*originally *Soviet Physics JETP)*
Journal of High Energy Physics (electronic)
Modern Physics Letters A
Nature
New Astronomy
Nuclear Physics B

Nuovo Cimento B
Pramana
Progress of Theoretical Physics
Publications of the Astronomical Society of the Pacific
Science

Review journals:
Annual Review of Astronomy and Astrophysics
Annual Review of Nuclear and Particle Science
Astronomy and Astrophysics Review
Living Reviews in Relativity (electronic)
New Astronomy Reviews
Physics Reports
Reports on Progress in Physics
Reviews of Modern Physics

Less technical surveys and articles have appeared in:
Discover
New Scientist
Physics Today
Physics World
Sky and Telescope
SLAC Beam Line (Stanford Linear Accelerator Center)
Science News
Scientific American

III. TECHNICAL BOOKS

This section lists articles and books at a more advanced level. See Secs. IV and V for articles and books at a more introductory level. We only list more recent books here; some historically significant books are listed in Sec. IV.A and books listed below include references to earlier books. The subsections below are somewhat artificial, e.g., Ref. [4] listed in the Physical cosmology subsection below probably has the best introduction to general relativity for cosmology, and some of the books listed in the General relativity and Astroparticle physics subsections contain fairly detailed discussions of physical cosmology.

A. Textbooks

1. Physical cosmology

1. **The Large-Scale Structure of the Universe**, P. J. E. Peebles (Princeton Univ., Princeton, 1980). (I, A)

2. **Relativistic Astrophysics. Vol. 2: The Structure and Evolution of the Universe**, Ya. B. Zel'dovich and I. D. Novikov (Univ. of Chicago, Chicago, 1983). (A)

3. **Structure Formation in the Universe**, T. Padmanabhan (Cambridge Univ., Cambridge, 1993). (A)

4. **Principles of Physical Cosmology**, P. J. E. Peebles (Princeton Univ., Princeton, 1993). (I, A)

5. **Cosmology**, P. Coles and F. Lucchin (Wiley, Chichester, 1995). (I, A)

6. **Cosmology and Astrophysics through Problems**, T. Padmanabhan (Cambridge Univ., Cambridge, 1996). (A)

7. **First Principles of Cosmology**, E. V. Linder (Addison-Wesley, Harlow, 1997). (I, A)

8. **Modern Cosmological Observations and Problems**, G. Bothun (Taylor & Francis, London, 1998). (I, A)

9. **Cosmological Physics**, J. A. Peacock (Cambridge Univ., Cambridge, 1999). (I, A)

10. **Cosmology: The Science of the Universe**, 2nd ed. (corrected), E. Harrison (Cambridge Univ., Cambridge, 2001). (E, I)

11. **Fundamentals of Cosmology**, J. Rich (Springer, Berlin, 2001). (I, A)

12. **An Introduction to Cosmology**, J. V. Narlikar (Cambridge Univ., Cambridge, 2002). (I, A)

13. **Theoretical Astrophysics Vol. III: Galaxies and Cosmology**, T. Padmanabhan (Cambridge Univ., Cambridge, 2002). (A)

14. **Modern Cosmology**, S. Dodelson (Academic, San Diego, 2003). (I, A)

15. **An Introduction to Modern Cosmology**, 2nd ed., A. Liddle (Wiley, Chichester, 2003). (E, I)

16. **Introduction to Cosmology**, 3rd ed., M. Roos (Wiley, Chichester, 2003). (I)

17. **Introduction to Cosmology**, B. Ryden (Addison Wesley, San Francisco, 2003). (I)

18. **An Introduction to Galaxies and Cosmology**, D. J. Adams, et al., edited by M. H. Jones and R. J. A. Lambourne (Cambridge Univ., Cambridge, 2004). (E, I)

19. **Cosmology**, 4th ed., M. Rowan-Robinson (Clarendon, Oxford, 2004). (I)

2. Specific topics in physical cosmology

20. **The Cosmological Distance Ladder: Distance and Time in the Universe**, M. Rowan-Robinson (Freeman, New York, 1985). A nice review of the more classical methods used to establish the distance scale. (I, A)

21. **3 K: The Cosmic Microwave Background Radiation**, R. B. Partridge (Cambridge Univ., Cambridge, 1995). (I)

22. **Measuring the Universe: The Cosmological Distance Ladder**, S. Webb (Springer-Praxis, Chichester, 1999). A very nice discussion of measuring distances. (E, I)

23. **Galaxy Formation**, M. S. Longair (Springer, Berlin, 2000). (I, A)

24. **Galaxy Formation and Evolution**, H. Spinrad (Springer, Berlin, 2005). A nice introduction that could have been proofread a little more carefully. (I, A)

3. General relativity

We are biased here, in that we have picked treatments relevant for cosmology.

25. **Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity**, S. Weinberg (Wiley, New York, 1972). (I, A)

26. **Gravitation**, C. W. Misner, K. S. Thorne, and J. A. Wheeler (Freeman, San Francisco, 1973). (I, A)

27. **Problem Book in Relativity and Gravitation**, A. P. Lightman, W. H. Press, R. H. Price, and S. A. Teukolsky (Princeton Univ., Princeton 1975). (I, A)

28. **Principles of Cosmology and Gravitation**, M. Berry (Cambridge Univ., Cambridge, 1976). (I)

29. **The Classical Theory of Fields**, 4th ed. (corrected), L. D. Landau and E. M. Lifshitz (Pergamon, Oxford, 1979). (A)

30. **A First Course in General Relativity**, B. F. Schutz (Cambridge Univ., Cambridge, 1985). (I, A)

31. **Introducing Einstein's Relativity**, R. d'Inverno (Oxford Univ., Oxford, 1992). (I, A)

32. **Gravitation and Spacetime**, 2nd ed., H. C. Ohanian and R. Ruffini (Norton, New York, 1994). (I, A)

33. **General Theory of Relativity**, P. A. M. Dirac (Princeton Univ., Princeton, 1996). (I, A)

34. **Exploring Black Holes: Introduction to General Relativity**, E. F. Taylor and J. A. Wheeler (Addison Wesley Longman, San Francisco, 2000). (E, I)

35. **Relativity: Special, General, and Cosmological**, W. Rindler (Oxford Univ., Oxford, 2001). (I, A)

36. **Gravity: An Introduction to Einstein's General Relativity**, J. B. Hartle (Addison Wesley, San Francisco, 2003). (I, A)

37. **Spacetime and Geometry: An Introduction to General Relativity**, S. M. Carroll (Addison Wesley, San Francisco, 2004). (I, A)

38. **General Relativity: An Introduction for Physicists**, M. P. Hobson, G. P. Efstathiou, and A. N. Lasenby (Cambridge Univ., Cambridge, 2006). (I, A)

4. Quantum field theory

39. **Quantum Field Theory**, C. Itzykson and J. B. Zuber (McGraw-Hill, New York, 1980). (A)

40. **Quantum Field Theory**, revised ed., F. Mandl and G. Shaw (Wiley, Chichester, 1993). (I)

41. **An Introduction to Quantum Field Theory**, M. E. Peskin and D. V. Schroeder (Addison-Wesley, Reading, 1995). (A)

42. **The Quantum Theory of Fields. Vol. I: Foundations**, S. Weinberg (Cambridge Univ., Cambridge, 1995). (A)

43. **Quantum Field Theory**, 2nd ed., L. H. Ryder (Cambridge Univ., Cambridge, 1996). (I)

44. **The Quantum Theory of Fields. Vol. II: Modern Applications**, S. Weinberg (Cambridge Univ., Cambridge, 1996). (A)

45. **Field Theory: A Modern Primer**, 2nd ed., P. Ramond (Perseus, Cambridge, 1997). (A)

46. **Quantum Field Theory in a Nutshell**, A. Zee (Princeton Univ., Princeton, 2003). (I, A)

5. Astroparticle physics

References [4, 9] also discuss some astroparticle physics.

47. **The Early Universe**, E. W. Kolb and M. S. Turner (Addison-Wesley, Redwood City, 1990). (I, A)

48. **Particle Physics and Inflationary Cosmology**, A. Linde (Harwood Academic, Chur, 1990). (A)

49. **Cosmic Strings and Other Topological Defects**, A. Vilenkin and E. P. S. Shellard (Cambridge Univ., Cambridge, 1994). (A)

50. **An Introduction to Cosmology**, J. Bernstein (Prentice-Hall, Englewood Cliffs, 1995). (I, A)

51. **Cosmology and Particle Astrophysics**, L. Bergström and A. Goobar (Wiley, Chichester, 1999). (I, A)

52. **Cosmological Inflation and Large-Scale Structure**, A. R. Liddle and D. H. Lyth (Cambridge Univ., Cambridge, 2000). (A)

53. **Particle Astrophysics**, D. Perkins (Oxford Univ., Oxford, 2003). (I, A)

54. **Physical Foundations of Cosmology**, V. Mukhanov (Cambridge Univ., Cambridge, 2005). (A)

6. Quantum fields in curved spacetime

55. **Quantum Fields in Curved Space**, N. D. Birrell and P. C. W. Davies (Cambridge Univ., Cambridge, 1982). (A)

56. **Aspects of Quantum Field Theory in Curved Space-Time**, S. A. Fulling (Cambridge Univ., Cambridge, 1989). (A)

7. Standard model of particle physics and proposed extensions

57. **Gauge Theories of the Strong, Weak, and Electromagnetic Interactions**, C. Quigg (Benjamin-Cummings, Reading, 1983). (I, A)

58. **Gauge Theory of Elementary Particle Physics**, T. P. Cheng and L. F. Li (Oxford, New York, 1984). (A)

59. **Grand Unified Theories**, G. G. Ross (Benjamin-Cummings, Menlo Park, 1984). (I, A)

60. **Superstring Theory**, M. B. Green, J. H. Schwarz, and E. Witten (Cambridge Univ., Cambridge, 1987). (A)

61. **Particle Physics and Cosmology**, P. D. B. Collins, A. D. Martin, and E. J. Squires (Wiley, New York, 1989). (I, A)

62. **Dynamics of the Standard Model**, J. F. Donoghue, E. Golowich, and B. R. Holstein (Cambridge Univ., Cambridge, 1992). (I)

63. **Modern Elementary Particle Physics: The Fundamental Particles and Forces?**, updated ed., G. Kane (Addison-Wesley, Reading, 1993). (I)

64. **Gauge Theories in Particle Physics**, 2nd ed. (corrected), I. J. R. Aitchison and A. J. Hey (Institute of Physics, Bristol, 1996). (I, A)

65. **An Introduction to the Standard Model of Particle Physics**, W. N. Cottingham and D. A. Greenwood (Cambridge Univ., Cambridge, 1998). (I)

66. **String Theory**, J. Polchinski (Cambridge Univ., Cambridge, 1998). (A)

67. **Journeys beyond the Standard Model**, P. Ramond (Perseus, Cambridge, 1999). (A)

68. **Introduction to High-Energy Physics**, 4th ed., D. H. Perkins (Cambridge Univ., Cambridge, 2000). (I)

69. **The Quantum Theory of Fields. Vol. III: Supersymmetry**, S. Weinberg (Cambridge Univ., Cambridge, 2000). (A)

B. Collections of reprints of papers

References [70, 72] reprint many of the classic cosmology papers and Ref. [71] includes some classic inflation reprints.

70. **A Source Book in Astronomy and Astrophysics, 1900–1975**, edited by K. R. Lang and O. Gingerich (Harvard Univ., Cambridge, 1979). (E, I, A)

71. **Inflationary Cosmology**, edited by L. F. Abbott and S.-Y. Pi (World Scientific, Singapore, 1986). (I, A)

72. **Cosmological Constants: Papers in Modern Cosmology**, edited by J. Bernstein and G. Feinberg (Columbia Univ., New York, 1986). (E, I, A)

C. Summer school and other lecture notes

The following works include lecture notes from summer schools and similar programs that are of a pedagogical nature at a level suitable for beginning graduate students. We restrict this list to the last decade or so as earlier developments in cosmology and astroparticle physics are discussed in detail in the textbooks listed in Secs. III.A.1, 2, and 5 above.

73. **The Deep Universe: Saas-Fee Advanced Course 23**, A. R. Sandage, R. G. Kron, and M. S. Longair, edited by B. Binggeli and R. Buser (Springer, Berlin, 1995). (A)

74. **The Universe at High-z, Large-Scale Structure and the Cosmic Microwave Background**, edited by E. Martínez-González and J. L. Sanz (Springer, Berlin, 1996). (A)

75. **Cosmology and Large Scale Structure: Les Houches Session LX**, edited by R. Schaeffer, J. Silk, M. Spiro, and J. Zinn-Justin (Elsevier, Amsterdam, 1996). (A)

76. **Formation of Structure in the Universe**, edited by A. Dekel and J. P. Ostriker (Cambridge Univ., Cambridge, 1999). (I, A)

77. **Theoretical and Observational Cosmology**, edited by M. Lachièze-Rey (Kluwer, Dordrecht, 1999). Particularly useful for lectures on general relativity and gravitational lensing. (A)

78. **The Primordial Universe: Les Houches Session LXXI**, edited by P. Binétruy, R. Schaeffer, J. Silk, and F. David (Springer, Berlin, 2000). (A)

79. **Structure Formation in the Universe** edited by R. G. Crittenden and N. G. Turok (Kluwer, Dordrecht, 2001). (A)

80. **Galaxies at High Redshift**, edited by I. Pérez-Fournon, M. Balcells, F. Moreno-Insertis, and F. Sánchez (Cambridge Univ., Cambridge, 2003). Lectures by leading high-redshift galaxy observers. (A)

81. **Frontiers of the Universe: Cosmology 2003**, edited by A. Blanchard and M. Signore (Springer, Berlin, 2005). (I, A)

82. **The New Cosmology**, edited by M. Colless (World Scientific, Singapore, 2005). (A)

83. **Gravitational Lensing: Strong, Weak and Micro: Saas-Fee Advanced Course 33**, C. S. Kochanek, P. Schneider, and J. Wambsganss, edited by G. Meylan, P. Jetzer, and P. North (Springer, Berlin, 2006). (I, A)

IV. HISTORICAL AND BIOGRAPHICAL REFERENCES

A. Historically significant books

We list here some books that played an important role in the development of the field.

84. **The Creation of the Universe**, G. Gamow (Viking, New York, 1952). A semi-popular, fascinating, and widely-read introduction to the cosmology and astrophysics of the 1940's. Also discusses a number of ideas that are now known to be incorrect. (E)

85. **The Realm of the Nebulae**, E. Hubble (Dover, New York, 1958). (E, I)

86. **Cosmology**, 2nd ed., H. Bondi (Cambridge Univ., Cambridge, 1960). (E, I)

87. **Gravitation and the Universe**, R. H. Dicke (American Philo. Soc., Philadelphia, 1970). (E, I)

88. **Physical Cosmology**, P. J. E. Peebles (Princeton Univ., Princeton, 1971). (E, I)

B. History of cosmology

Reference [92] is an early review of the history of the classical period of modern cosmology, through to the 1950's. References [89] and [90] are less technical accounts of how the scale of our Milky Way galaxy and the distances to external galaxies came to be established, with emphasis on the developments during the first three decades of the 1900's. Reference [93] is a history of modern cosmology through to the late 1960's, with some emphasis on how the Big Bang Model gradually gained acceptance and the steady state model faded away. Reference [91] contains articles from a conference on the history of cosmology; McCrea's article is beautiful.

89. **Man Discovers the Galaxies**, R. Berendzen, R. Hart, and D. Seeley (Science History, New York, 1976). (E, I)

90. **The Expanding Universe: Astronomy's 'Great Debate' 1900–1931**, R. W. Smith (Cambridge Univ., Cambridge, 1982). (E, I)

91. **Modern Cosmology in Retrospect**, edited by B. Bertotti, R. Balbinot, S. Bergia, and A. Messina (Cambridge Univ., Cambridge, 1990). (I)

92. **The Measure of the Universe: A History of Modern Cosmology**, J. D. North (Dover, New York, 1990). (E, I)

93. **Cosmology and Controversy: The Historical Development of Two Theories of the Universe**, H. Kragh (Princeton Univ., Princeton, 1996). (E, I)

C. Biography

The history and context of some of the developments in cosmology may be traced through biographies of some of the prominent developers of the field.

94. **My World Line: An Informal Autobiography**, G. Gamow (Viking, New York, 1970). (E)

95. "Willem de Sitter 1872-1934," W. H. McCrea, J. Brit. Astron. Assoc. **82** (3) 178-181 (1972). De Sitter, along with Einstein, Friedmann, and Lemaître, laid the foundations of theoretical cosmology; he was the only astronomer of the four. A book-length biography of de Sitter does not appear to exist; this would be of some interest. (E)

96. **'Subtle is the Lord...'—The Science and the Life of Albert Einstein**, A. Pais (Clarendon, Oxford, 1982). (I)

97. **Eddington: The Most Distinguished Astrophysicist of his Time**, S. Chandrasekhar (Cambridge Univ., Cambridge, 1983). (I)

98. **Three Degrees Above Zero: Bell Labs in the Information Age**, J. Bernstein (Scribner's Sons, New York, 1984). Includes chapters on Penzias and Wilson. (E)

99. **Cosmology of Lemaître**, O. Godart and M. Heller (Pachart Publishing House, Tucson, 1985). A scientific biography of Lemaître's cosmological research; O. G. was a scientific colleague and collaborator. (I)

100. **Origins: The Lives and Worlds of Modern Cosmologists**, A. Lightman and R. Brawer (Harvard Univ., Cambridge, 1990). A collection of interviews of some modern cosmologists, astronomers, astroparticle physicists, and general relativists. Gives an interesting overview of cosmology in the mid- to late-1980's when the inflation picture started to profoundly affect cosmological thought. (E, I)

101. **Edwin Hubble, the Discoverer of the Big Bang Universe**, A. S. Sharov and I. D. Novikov (Cambridge Univ., Cambridge, 1993). (E)

102. **Alexander A. Friedmann: The Man Who Made the Universe Expand**, E. A. Tropp, V. Ya. Frenkel, and A. D. Chernin (Cambridge Univ., Cambridge, 1993). (E, I)

103. **Home is Where the Wind Blows: Chapters from a Cosmologist's Life**, F. Hoyle (University Science Books, Mill Valley, 1994). (E)

104. **Edwin Hubble: Mariner of the Nebulae**, G. E. Christianson (Farrar, Strauss and Giroux, New York, 1995). (E)

105. **Fred Hoyle: A Life in Science**, S. Mitton (Aurum, London, 2005). (E, I)

106. **Fred Hoyle's Universe**, J. Gregory (Oxford Univ., Oxford, 2005). (E)

V. SEMI-POPULAR LITERATURE

A. Books

There are a number of good semi-popular books that provide a fast introduction to cosmology and related fields.

1. General relativity

107. **General Relativity from A to B**, R. Geroch (Univ. Chicago, Chicago, 1978). (E, I)

108. **Einstein's Legacy**, J. Schwinger (Freeman, New York, 1986). (E, I)

109. **Was Einstein Right? Putting General Relativity to the Test**, C. M. Will (Basic Books, New York, 1986). (E)

110. **A Journey into Gravity and Spacetime**, J. A. Wheeler (Freeman, New York, 1990). (E, I)

111. **From Black Holes to Time Warps: Einstein's Outrageous Legacy**, K. S. Thorne (Norton, New York, 1994). No Kip, Stephen Hawking did not break into your Caltech office in June 1990 (p. 315) to affix a thumbprint to, and so concede losing, the bet: one of us let him in! (E, I)

2. Cosmology

References [118], [121], and [128] discuss the COBE CMB observations and should be read together. References [132] and [138] review the discovery of accelerated expansion of the Universe and so dark energy. References [112] and [123] are nice introductions to the more established parts of cosmology; out of date in parts but still worth the read.

112. **Cosmic Horizons: Understanding the Universe**, R. V. Wagoner and D. W. Goldsmith (Freeman, New York, 1983). (E)

113. **Darkness at Night: A Riddle of the Universe**, E. Harrison (Harvard Univ., Cambridge, 1987). An introduction to cosmology from the viewpoint of Olbers's paradox. (E, I)

114. **From Quarks to the Cosmos: Tools of Discovery**, L. M. Lederman and D. N. Schramm (Freeman, New York, 1989). An overview of particle physics, astroparticle physics, and cosmology to the end of the 1980's. Out of date in parts. (E)

115. **An Old Man's Toy: Gravity at Work and Play in Einstein's Universe**, A. Zee (Macmillan, New York, 1989). A nice, somewhat heuristic, description of gravitational physics underlying cosmology. Out of date in parts. (E)

116. **Ancient Light: Our Changing View of the Universe**, A. Lightman (Harvard Univ., Cambridge, 1991). A broad overview of historical cosmologies. (E)

117. **Through a Universe Darkly: A Cosmic Tale of Ancient Ethers, Dark Matter, and the Fate of the Universe**, M. Bartusiak (Harper Collins, New York, 1993). A popular, informal, introductory discussion of dark matter and related issues. Out of date in parts. (E)

118. **Afterglow of Creation: From the Fireball to the Discovery of Cosmic Ripples**, M. Chown (Arrow Books, London, 1993). (E)

119. **The Universe for Beginners**, F. Pirani and C. Roche (Icon Books, Cambridge, 1993). Cartoon history! (E)

120. **Ripples in the Cosmos: A View Behind the Scenes of the New Cosmology**, M. Rowan-Robinson (Freeman, Oxford, 1993). A personal introduction to some of the results from the IRAS and COBE space missions. Out of date in parts. (E)

121. **Wrinkles in Time**, G. Smoot and K. Davidson (Morrow, New York, 1993). (E)

122. **The Birth of the Universe: The Big Bang and Beyond**, T. X. Thuan (Abrams, New York, 1993). A beautifully illustrated introduction to cosmology and astrophysics. (E)

123. **The First Three Minutes: A Modern View of the Origin of the Universe**, updated ed., S. Weinberg (Basic Books, New York, 1993). (E, I)

124. **A Short History of the Universe**, J. Silk (Freeman, New York, 1994). A good book to use as a base for a "Cosmology for Poets" course although it is a little out of date. See comment on the related Ref. [141] below. (E, I)

125. **Poetry of the Universe: A Mathematical Exploration of the Cosmos**, R. Osserman (Doubleday, New York, 1995). An introduction to some of the geometrical ideas used in cosmology. (E, I)

126. **The Secret Melody: And Man Created the Universe**, T. X. Thuan (Oxford Univ., Oxford, 1995). An introduction to cosmology as well as some astrophysics and particle physics. Also touches on some philosophical issues, more so than any other book cited in this subsection. (E)

127. **Our Evolving Universe**, M. S. Longair (Cambridge Univ., Cambridge, 1996). Beautifully illustrated introductory review of modern astronomy and cosmology. (E, I)

128. **The Very First Light: The True Inside Story of the Scientific Journey Back to the Dawn of the Universe**, J. C. Mather and J. Boslough (Basic Books, New York, 1996). (E)

129. **The Inflationary Universe**, A. H. Guth (Helix Books, Reading, 1997). An introduction to cosmology and astroparticle physics and a personal account of the development of inflation. (E, I)

130. **After the First Three Minutes: The Story of our Universe**, T. Padmanabhan (Cambridge Univ., Cambridge, 1998). A thorough, but challenging, introduction to cosmology. Out of date in parts. (I)

131. **The Routledge Critical Dictionary of the New Cosmology**, edited by P. Coles (Routledge, New York, 1999). Six extended articles reviewing various parts of cosmology followed by an extensive alphabetized summary of terms, themes, and people in cosmology. (E, I)

132. **The Runaway Universe: The Race to Find the Future of the Cosmos**, D. Goldsmith (Perseus Books, Cambridge, 2000). (E)

133. **Quintessence: The Mystery of the Missing Mass in the Universe**, L. Krauss (Basic Books, New York, 2000). A detailed and challenging introduction to dark matter, including experiments designed to detect it. (I)

134. **Genesis of the Big Bang**, R. A. Alpher and R. Herman (Oxford Univ., Oxford, 2001). The story of the Big Bang Model, told by two of the earliest explorers. (E, I)

135. **The Magic Furnace: The Search for the Origins of Atoms**, M. Chown (Oxford Univ., Oxford, 2001). An introduction to stellar and cosmological nucleosynthesis. (E, I)

136. **Cosmology: A Very Short Introduction**, P. Coles (Oxford Univ., Oxford, 2001). (E)

137. **The Big Bang**, 3rd ed., J. Silk (Freeman, New York, 2001). A good text for a "Cosmology for Poets with some Mathematical Ability" course. (E, I)

138. **The Extravagant Universe: Exploding Stars, Dark Energy and the Accelerating Cosmos**, R. P. Kirshner (Princeton Univ., Princeton, 2002). Those who have not had the good fortune of attending a famous east coast university which started out as a divinity school might need a dictionary to help read this one! (E)

139. **Big Bang: Origin of the Universe**, S. Singh (HarperCollins, New York, 2004). A survey of the development of the Big Bang Model focusing on the period up to and including the mid 1960's discovery of the CMB. (E)

140. **Miss Leavitt's Stars: The Untold Story of the Woman Who Discovered How to Measure the Universe**, G. Johnson (Norton, New York, 2005). An accessible discussion of how Leavitt's Cepheid variable star period-luminosity relation allowed large cosmological distances to be measured. (E)

141. **On the Shores of the Unknown: A Short History of the Universe**, J. Silk (Cambridge Univ., Cambridge, 2005). This is in large part an updated version of Ref. [124], but with significantly fewer illustrations. (E, I)

B. Scientific American Articles

Scientific American has a tradition of publishing excellent introductory articles, written at a level accessible by a well-educated lay person, on many topics relevant for cosmology. Listed below in chronological order are some of the more recent ones; these often cite earlier Scientific American articles. Scientific American occasionally publishes compilations where some of these articles have been updated; we do not list these here, except for one article that has not appeared elsewhere.

142. "The Evolution of the Universe," P. J. E. Peebles, D. N. Schramm, E. L. Turner, and R. G. Kron, Sci. Am. **271** (4), 53-57 (1994). (E)

143. "Primordial Deuterium and the Big Bang," C. J. Hogan, Sci. Am. **275** (6), 68-73 (1996). While the technique Hogan reviews is still of great use, most of the early observations he discusses are now felt to be unreliable. (E)

144. "The Ghostliest Galaxies," G. D. Bothun, Sci. Am. **276** (2), 56-61 (1997). Low-surface-brightness galaxies. (E)

145. "Galaxies in the Young Universe," D. Macchetto and M. Dickinson, Sci. Am. **276** (5), 92-99 (1997). (E)

146. "Dark Matter in the Universe," V. Rubin, Sci. Am. Presents **9** (1), 106-110 (1998). (E)

147. "The Asymmetry Between Matter and Antimatter," H. R. Quinn and M. S. Witherell, Sci. Am. **279** (4), 76-81 (1998). (E)

148. "The Evolution of Galaxy Clusters," J. P. Henry, U. G. Briel, and H. Böhringer, Sci. Am. **279** (6), 52-57 (1998). (E)

149. "Surveying Space-Time with Supernovae," C. J. Hogan, R. P. Kirshner, and N. B. Suntzeff, Sci. Am. **280** (1) 28-33 (1999). (E)

150. "Cosmological Antigravity," L. M. Krauss, Sci. Am. **280** (1), 52-58 (1999). Dark energy. (E)

151. "Mapping the Universe," S. D. Landy, Sci. Am. **280** (6), 38-45 (1999). Large-scale structure. (E)

152. "Echoes from the Big Bang," R. R. Caldwell and M. Kamionkowski, Sci. Am. **284** (1), 37-42 (2001). Gravity waves from the early universe. (E)

153. "The Quintessential Universe," J. P. Ostriker and P. J. Steinhardt, Sci. Am. **284** (1), 46-53 (2001). Dark energy. (E)

154. "Making Sense of Modern Cosmology," P. J. E. Peebles, Sci. Am. **284** (1), 54-55 (2001). (E)

155. "Gravity's Kaleidoscope," J. Wambsganss, Sci. Am. **285** (5), 64-71 (2001). Gravitational lensing. (E)

156. "The First Stars in the Universe," R. B. Larson and V. Bromm, Sci. Am. **285** (6), 64-71 (2001). (E)

157. "The Life Cycle of Galaxies," G. Kauffmann and F. van den Bosch, Sci. Am. **286** (6), 46-55 (2002). (E)

158. "Does Dark Matter Really Exist?" M. Milgrom, Sci. Am. **287** (2), 42-48 (2002). (E)

159. "The Emptiest Places," E. Scannapieco, P. Petitjean, and T. Broadhurst, Sci. Am. **287** (4), 56-63 (2002). Intergalactic medium. (E)

160. "The Search for Dark Matter," D. B. Cline, Sci. Am. **288** (3), 50-59 (2003). (E)

161. "The Cosmic Symphony," W. Hu and M. White, Sci. Am. **290** (2), 44-53 (2004). Cosmic microwave background. (E)

162. "Reading the Blueprints of Creation," M. A. Strauss, Sci. Am. **290** (2), 54-61 (2004). Large-scale structure. (E)

163. "From Slowdown to Speedup," A. G. Riess and M. S. Turner, Sci. Am. **290** (2), 62-67 (2004). Dark energy. (E)

164. "A Cosmic Conundrum," L. M. Krauss and M. S. Turner, Sci. Am. **291** (3), 70-77 (2004). Dark energy. (E)

165. "The Midlife Crisis of the Cosmos," A. J. Barger, Sci. Am. **292** (1), 46-53 (2005). The more recent past Universe. (E)

166. "Misconceptions About the Big Bang," C. H. Lineweaver and T. M. Davis, Sci. Am. **292** (3), 36-45 (2005). Distant galaxies recede faster than the speed of light without violating relativity. (E)

167. "The Dark Ages of the Universe," A. Loeb, Sci. Am. **295** (5), 46-53 (2006). The "dark age" between when the CMB decoupled from the matter and before the first stars and quasars formed. (E)

168. "The Universe's Invisible Hand," C. J. Conselice, Sci. Am. **296** (2), 34-41 (2007). Effect of dark energy on the formation of structure. (E)

VI. INTERNET RESOURCES

A. Preprints and published journal articles

Preprints on cosmology, astrophysics, general relativity, and particle physics may be found at `http://arXiv.org/`, including cosmology and astrophysics preprints at `http://arXiv.org/archive/astro-ph`, general relativity and gravitation preprints at `http://arXiv.org/archive/gr-qc`, phenomenological particle physics preprints at `http://arXiv.org/archive/hep-ph`, and more theoretical particle physics preprints at `http://arXiv.org/archive/hep-th`. The Astrophysics Data System (ADS) at `http://adsabs.harvard.edu/` includes links to many published cosmology papers. The SPIRES database at SLAC at `http://www.slac.stanford.edu/spires/` includes links to many preprints and published papers. Most journals also have their own webpages.

B. Observatories and satellite missions

In Tables 1-6 we list observatories and satellite missions most relevant for cosmology. Details of future observatories and missions are, of course, subject to change. In certain cases, to save space, national observatory facilities (e.g., NOAO) that include multiple telescopes or sites are given a single listing. The listing of CMB experiments is maintained by NASA.

C. Some other useful webpages

The NASA Extragalactic Database (NED), `http://nedwww.ipac.caltech.edu/`, includes searchable extragalactic data, literature, and cosmological tools. Tony Banday's CMB Resources at `http://www.mpa-garching.mpg.de/~banday/CMB.html`; Ned Wright's Tutorial, `http://www.astro.ucla.edu/~wright/cosmolog.htm`; and the Legacy Archive for Microwave Background Data Analysis (LAMBDA) website at `http://lambda.gsfc.nasa.gov/` provide useful content and links to other pages. Physical constants are tabulated at `http://physics.nist.gov/cuu/constants/` and at the Particle Data Group site at `http://pdg.lbl.gov/`.

Table 1: Current ground-based optical/infrared observatories.

Observatory	Location	Web address
2 Micron All Sky Survey (2MASS)	Mt. Hopkins, Arizona; Cerro Telolo, Chile	http://pegasus.phast.umass.edu/
Anglo-Australian Observatory (AAO)	Siding Spring, Australia	http://www.aao.gov.au/
Canada-France-Hawaii Telescope (CFHT)	Mauna Kea, Hawaii	http://www.cfht.hawaii.edu/
European Southern Observatory (ESO)	Cerro Paranal and La Silla, Chile	http://wwww.eso.org/
Hobby-Eberly Telescope	Mt. Fowlkes, Texas	http://www.as.utexas.edu/mcdonald/het/
W. M. Keck Observatory	Mauna Kea, Hawaii	http://www.keckobservatory.org/
Lick Observatory	Mt. Hamilton, California	http://mthamilton.ucolick.org/
Magellan Telescopes	Las Campanas, Chile	http://www.ociw.edu/magellan/
MMT Observatory	Mt. Hopkins, Arizona	http://www.mmto.org/
National Optical Astronomy Observatory (NOAO)	Arizona, Hawaii; Chile	http://www.noao.edu/
Palomar Observatory	Mt. Palomar, California	http://www.astro.caltech.edu/palomar/
Sloan Digital Sky Survey (SDSS)	Apache Point, New Mexico	http://www.sdss.org/
Southern African Large Telescope (SALT)	Sutherland, South Africa	http://www.salt.ac.za/
Subaru Telescope	Mauna Kea, Hawaii	http://www.naoj.org/

Table 2: Future ground-based optical/infrared observatories.

Observatory	Location	Web address
Dark Energy Survey (DES, on CTIO 4 m telescope)	Cerro Telolo, Chile	http://www.darkenergysurvey.org/
Large Sky Area Multi-Object Fiber Spectroscopic Telescope (LAMOST)	Xinglong, China	http://www.lamost.org/en/
Large Synoptic Survey Telescope (LSST)	Cerro Pachón, Chile	http://www.lsst.org/
Pan-STARRS	Mauna Kea, Hawaii	http://pan-starrs.ifa.hawaii.edu/
Thirty Meter Telescope	TBD	http://tmt.ucolick.org/

Table 3: Current and future ground-based radio/submm/mm observatories.

Observatory	Location	Web address
Arecibo Observatory	Arecibo, Puerto Rico	http://www.naic.edu/
Australia Telescope National Facility (ATNF)	Australia	http://www.atnf.csiro.au/
Caltech Submillimeter Observatory	Mauna Kea, Hawaii	http://www.astro.caltech.edu/~cso/
Combined Array for Research in Millimeter-Wave Astronomy (CARMA)	Cedar Flat, California	http://www.mmarray.org/
Giant Meterwave Radio Telescope (GMRT)	Pune, India	http://www.gmrt.ncra.tifr.res.in/
James Clerk Maxwell Telescope (JCMT)	Mauna Kea, Hawaii	http://www.jach.hawaii.edu/JCMT/
Jodrell Bank Observatory	United Kingdom	http://www.jb.man.ac.uk/
National Radio Astronomy Observatory (NRAO)	Virginia, New Mexico; Chile	http://www.nrao.edu/
Westerbork Observatory	Drenthe, Netherlands	http://www.astron.nl/p/observing.htm
Atacama Cosmology Telescope (ACT)	Cerro Toco, Chile	http://www.hep.upenn.edu/act/
Low-Frequency Array (LOFAR)	Netherlands and Germany	http://www.lofar.org/
South Pole Telescope	Antarctica	http://spt.uchicago.edu/
Square Kilometer Array (SKA)	TBD	http://www.skatelescope.org/

Table 4: Comprehensive lists of and links to satellite and suborbital (ground-based and balloon-borne) CMB experiments.

Satellite:	http://lambda.gsfc.nasa.gov/product/space/
Suborbital:	http://lambda.gsfc.nasa.gov/product/suborbit/su_experiments.cfm

Table 5: Ongoing and recent satellite missions.

Satellite	Duration	Web address
Chandra X-ray Observatory	1999–	http://chandra.harvard.edu/
Cosmic Background Explorer (COBE)	1989–1993	http://lambda.gsfc.nasa.gov/product/cobe/
Far Ultraviolet Spectroscopic Explorer (FUSE)	1999–	http://fuse.pha.jhu.edu/
Galaxy Evolution Explorer (GALEX)	2003–	http://www.galex.caltech.edu/
Hubble Space Telescope (HST)	1990–	http://www.stsci.edu/hst/
Infrared Astronomical Satellite (IRAS)	1983-1984	http://irsa.ipac.caltech.edu/Missions/iras.html
Infared Space Observatory	1996–1998	http://www.iso.vilspa.esa.es/
International Gamma-Ray Astrophysics Laboratory (INTEGRAL)	2002–	http://www.esa.int/SPECIALS/Integral/
ROSAT	1990–1999	http://wave.xray.mpe.mpg.de/rosat/
Spitzer Space Telescope	2003–	http://ssc.spitzer.caltech.edu/
Swift	2004–	http://www.nasa.gov/mission_pages/swift/main/
Wilkinson Microwave Anisotropy Probe (WMAP)	2001–	http://wmap.gsfc.nasa.gov/
XMM-Newton	1999–	http://xmm.vilspa.esa.es/

Table 6: Future satellite missions.

Satellite	Launch Date	Web address
James Webb Space Telescope (JWST)	2013?	http://www.jwst.nasa.gov/
Laser Interferometer Space Antenna (LISA)	2015?	http://lisa.jpl.nasa.gov/
Planck	2008?	http://planck.esa.int/

VII. FOUNDATIONS OF THE BIG BANG MODEL

A. General relativity and the expansion of the Universe

Modern cosmology begins with Einstein's 1917 paper [169] where he applies his general relativity theory to cosmology. At this point in time our Milky Way galaxy was thought by most to be the Universe. To make progress Einstein assumed the Universe was spatially homogeneous and isotropic; this was enshrined as the "Copernican" cosmological principle by Milne [170]. Reference [4] (Sec. 3) reviews the strong observational evidence for large-scale statistical isotropy; observational tests of homogeneity are not as straightforward. Einstein knew that the stars in the Milky Way moved rather slowly and decided, as everyone had done before him, that the Universe should not evolve in time. He could come up with a static solution of his equations if he introduced a new form of energy, now called the cosmological constant. It turns out that Einstein's static model is unstable. In the same year de Sitter [171] found the second cosmological solution of Einstein's general relativity equations; Lemaître [172] and Robertson [173] re-expressed this solution in the currently more familiar form of the exponentially expanding model used in the inflation picture. Weyl [174] noted the importance of prescribing initial conditions such that the particle geodesics diverge from a point in the past. Friedmann [175, 176], not bound by the desire to have a static model, discovered the evolving homogeneous solutions of Einstein's equations; Lemaître [177] rediscovered these "Friedmann-Lemaître" models. Robertson [178] initiated the study of metric tensors of spatially homogeneous and isotropic spacetimes, and continuing study by him and A. G. Walker (in the mid 1930's) led to the "Robertson-Walker" form of the metric tensor for homogeneous world models. Of course, in the evolving cosmological model solutions only observers at rest with respect to the expansion/contraction see an isotropic and homogeneous Universe; cosmology thus re-introduces preferred observers! North [92] provides a comprehensive historical review. See the standard cosmology textbooks cited above for the modern formalism.

169. "Cosmological Considerations on the General Theory of Relativity," A. Einstein (1917), English translation in **The Principle of Relativity**, H. A. Lorentz, A. Einstein, H. Minkowski, and H. Weyl (Dover, New York, 1952), 175-188. (E, I)

170. "World-Structure and the Expansion of the Universe," E. A. Milne, Zeit. Astrophys. **6**, 1-35 (1933). (E, I)

171. "On Einstein's Theory of Gravitation, and its Astronomical Consequences. Third Paper," W. de Sitter, Mon. Not. R. Astron. Soc. **78**, 3-28 (1917). (I)

172. "Note on de Sitter's Universe," G. Lemaître, J. Math. Phys. (Cambridge, MA) **4**, 188-192 (1925). (E, I)

173. "On Relativistic Cosmology," H. P. Robertson, Philo. Mag. **5**, 835-848 (1928). (E, I)

174. “Zur Allgemeinen Relativitätstheorie,” H. Weyl, Phys. Zeit. **24**, 230-232 (1923). An English translation should appear in Gen. Relativ. Gravit. in 2007. (E, I)

175. “On the Curvature of Space,” A. Friedmann (1922), English translation in Ref. [72], 49-58. (E, I)

176. “On the Possibility of a World with Constant Negative Curvature,” A. Friedmann (1924), English translation in Ref. [72], 59-65. (I)

177. “A Homogeneous Universe of Constant Mass and Increasing Radius Accounting for the Radial Velocity of Extra-galactic Nebulae,” G. Lemaître (1927), abridged and updated English translation in Mon. Not. R. Astron. Soc. **91**, 483-490 (1931). (E, I)

178. “On the Foundations of Relativistic Cosmology,” H. P. Robertson, Proc. Natl. Acad. Sci. (USA) **15**, 822-829 (1929). (E, I)

B. Galaxy redshift and distance measurements

Meanwhile, with first success in 1912, Slipher [179] finds that most of the “white spiral nebulae” (so-called because they have a continuum spectrum, and what we now term spiral galaxies) emit light that is redshifted (we now know that the few, including M31 (Andromeda) and some in the Virgo cluster, that emit blue-shifted light are approaching us), and Eddington in 1923 [180] identifies this with a redshift effect in de Sitter’s [171] model (not the cosmological redshift effect). In 1925 Lemaître [172] and Robertson [173] (in 1928) derive Hubble’s velocity-distance law $v = H_0 r$ (relating the galaxy’s speed of recession v to its distance r from us, where H_0 is the Hubble constant, the present value of the Hubble parameter) in the Friedmann-Lemaître models. The velocity-distance Hubble law is a consequence of the cosmological principle, is exact, and implies that galaxies further away than the current Hubble distance $r_H = c/H_0$ are moving away faster than the speed of light c. Hubble [181] uses Leavitt’s [182, 140] quantitative Cepheid variable star period-luminosity relation to establish that M31 and M33 are far away (confirming the earlier somewhat tentative conclusion of Öpik [183]), and does this for more galaxies, conclusively establishing that the white nebulae are other galaxies outside our Milky Way galaxy (there was some other earlier observational evidence for this position but Hubble’s work is what convinces people). Hubble gets Humason (middle school dropout and one time muleskinner and janitor) to re-measure some Slipher spectra and measure more spectra, and Hubble [184] establishes Hubble’s redshift-distance law $cz = H_0 r$, where the redshift z is the fractional change in the wavelength of the spectral line under study (although in the paper Hubble calls cz velocity and does not mention redshift). The redshift-distance Hubble law is approximate — it is an approximation to the velocity-distance law, valid only on short distances and at low redshifts. North [92] provides a comprehensive historical review; Refs. [89, 90] are more accessible historical summaries. See the standard cosmology textbooks cited above for the modern formalism.

Reference [185] discusses the use of type Ia supernovae as standard candles for measuring the Hubble constant. See Fig. 1 of Ref. [186] for a recent plot of the Hubble law. References [187, 188, 166] provide pedagogical discussions of galaxies-moving-away-faster-than-the-speed-of-light type issues.

179. "Nebulae," V. M. Slipher, Proc. Am. Philo. Soc. **56**, 403-409 (1917). Although the "canals" on Mars are not really canals, they had an indirect but profound influence on cosmology. Percival Lowell built Lowell Observatory to study the Solar System and Mars in particular, and closely directed the research of his staff. Slipher was instructed to study M31 and the other white nebulae under the hope that they were proto-solar-systems. (E)

180. **The Mathematical Theory of Relativity**, A. S. Eddington (Cambridge Univ., Cambridge, 1923), 161-2. (I)

181. "Cepheids in Spiral Nebulae," E. P. Hubble, Publ. Am. Astron. Soc. **5**, 261-264 (1925). Duncan had earlier found evidence for variable stars in M33, the spiral galaxy in Triangulum. (E)

182. "Periods of Twenty-five Variable Stars in the Small Magellanic Cloud", H. S. Leavitt, Harvard Coll. Obs. Circ. **173**, 1-3 (1912). Leavitt published a preliminary result in 1908 and Hertzsprung and Shapley helped develop the relation, but it would be another 4 decades (1952) before a reasonably accurate version became available (which led to a drastic revision of the distance scale). (E)

183. "An Estimate of the Distance of the Andromeda Nebula," E. Öpik, Astrophys. J. **55**, 406-410 (1922). (I)

184. "A Relation between Distance and Radial Velocity among Extra-Galactic Nebulae," E. Hubble, Proc. Natl. Acad. Sci. (USA) **15**, 168-173 (1929). In the mid 1920's Lundmark and Strömberg had already noted that more distant galaxies seemed to have spectra that were more redshifted. (E)

185. "Type Ia Supernovae and the Hubble Constant," D. Branch, Annu. Rev. Astron. Astrophys. **36**, 17-55 (1998). (I)

186. "Cosmological Implications from Observations of Type Ia Supernovae," B. Leibundgut, Annu. Rev. Astron. Astrophys. **39**, 67-98 (2001). (I)

187. "The Redshift-Distance and Velocity-Distance Laws," E. Harrison, Astrophys. J. **403**, 28-31 (1993). (E, I)

188. "Expanding Confusion: Common Misconceptions of Cosmological Horizons and the Superluminal Expansion of the Universe," T. M. Davis and C. H. Lineweaver, Pub. Astron. Soc. Australia **21**, 97-109 (2004). (I)

C. The hot early Universe and nucleosynthesis

As one looks out further in space (and so back in time, since light travels at finite speed) wavelengths of electromagnetic radiation we receive now have been redshifted further by the expansion and so Wien's law tells us (from the blackbody CMB) that the temperature was higher in the past. The younger Universe was a hotter, denser place. Lemaître ("the father of the Big Bang") emphasized the importance of accounting for the rest of known physics in the general relativistic cosmological models.

Early work on explaining the astrophysically observed abundances of elements assumed that they were a consequence of rapid thermal equilibrium reactions and that a rapidly falling temperature froze the equilibrium abundances. Tolman, Suzuki, von Weizsäcker, and others in the 1920's and 1930's argued that the observed helium-hydrogen ratio in this scenario required that at some point the temperature be at least 10^9 K (and possibly as much as 10^{11} K). Chandrasekhar and Henrich [189] performed the first detailed, correct equilibrium computation and concluded that no single set of temperature and density values can accommodate all the observed abundances; they suggested that it would be useful to consider a non-equilibrium process. Gamow, in 1946 [190] (building on his earlier work), makes the crucial point that in the Big Bang Model "the conditions necessary for rapid nuclear reactions were existing only for a very short time, so that it may be quite dangerous to speak about an equilibrium state", i.e., the Big Bang was the place to look for this non-equilibrium process.

Gamow [191], a student of Friedmann, and Alpher [192], a student of Gamow, estimated the radiation (photon) temperature at nucleosynthesis, and from the Stefan-Boltzmann law for blackbody radiation noted that the energy budget of the Universe must then have been dominated by radiation. Gamow [191] evolved the radiation to the much later epoch of matter-radiation equality (the matter and radiation energy densities evolve in different ways and this is the time at which both had the same magnitude), a concept also introduced by Gamow, while Alpher and Herman [193] predicted a residual CMB radiation at the present time from nucleosynthesis and estimated its present temperature to be 5 K (since the zero-redshift baryon density was not reliably known then, it is somewhat of a coincidence that this temperature estimate is close to the observed modern value). Hayashi [194] pointed out that at temperatures about 10 times higher than during nucleosynthesis rapid weak interactions lead to a thermal equilibrium abundance ratio of neutrons and protons determined by the neutron-proton mass difference, which becomes frozen in as the expansion decreases the temperature, thus establishing the initial conditions for nucleosynthesis. This is fortunate, in that an understanding of higher energy physics is not needed to make firm nucleosynthesis predictions; this is also unfortunate, since element abundance observations cannot be used to probe higher energy physics.

Alpher et al. [195] conclude the early period of the standard model of nucleosynthesis. By this point it was clear that initial hopes to explain all observed abundances in this manner must fail, because of the lack of stable nuclei at mass numbers 5 and 8 and because as the temperature drops with the expansion it becomes more difficult to penetrate the Coulomb barriers. Cosmological nucleosynthesis can only generate the light elements and the heavier elements are generated from these light elements by further processing in the stars.

Zel'dovich [196] and Smirnov [197] noted that the ^{4}He and D abundances are sensitive to the baryon density: the observed abundances can be used to constrain the baryon density. Hoyle and Tayler [198] carried out a detailed computation of the ^{4}He abundance and on comparing to measurements concluded "most, if not all, of the material of our .... Universe, has been 'cooked' to a temperature in excess of 10^{10} K". They were the first to note that the observed light element abundances were sensitive to the expansion rate during nucleosynthesis and that this could constrain new physics at that epoch (especially the number of light, relativistic, neutrino families).

After Penzias and Wilson measured the CMB (see below), Peebles [199, 200] computed the abundances of D, ^{3}He, and ^{4}He, and their dependence on, among other things, the baryon density and the expansion rate during nucleosynthesis. The monumental Wagoner et al. [201] paper established the ground rules for future work. For a history of these developments see pp. 125-128 and 240-241 of Ref. [88], the articles by Alpher and Herman and Wagoner on pp. 129-157 and 159-185, respectively, of Ref. [91], and Ch. 3 and Sec. 7.2 of Ref. [93]. See the cosmology textbooks cited above for the modern formalism. Accurate abundance predictions require involved numerical analysis; on the other hand pedagogy could benefit from the approximate semi-analytical models of Refs. [202, 203].

In the simplest nucleosynthesis scenario, the baryon density estimated from the observed D abundance is consistent with that estimated from WMAP CMB anisotropy data, and higher than that estimated from the ^{4}He and ^{7}Li abundances. This is further discussed in Sec. XII. References [204, 205, 206] are recent reviews of nucleosynthesis.

In addition to residual CMB radiation, there is also a residual neutrino background. Above a temperature of about 10^{10} K the CMB photons have enough energy to produce a thermal equilibrium abundance of neutrinos. Below this temperature the neutrinos decouple and freely expand, resulting in about 300 neutrinos per cubic centimeter now (with three families, and this number also includes antineutrinos), at a temperature of about 2 K, lower than that of the CMB because electron-positron annihilation heats the CMB a little. See Refs. [207, 208] and the more recent textbooks cited above for more detailed discussions of the (as yet undetected) neutrino background. We touch on neutrinos again in Sec. X.A.

189. "An Attempt to Interpret the Relative Abundances of the Elements and their Isotopes," S. Chandrasekhar and L. R. Henrich, Astrophys. J. **95**, 288-298 (1942). (I, A)

190. "Expanding Universe and the Origin of Elements," G. Gamow, Phys. Rev. **70**, 572-573 (1946). (E)

191. "Origin of Elements and the Separation of Galaxies," G. Gamow, Phys. Rev. **74**, 505-506 (1948). (E)

192. "A Neutron-Capture Theory of the Formation and Relative Abundance of the Elements," R. A. Alpher, Phys. Rev. **74**, 1577-1589 (1948). (I, A)

193. “Evolution of the Universe,” R. A. Alpher and R. Herman, Nature **162**, 774-775 (1948). (E)

194. “Proton-Neutron Concentration Ratio in the Expanding Universe at the Stages Preceding the Formation of the Elements,” C. Hayashi, Prog. Theor. Phys. **5**, 224-235 (1950). (I)

195. “Physical Conditions in the Initial Stages of the Expanding Universe,” R. A. Alpher, J. W. Follin, Jr., and R. Herman, Phys. Rev. **92**, 1347-1361 (1953). (I, A)

196. “The Theory of the Expanding Universe as Originated by A. A. Friedman,” Ya. B. Zel’dovich (1963), English translation in Sov. Phys. Usp. **6**, 475-494 (1964). (E, I)

197. “Hydrogen and He^4 Formation in the Prestellar Gamow Universe,” Yu. N. Smirnov (1964), English translation in Sov. Astron.–AJ **8**, 864-867 (1965). (I)

198. “The Mystery of the Cosmic Helium Abundance,” F. Hoyle and R. J. Tayler, Nature **203**, 1108-1110 (1964). (E, I)

199. “Primeval Helium Abundance and the Primeval Fireball,” P. J. E. Peebles, Phys. Rev. Lett. **16**, 410-413 (1966). (E, I)

200. “Primordial Helium Abundance and the Primordial Fireball. II,” P. J. E. Peebles, Astrophys. J. **146**, 542-552 (1966). (I, A)

201. “On the Synthesis of Elements at Very High Temperatures,” R. V. Wagoner, W. A. Fowler, and F. Hoyle, Astrophys. J. **148**, 3-49 (1967). (I, A)

202. “Cosmological Helium Production Simplified,” J. Bernstein, L. S. Brown, and G. Feinberg, Rev. Mod. Phys. **61**, 25-39 (1989). (I, A)

203. “Primordial Nucleosynthesis without a Computer,” R. Esmailzadeh, G. D. Starkman, and S. Dimopoulos, Astrophys. J. **378**, 504-518 (1991). (I, A)

204. “20. Big-Bang Nucleosynthesis,” B. D. Field and S. Sarkar, J. Phys. G **33**, 220-223 (2006). The latest version is available at `http://pdg.lbl.gov/`. (I)

205. “Primordial Nucleosynthesis for the New Cosmology: Determining Uncertainties and Examining Concordance,” R. H. Cyburt, Phys. Rev. D **70**, 023505 (2004). (I, A)

206. “Primordial Nucleosynthesis: Successes and Challenges,” G. Steigman, Int. J. Mod. Phys. E **15**, 1-36 (2006). (I, A)

207. “Neutrinos in Cosmology,” A. D. Dolgov, Phys. Rept. **370**, 333-535 (2002). (I, A)

208. "Primordial Neutrinos," S. Hannestad, Annu. Rev. Nucl. Part. Sci. **56**, 137-161 (2006). (I, A)

D. Theory and observations of the CMB

The CMB radiation contributes of order a percent of the static/"snow" seen when switching between channels on a television with a conventional VHF antenna; it is therefore not surprising that it had been detected a number of times before its 1965 discovery/identification. For instance, it is now known that in 1941 McKellar [209] deduced a CMB temperature of 2.3 K at a wavelength of 2.6 mm by estimating the ratio of populations in the first excited rotational and ground states of the interstellar cyanogen (CN) molecule (determined from absorption line measurements of Adams). It is now also known that the discrepancy of 3.3 K between the measured and expected temperature of the Bell Labs horn antenna (for communicating with the Echo I satellite) at a wavelength of 12.5 cm found in 1961 by Ohm [210] is due to the CMB. Ohm also notes that an earlier (1959) measurement with this telescope [211] ascribes a temperature of 2 ± 1 K to back and side lobe pick up, that this is "... temperature "not otherwise accounted for"...", and that "it is somewhat larger than the calculated temperature expected". Of course, McKellar had the misfortune of performing his analyses well before Gamow and collaborators had laid the nucleosynthesis foundations that would eventually explain the CN measurements (and allow the CMB interpretation) and Ohm properly did not overly stress the discrepancy beyond its weak statistical significance.

While Alpher and Herman (e.g., pp. 114-115 of Ref. [134] and p. 130 of Ref. [123]) privately raised the issue of searching for the CMB, and Hoyle came close to correctly explaining McKellar's CN measurements (see pp. 345-346 of Ref. [93]), Zel'dovich (1963-1965; p. 89, p. 491, and p. 315, respectively) [212, 196, 213], Doroshkevich and Novikov (1964) [214], and Dicke and Peebles (1965, p. 448) [215], are the first published discussions of possible observational consequences of the (then still hypothetical) CMB in the present Universe. The relevant discussions in Zel'dovich and Doroshkevich and Novikov are motivated by the same nucleosynthesis considerations that motivated Gamow and collaborators; Dicke and Peebles favored an oscillating Universe and needed a way to destroy heavy elements from the previous cycle and so postulated an initial hotter stage in each cycle. Both Doroshkevich and Novikov (1964) and Zel'dovich (1965) refer to Ohm [210] but neither appear to notice Ohm's 3.3 K discrepancy; in fact Zel'dovich (1965) (incorrectly) argues that Ohm constrains the temperature to be less than 1 K and given the observed helium abundance this rules out the hot Big Bang Model!

Working with the same antenna as Ohm, using the Dicke switching technique to compare the antenna temperature to a liquid helium load at a known temperature, and paying very careful attention to possible systematic effects, Penzias and Wilson (1965) [216] measure the excess temperature to be 3.5 ± 1 K at 7.35 cm wavelength; Dicke et al. (1965) [217] identify this as the CMB radiation left over from the hot Big Bang.

The CMB is the dominant component of the radiation density of the Universe, with a density now of about 400 CMB photons per cubic centimeter at a temperature of about 2.7 K now. As noted in the previous subsection, observed light element abundances in conjunction with nucleosynthesis theory allows for constraints on the density of baryonic matter. Thus, there are a few billion CMB photons for every baryon; the CMB photons carry most of the cosmological entropy.

To date there is no observational indication of any deviation of the CMB spectrum from a Planckian blackbody. Reference [21] reviews early measurements of the CMB spectrum. A definitive observation of the CMB spectrum was made by COBE (see Ref. [218] for a contemporaneous rocket-based measurement), which measured a temperature of 2.725 ± 0.002 K (95 % confidence) [219] and 95 % confidence upper limits on possible spectral distortions: $|\mu| < 9 \times 10^{-5}$ for the chemical potential of early ($10^5 < z < 3 \times 10^6$) energy release and $|y| < 1.5 \times 10^{-5}$ for Comptonization of the spectrum at later times [220]. Reference [221] shows that these constraints strongly rule out many alternatives to the Big Bang Model, including the steady state model and explosive galaxy formation.

Anisotropy of the CMB temperature, first detected by COBE [222], reveals important features of the formation and evolution of structure in the Universe. A small dipole anisotropy (discovered in the late 1960's and early 1970's by Conklin and Henry and confirmed by Corey and Wilkinson and Smoot, Gorenstein, and Muller) is caused by our peculiar motion: the CMB establishes a preferred reference frame. Higher multipole anisotropies in the CMB reflect the effect of primordial inhomogeneities on structure at the epoch of recombination and more complex astrophysical effects along the past light cone that alter this primordial anisotropy. We discuss these anisotropies, as well as the recently-detected polarization anisotropy of the CMB in Sec. X.B. The anisotropy signal from the recombination epoch allows precise estimation of cosmological parameters (see Sec. XII).

In addition to references cited above, Refs. [87] (pp. 64-70), [223, 224], Ch. 2 of [21], and [93] (Sec. 7.2) review the history. For the modern formalism see the more recent cosmology textbooks cited above and Ref. [225].

209. "Molecular Lines from the Lowest States of the Diatomic Molecules Composed of Atoms Probably Present in Interstellar Space," A. McKellar, Publ. Dominion Astrophys. Obs. **7**, 251-272 (1941). (E)

210. "Receiving System," E. A. Ohm, Bell Syst. Tech. J. **40**, 1065-1094 (1961). (I)

211. "Ultra-Low-Noise Antenna and Receiver Combination for Satellite or Space Communication," R. W. DeGrasse, D. C. Hogg, E. A. Ohm, and H. E. D. Scovil, Proc. Natl. Electronics Conf. **15**, 370-379 (1959). (I)

212. "The Initial Stages of the Evolution of the Universe," Ya. B. Zel'dovich (1963), English translation in Atomic Energy **14**, 83-91 (1963). (I)

213. "Survey of Modern Cosmology," Ya. B. Zel'dovich, Adv. Astron. Astrophys. **3**, 241-379 (1965). (E, I)

214. "Mean Density of Radiation in the Metagalaxy and Certain Problems in Relativistic Cosmology," A. G. Doroshkevich and I. D. Novikov (1964), English translation in Sov. Phys.–Dokl. **9**, 111-113 (1964). (E, I)

215. "Gravitation and Space Science," R. H. Dicke and P. J. Peebles, Space Sci. Rev. **4**, 419-460 (1965). (E, I)

216. "A Measurement of Excess Antenna Temperature at 4080 Mc/s," A. A. Penzias and R. W. Wilson, Astrophys. J. **142**, 419-421 (1965). (E)

217. "Cosmic Black-Body Radiation," R. H. Dicke, P. J. E. Peebles, P. G. Roll, and D. T. Wilkinson, Astrophys. J. **142**, 414-419 (1965). (E)

218. "Rocket Measurement of the Cosmic-Background-Radiation mm-Wave Spectrum," H. P. Gush, M. Halpern, and E. H. Wishnow, Phys. Rev. Lett. **65**, 537-540 (1990). (I, A)

219. "Calibrator Design for the COBE Far Infrared Absolute Spectrophotometer (FIRAS)," J. C. Mather, et al., Astrophys. J. **512**, 511-520 (1999). (I, A)

220. "The Cosmic Microwave Background Spectrum from the Full COBE FIRAS Data Set," D. J. Fixsen, et al., Astrophys. J. **473**, 576-587 (1996). (I)

221. "Interpretation of the COBE FIRAS CMBR Spectrum," E. L. Wright, et al., Astrophys. J. **420**, 450-456 (1994). (I)

222. "Structure in the COBE Differential Microwave Radiometer First-Year Maps," G. F. Smoot, et al., Astrophys. J. Lett. **396**, L1-L5 (1992). (I)

223. "Discovery of the Cosmic Microwave Background," R. W. Wilson, in **Serendipitous Discoveries in Radio Astronomy**, edited by K. Kellermann and B. Sheets (NRAO, Green Bank, 1983), 185-195. (E)

224. "Discovery of the 3 °K Radiation," D. T. Wilkinson and P. J. E. Peebles, in **The Cosmic Microwave Background: 25 Years Later**, edited by N. Mandolesi and N. Vittorio (Kluwer, Dordrecht, 1990), 17-31. (E)

225. "The Cosmic Microwave Background and Particle Physics," M. Kamionkowski and A. Kosowsky, Annu. Rev. Nucl. Part. Sci. **49**, 77-123 (1999). (I)

E. Challenges for the Big Bang Model

Since the Universe is now expanding, at earlier times it was denser and hotter. A naive extrapolation leads to a (mathematical) singularity at the beginning, with infinite density and temperature, at the initial instant of time, and over all space. This naive extrapolation is unjustified since the model used to derive it breaks down physically before the mathematical singularity is reached. Deriving the correct equations of motion for the very early Universe is an important area of current research.

While there has been much work, there is as yet no predictive model which unifies gravity and quantum mechanics — and this appears essential for an understanding of the very very early Universe, because as one goes back in time the gravitational expansion of the Universe implies that large cosmological length scales now correspond to tiny quantum mechanical scales in the very very early Universe. There is a small but active group of workers who believe that only a resolution of this issue (i.e., the derivation of a full quantum theory of gravity) will allow for progress on the modeling of the very early Universe. But most others now, perhaps inspired by the wonderful successes of particle physics models which have successively successfully described shorter and shorter distance physics, believe that it is important to try to solve some of the "problems" of the Big Bang Model by attempting to model the cosmophysical world at a higher energy density than is probed by nucleosynthesis and other lower redshift physics, but still well below the Planck energy density where quantum gravitational effects are important. And this is the approach we take in the following discussion, by focusing on "problems" that could be resolved below the Planck density. Whether Nature has chosen this path is as yet unclear, but at least the simplest versions of the inflation scenario (discussed in the next section) are compatible with current observations and will likely be well tested by data acquired within this decade.

With just nonrelativistic matter and radiation (CMB and neutrinos) in order of magnitude agreement with observations, the distance over which causal contact is possible grows with the age of the Universe. That is, if one assumes that in this model the cosmological principle is now valid because of "initial conditions" at an earlier time, then those initial conditions must be imposed over distances larger than the distance over which causal communication was possible. (And maybe this is what a quantum theory of gravitation will do for cosmology, but in the spirit of the earlier discussion we will view this as a "problem" of the Big Bang Model that should be resolved by physics at energies below the Planck scale.) Alpher et al. (p. 1349) [195] contains the earliest remarks (in passing) that we are aware of about this particle horizon problem. The terminology is due to Rindler (1956) [226] which is an early discussion of horizons in general. Harrison [227] also mentions the particle horizon problem in passing, but McCrea (1968) [228] and Misner (1969) [229] contain the first clear statements we are aware of, with Misner stating: "These Robertson-Walker models therefore give no insight into why the observed microwave radiation from widely different angles in the sky has ... very precisely ... the same temperature". Other early discussions are in Dicke (1970, p. 61) [87], Doroshkevich and Novikov [230], and the text books of Weinberg (pp. 525-526) [25] and Misner et al. (pp. 815-816) [26], This issue was discussed in many papers and books starting in the early 1970's, but the celebrated Dicke and Peebles review [231] is often credited with drawing prime-time attention to the particle horizon "problem".

The large entropy of the Universe (as discussed above, there now are a few billion CMB photons for every baryon) poses another puzzle. When the Universe was younger and hotter there had to have been a thermal distribution of particles and antiparticles, and as the Universe expanded and cooled particles and antiparticles annihilated into photons, resulting in the current abundance of CMB photons and baryons. Given the lack of a significant amount of antibaryons now, and the large photon to baryon ratio now, at early times there must have been a slight (a part in a few billion) excess of baryons over antibaryons. We return to this issue in the next section.

226. "Visual Horizons in World-Models," W. Rindler, Mon. Not. R. Astron. Soc. **116**, 662-677 (1956). (E, I)

227. "Baryon Inhomogeneity in the Early Universe," E. R. Harrison, Phys. Rev. **167**, 1170-1175 (1968). (E)

228. "Cosmology after Half a Century," W. H. McCrea, Science **160**, 1295-1299 (1968). (E)

229. "Mixmaster Universe," C. W. Misner, Phys. Rev. Lett. **22**, 1071-1074 (1969). (I, A)

230. "Mixmaster Universe and the Cosmological Problem," A. G. Doroshkevich and I. D. Novikov (1970), English translation in Sov. Astron.–AJ **14**, 763-769 (1971). (I, A)

231. "The Big Bang Cosmology — Enigmas and Nostrums," R. H. Dicke and P. J. E. Peebles, in **General Relativity: An Einstein Centenary Survey**, edited by S. W. Hawking and W. Israel (Cambridge Univ., Cambridge, 1979), 504-517. (E, I)

VIII. INFLATION

It is possible to trace a thread in the particle horizon problem tapestry back to the singularity issue and early discussions of Einstein, Lemaître, and others that viewed the singularity as arising from the unjustified assumption of exact isotropy, and led to the intensive study of homogeneous but anisotropic cosmological models in the late 1960's and early 1970's. This failed to tame the singularity but did draw attention to isotropy and the particle horizon problem of the standard Big Bang Model. It is interesting that this singularity issue also drove the development of the steady state picture, which in its earliest version was just a de Sitter model. While observations soon killed off the original steady state model (a more recent variant, the quasi-steady state model, can be adjusted to accommodate the data, see, e.g., Ref. [232]), the idea of a possible early, pre-Big-Bang, nonsingular de Sitter epoch thrived. It appears that Brout et al. (1978) [233] were the first to note that such a cosmological model was free of a particle horizon. However, they do not seem to make the connection that this could allow for isotropy by ensuring that points well separated now shared some common events in the past and thus causal physics could in principle make the Universe isotropic. Zee [234] noted that if one modifies the early Universe by speeding up the expansion rate enough over the expansion rate during the radiation dominated epoch, the particle horizon problem is resolved (but he does not go to the exponentially expanding de Sitter solution characteristic of the early inflation scenario). Sato [235, 236], Kazanas [237], and Guth [238] are the ones who make the (now viewed to be crucial) point that during a phase transition at very high temperature in grand unified models it is possible for the grand unified Higgs scalar field energy density to behave like a cosmological constant, driving a de Sitter exponential cosmological expansion, which results in a particle-horizon-free cosmological model. And the tremendous expansion during the de Sitter epoch will smooth out wrinkles in the matter distribution, by stretching them to very large scales, an effect alluded to earlier by Hoyle and Narlikar [239] in the context of the steady state model, which could result in an isotropic Universe now, provided the initial wrinkles satisfy certain conditions. See Refs. [240, 241] for caveats and criticism. Of course, to get the inflationary expansion started requires a large enough, smooth enough initial patch; the contemporary explanation appeals to probability: loosely, such a patch will exist somewhere and inflation will start there. In addition the initial conditions issue is not completely resolved by inflation, only greatly alleviated: since inflation stretches initially small length scales to length scales of contemporary cosmological interest, the cosmological principle requires there not be very large irregularities on very small length scales in the very early Universe. This could be a clue to what might be needed from a model of very high energy, pre-inflation, physics. For reviews of inflation see the more recent books listed in Sec. III.A.1 as well as those in Sec. III.A.5.

Building on ideas of Brout, Englert, and collaborators, Gott [242] noted that it was possible to have inflation result in a cosmological model with open spatial hypersurfaces at the present time, in contrast to the Sato-Kazanas-Guth discussion which focused on flat spatial hypersurfaces. This open-bubble inflation model, in which the observable part of the contemporary Universe resides inside a bubble nucleated (because of a small upward "bump" in the potential energy density function) between two distinct epochs of inflation, is a clear counter-example to the oft-repeated (but incorrect) claim that inflation explains why the Universe appears to have negligible space curvature. See Refs. [243, 244] for a more detailed discussion of this model.

The open-bubble inflation model was the first consistent inflation model: in the Guth model as the phase transition completes and one hopes to have a smooth transition to the more familiar radiation-dominated expansion of the hot Big Bang Model one finds that the potential Guth worked with results in many small bubbles forming with most of energy density residing in the bubble walls, a very inhomogeneous distribution. In this model the Universe at the end of inflation was very inhomogeneous since the bubble collisions were not rapid enough to thermalize the bubble wall energy density (i.e., the bubbles did not "percolate"). Linde [245] and Albrecht and Steinhardt [246] used a specific potential energy density function for the Higgs field in a grand unified model and implemented Gott's scenario in the Sato-Kazanas-Guth picture, except they argued that the second epoch of inflation lasts much longer than Gott envisaged and so stretches the bubble to length scales much larger than the currently observable part of the Universe, thus resulting in flat spatial hypersurfaces now. The great advantage of the Gott scenario is that it uses the first epoch of inflation to resolve the particle horizon/homogeneity problem and so this problem does not constrain the amount of inflation after the bubble nucleates. References [233, 247] note that symmetry forces the nucleating bubble to have an open geometry, and this is why inflation requires open spatial hypersurfaces, but with significant inflation after bubble nucleation the radius of curvature of these hypersurfaces can be huge. Thus the amount of space curvature in the contemporary Universe is a function of the amount of inflation after bubble nucleation, and it is now widely accepted that observational data (as discussed below in Sec. X.B) are consistent with an insignificant amount of space curvature and thus significant inflation after bubble nucleation.

It is well known that phase transitions can create topological defects; grand unified phase transitions are no exception and often create monopoles and other topological defects. If the Universe is also inflating through this phase transition then the density of such topological defects can be reduced to levels consistent with the observations. This is not another argument in support of inflation, although it is often claimed to be: it is just a way of using inflation to make viable a grand unified theory that is otherwise observationally inconsistent. One major motivation for grand unification is that it allows for a possible explanation of the observed excess of matter over antimatter (or the baryon excess) mentioned in the previous section. There are other possible explanations of how this baryon excess might have come about. One much discussed alternative is the possibility of forming it at the much lower temperature electroweak phase transition, through a non-perturbative process, but this might raise particle horizon/homogeneity issues. However, at present there is no convincing, numerically satisfying explanation of the baryon excess, from any process. References [147, 248, 249, 250] review models now under discussion for generating the baryon excess.

At the end of inflation, as the phase transition completes and the Universe is said to reheat, one expects the generation of matter and radiation as the Universe makes the transition from rapid inflationary expansion to the more sedate radiation-dominated expansion of the hot Big Bang Model. This is an area of ongoing research and it would be useful to have a convincing, numerically satisfying model of this epoch. The baryon excess might be generated during this reheating process.

While great effort has been devoted to it, resulting in a huge number of different models, at the present stage of development inflation is a very interesting general scenario desperately in need of a more precise and more convincing very high energy particle physics based realization. As far as large-scale cosmology is concerned, inflation in its simplest form is modeled by a scalar field (the inflaton) whose potential energy density satisfies certain properties that result in a rapid enough cosmological expansion at early times. It is interesting that cosmological observations within this decade might firm up this model of the very early Universe based on very high energy physics before particle physicists do so. For reviews see the more recent books in Secs. III.A.1 and III.A.5.

Assuming an early epoch of inflation, the cumulative effect of the expansion of the Universe from then to the present means that contemporary cosmological length scales (e.g., the length scale that characterizes the present galaxy distribution) correspond to very tiny length scales during inflation, so tiny that zero-point quantum-mechanical fluctuations must be considered in any discussion involving physics on these length scales.

As mentioned above, the idea of an early de-Sitter-like expansion epoch, pre-Big-Bang, was discussed in the 1970's, as a possible way of taming the initial singularity. While this de Sitter epoch was typically placed at very high energy, it differs significantly from the inflation scenario in that it was not driven by a scalar field potential energy density. Nevertheless since it was at energies close to the Planck energy there were many discussions of quantum mechanical fluctuations in de Sitter spacetime in the 1970's.

In the inflation case quantum mechanics introduces additional fluctuations, the zero-point fluctuations in the scalar field. This was noted by Hawking, Starobinsky, and Guth and Pi [251, 252, 253], and further studied in Ref. [254]. For a discussion of scalar field quantum fluctuations in de Sitter spacetime and their consequences see Ref. [255]. Fischler et al. [256] use the Dirac-Wheeler-DeWitt formalism to consistently semi-classically quantize both gravitation and the scalar field about a de Sitter background, and carry through a computation of the power spectrum of zero-point fluctuations. The simplest inflation models have a weakly coupled scalar field and so a linear perturbation theory computation suffices and the fluctuations obey Gaussian statistics and so can be completely characterized by their two-point correlation function or equivalently their power spectrum. Inflation models that give non-Gaussian fluctuations are possible (for a review see Ref. [257]), but the observations do not yet demand this, being almost completely consistent with Gaussianity (see discussion in Sec. X.B below). The simplest models give adiabatic or curvature (scalar) fluctuations; these are what result from adiabatically compressing or decompressing parts of an exactly spatially homogeneous Universe. More complicated models of inflation can produce fluctuations that break adiabaticity, such as (tensor) gravitational waves [258] and (vector) magnetic fields [259, 260], which might have interesting observational consequences (see Secs. X.A and X.B below).

The power spectrum of energy density fluctuations depends on the model for inflation. If the scalar field potential energy density during inflation is close to flat and dominates the scalar field energy density, the scale factor grows exponentially with time (this is the de Sitter model), and after inflation but at high redshift the power spectrum of (scalar) mass-energy density fluctuations with wavenumber magnitude k is proportional to k, or scale invariant, on all interesting length scales, i.e., curvature fluctuations diverge only as $\log k$. This was noted in Refs. [251, 252, 253] in the early 1980's for the inflation model, although the virtues of a scale-invariant spectrum were emphasized in the early 1970's, well before inflation, by Harrison [261], Peebles and Yu [262], and Zel'dovich [263]. When the scalar field potential energy density is such that the scalar field kinetic energy density is also significant during inflation a more general spectrum proportional to k^n can result (where the spectral index n depends on the slope of the potential energy density during inflation); for $n \neq 1$ the spectrum is said to be tilted [264, 265, 266]. Current observations appear to be reasonably well fit by $n = 1$. More complicated, non-power-law spectra are also possible.

We continue this discussion of fluctuations in Sec. X below.

232. "Inhomogeneities in the Microwave Background Radiation Interpreted within

the Framework of the Quasi-Steady State Cosmology," J. V. Narlikar, et al., Astrophys. J. **585**, 1-11 (2003). (I, A)

233. "The Creation of the Universe as a Quantum Phenomenon," R. Brout, F. Englert, and E. Gunzig, Ann. Phys. **115**, 78-106 (1978). (I)

234. "Horizon Problem and the Broken-Symmetric Theory of Gravity," A. Zee, Phys. Rev. Lett. **44**, 703-706 (1980). (I)

235. "Cosmological Baryon-Number Domain Structure and the First Order Phase Transition of a Vacuum," K. Sato, Phys. Lett. B **99**, 66-70 (1981). (I)

236. "First Order Phase Transition of a Vacuum and the Expansion of the Universe," K. Sato, Mon. Not. R. Astron. Soc. **195**, 467-479 (1981). (I, A)

237. "Dynamics of the Universe and Spontaneous Symmetry Breaking," D. Kazanas, Astrophys. J. Lett. **241**, L59-L63 (1980). (I, A)

238. "Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems," A. H. Guth, Phys. Rev. D **23**, 347-356 (1981). (I, A)

239. "Mach's Principle and the Creation of Matter," F. Hoyle and J. V. Narlikar, Proc. R. Soc. London A **270**, 334-339 (1962). (I)

240. "Horizons in Inflationary Universes," G. F. R. Ellis and W. Stoeger, Class. Quantum Grav. **5**, 207-220 (1988). (I)

241. "Inflation For Astronomers," J. V. Narlikar and T. Padmanabhan, Annu. Rev. Astron. Astrophys. **29**, 325-362 (1991). (I)

242. "Creation of Open Universes from De Sitter Space," J. R. Gott, III, Nature **295**, 304-307 (1982). (I)

243. "Cold Dark Matter Cosmogony in an Open Universe," B. Ratra and P. J. E. Peebles, Astrophys. J. Lett. **432**, L5-L9 (1994). (I)

244. "Inflation in an Open Universe," B. Ratra and P. J. E. Peebles, Phys. Rev. D **52**, 1837-1894 (1995). (I, A)

245. "A New Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy, and Primordial Monopole Problems," A. D. Linde, Phys. Lett. B **108**, 389-393 (1982). (I, A)

246. "Cosmology for Grand Unified Theories with Radiatively Induced Symmetry Breaking," A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220-1223 (1982). (I, A)

247. "Gravitational Effects on and of Vacuum Decay," S. Coleman and F. De Luccia, Phys. Rev. D **21**, 3305-3315 (1980). (A)

248. "Origin of the Matter-Antimatter Asymmetry," M. Dine and A. Kusenko, Rev. Mod. Phys. **76**, 1-30 (2004). (I, A)

249. "Baryogenesis and Leptogenesis," M. Trodden, eConf **C040802**, L018 (2004). Available at `www.stanford.slac.edu/econf/c040802/proceedings.htm`. (I, A)

250. "Baryogenesis," J. M. Cline, hep-ph/0609145. (I, A)

251. "The Development of Irregularities in a Single Bubble Inflationary Universe," S. W. Hawking, Phys. Lett. B **115**, 295-297 (1982). (I, A)

252. "Dynamics of Phase Transition in the New Inflationary Universe Scenario and Generation of Perturbations," A. A. Starobinsky, Phys. Lett. B **117**, 175-178 (1982). (I, A)

253. "Fluctuations in the New Inflationary Universe," A. H. Guth and S.-Y. Pi, Phys. Rev. Lett. **49**, 1110-1113 (1982). (I, A)

254. "Spontaneous Creation of Almost Scale-Free Density Perturbations in an Inflationary Universe," J. M. Bardeen, P. J. Steinhardt, and M. S. Turner, Phys. Rev. D **28**, 679-693 (1983). (A)

255. "Restoration of Spontaneously Broken Continuous Symmetries in De Sitter Spacetime," B. Ratra, Phys. Rev. D **31**, 1931-1955 (1985). (I, A)

256. "Quantum Mechanics of Inflation," W. Fischler, B. Ratra, and L. Susskind, Nucl. Phys. B **259**, 730-744 (1985). (A)

257. "Non-Gaussianity from Inflation: Theory and Observations," N. Bartolo, E. Komatsu, S. Matarrese, and A. Riotto, Phys. Rept. **402**, 103-266 (2004). (A)

258. "Graviton Creation in the Inflationary Universe and the Grand Unification Scale," V. A. Rubakov, M. V. Sazhin, and A. V. Veryaskin, Phys. Lett. B **115**, 189-192 (1982). (A)

259. "Inflation-Produced, Large-Scale Magnetic Fields," M. S. Turner and L. M. Widrow, Phys. Rev. D **37**, 2743-2754 (1988). (A)

260. "Cosmological "Seed" Magnetic Field From Inflation," B. Ratra, Astrophys. J. Lett. **391**, L1-L4 (1992). (A)

261. "Fluctuations at the Threshold of Classical Cosmology," E. R. Harrison, Phys. Rev. D **1**, 2726-2730 (1970). (A)

262. "Primeval Adiabatic Perturbation in an Expanding Universe," P. J. E. Peebles and J. T. Yu, Astrophys. J. **162**, 815-836 (1970). (A)

263. "A Hypothesis, Unifying the Structure and the Entropy of the Universe," Ya. B. Zel'dovich, Mon. Not. R. Astron. Soc. **160**, 1P-3P (1972). (A)

264. "Constraints on Generalized Inflationary Cosmologies," L. F. Abbott and M. B. Wise, Nucl. Phys. B **244**, 541-548 (1984). (A)

265. "Power-Law Inflation," F. Lucchin and S. Matarrese, Phys. Rev. D **32**, 1316-1322 (1985). (A)

266. "Inflation in an Exponential-Potential Scalar Field Model," B. Ratra, Phys. Rev. D **45**, 1913-1952 (1992). (A)

IX. DARK MATTER AND DARK ENERGY

Most cosmologists are of the firm opinion that observations indicate the energy budget of the contemporary Universe is dominated by far by dark energy, with the next most significant contributor being dark matter, with ordinary baryonic matter in distant third place. Dark energy and dark matter are hypothetical constructs generated to explain observational data, and the current model provides a good, but not perfect, explanation of contemporary cosmological observations. However, dark energy and dark matter have not been directly detected (in the lab or elsewhere).

In 1926 Hubble [267] presented the first systematic estimate of masses of the luminous part of galaxies (based on studying the motion of stars in galaxies), as well as an estimate of the mass density of the Universe (using counts of galaxies in conjunction with the estimated masses of galaxies).

Under similar assumptions (the validity of Newton's second law of motion and Newton's inverse-square law of gravitation, and that the large-scale structure under investigation is in gravitational equilibrium), Zwicky (1933) [268], in what many now believe will prove to be one of the more significant discoveries of the previous century, found that galaxies in the Coma cluster of galaxies were moving with surprisingly high speeds, indicative, in modern terms, of a Coma cluster mass density at least an order of magnitude greater than what would be expected from spreading the mass associated with the luminous parts of the galaxies in the Coma cluster over the volume of the cluster. Zwicky's measurements probe larger length scales than Hubble's and so might be detecting mass that lies outside the luminous parts of the galaxies, i.e., mass that does not shine, or dark matter. Ordinary baryonic matter is largely nonrelativistic in the contemporary Universe and hence would be pulled in by the gravitational field of the cluster. Nucleosynthesis and CMB anisotropy measurements constrain the mass density of ordinary baryonic matter, and modern data indicate that not only is the amount of gravitating mass density detected in Zwicky-like observations significantly greater than what is shining, it is likely a factor of 3 to 5 times the mass density of ordinary baryonic matter. (It is also known that a large fraction of the expected baryonic matter can not significantly shine.) Smith [269] confirmed Zwicky's result, using Virgo cluster measurements, and Zwicky soon followed up with a more detailed paper [270].

Babcock's (1939) Ph.D. thesis [271] was the next major (with hindsight) development in the dark matter story: he measured the rotation speed of luminous objects in or near the disk of the Andromeda (M31) galaxy, out to, in modern terms, a distance of almost 20 kpc from the center, and found that the rotation speed was still rising, not exhibiting the $1/\sqrt{r}$ Keplerian fall off with distance r from the center expected if the mass distribution in M31 followed the distribution of the light. That is, Babcock found that the outer part of the luminous part of M31 was dominated by matter that did not shine. Soon thereafter Oort (1940) [272] noted a similar result for the galaxy NGC 3115. Almost two decades later, van de Hulst et al. [273] confirmed Babcock's result by using 21 cm wavelength observations of hydrogen gas clouds that extend beyond the luminous part of M31, finding a roughly flat rotation curve at the edge (no longer rising with distance as Babcock had found). While there was some early theoretical discussion of this issue, the much more detailed M31 flat rotation curve measured by Rubin and Ford in 1970 (Rubin was a student of Gamow) [274] forced this dark matter into the limelight.

Other early indications of dark matter came from measurements of the velocities of binary galaxies [275] and the dynamics of our Local Group of galaxies [276]. References [277, 278] found that the elliptical galaxy M87 in the Virgo cluster had a faint mass-containing halo. In 1973 Ostriker and Peebles [279] noted that one way of making the disk of a spiral galaxy stable against a bar-like instability is to embed it in a massive halo, and soon thereafter Einasto et al. [280] and Ostriker et al. [281] showed that this suggestion was consistent with the observational evidence. These early results have been confirmed by a number of different techniques, including measuring the X-ray temperature of hot gas in galaxy clusters (which is a probe of the gravitational potential — and the mass which generates it — felt by the gas), and measurements of gravitational lensing of background sources by galaxy clusters. See Sec. XII for further discussion of this.

For reviews of dark matter see Sec. IV of Ref. [88] (note the fascinating comment on p. 64 on the issue of dark matter in clusters: "This quantity" M/L or the mass to luminosity ratio "is suspect because when it is used to estimate the masses of groups or clusters of galaxies the result often appears to be unreasonable", i.e., large), Refs. [282, 283, 284], Sec. 18 of Ref. [4], and Ref. [285].

Much as van Maanen's measurements of large (but erroneous) rotation velocities for a number of galaxies prompted Jeans (1923) [286] to consider a modification of Newton's inverse-square law for gravity such that the gravitational force fell off slower with distance on large distances, the large (but not erroneous) velocities measured by Zwicky and others prompted Finzi (1963) [287], and many since then, to consider modifications of the law of gravity. The current observational indications are that this is not a very viable alternative to the dark matter hypothesis (see Secs. IV.A.1 and IV.B.13 of Ref. [288]). In some cases, modern high energy physics suggests possible motivations for modifications of the inverse square law on various length scales; this is beyond the scope of our review.

Milgrom [289, 158] proposes a related but alternate hypothesis: Newton's second law of motion is modified at low accelerations, modified Newtonian dynamics (MOND). MOND does a remarkable job at fitting the flat rotation curves of spiral galaxies, but most who have cared to venture an informed opinion believe that it cannot do away completely with dark matter, especially in low-surface-brightness dwarf galaxies and rich clusters of galaxies. More importantly, the lack of a well motivated extension of the small-length-scale phenomenological MOND hypothesis that is applicable on large cosmological length scales greatly hinders testing the hypothesis. (For a recent attempt at such an extension see Ref. [290]; for a preliminary sketch of cosmology in this context see Ref. [291].) For a review of MOND see Ref. [292].

Most cosmologists are convinced that dark matter exists. Nucleosynthesis constraints indicate that most of the dark matter is not baryonic. (Not all baryons shine; for a review of options for dark baryons see Ref. [293].) Galaxies are in general older than larger-scale structures (such as clusters); this indicates that the dark matter primeval velocity dispersion is small (for if it were large gravity would be able to overcome the corresponding pressure only on large mass — and so length — scales, first forming large-scale objects that fragment later into younger smaller-scale galaxies). Dark matter with low primeval velocity dispersion is known as cold dark matter (CDM). More precisely, the CDM model assumes that most of the non-relativistic matter-energy of the contemporary Universe is in the form of a gas of massive, non-baryonic, weakly-interacting particles with low primeval velocity dispersion. (One reason they must be weakly interacting is so they do not shine. References [294, 295, 296] review particle physics dark matter candidates and prospects for experimental detection.) Bond et al. [297] and Blumenthal et al. [298] note the advantages of CDM and that modern high energy physics models provide plausible hypothetical candidates for these particles. Peebles [299] casts the cosmological skeleton of the CDM model, emphasizing that in this model structure forms from the gravitational growth of primordial departures from homogeneity that are Gaussian, adiabatic, and scale invariant, consistent with what is expected from the simplest inflation models. Reference [300] is a first fleshing out of the CDM model. See Refs. [4, 52] for textbook discussions of the CDM model. More details about this model, including possible problems, are given in Sec. X below.

To set the numerical scale for cosmological mass densities, following Einstein and de Sitter (1932) [301], one notes that the simplest Friedmann-Lemaître model relevant to the contemporary universe is that with vanishing space curvature and with energy budget dominated by non-relativistic matter (and no cosmological constant). In this critical or Einstein-de Sitter case the Friedmann equation fixes the energy density of nonrelativistic matter for a given value of the Hubble constant; cosmologists then define the mass-energy density parameter Ω for each type of mass-energy (including that of the curvature of spatial hypersurfaces $\Omega_{\rm K}$, the cosmological constant Ω_Λ, and nonrelativistic matter $\Omega_{\rm M}$) as the ratio of that mass-energy density to the critical or Einstein-de Sitter model mass-energy density, and the Friedmann equation implies that the mass-energy density parameters sum to unity. (In general the Ω's are time dependent; in what follows numerical values for these parameters refer to the current epoch.)

As discussed in Sec. XII below, it has long been known that nonrelativistic matter (baryons and CDM) contributes about 25 or 30 % to this sum. After the development of the inflation picture for the very early Universe in the 1980's there was a widespread belief that space curvature could not contribute to the mass-energy budget (this is not necessary, as discussed above), and for this and a few other reasons (among others, the time scale problem arising from the large measured values of the Hubble constant and age of the Universe), Peebles (1984) [302] proposed that Einstein's cosmological constant contributed the remaining 70 or 75 % of the mass-energy of the Universe. This picture was soon generalized to allow the possibility of a slowly varying in time, close to homogeneous in space, scalar field energy density, what is now called dark energy [303, 304]. As discussed in Sec. XII below, these models predict that the expansion of the Universe is now accelerating and, indeed, it appears that this acceleration has been detected at about the magnitude predicted in these models [305, 306]. Consistent with this, CMB anisotropy observations indicate that the spatial hypersurfaces are flat, which in conjunction with the low mass-energy density parameter for non-relativistic matter also requires a significant amount of dark energy. These issues are discussed in more detail in Sec. XII below. See Refs. [288, 307, 308, 309, 310, 311, 312, 313] for reviews.

The following sections flesh out this "standard model" of cosmology, elaborating on the model as well as describing the measurements and observations on which it is based.

267. "Extra-Galactic Nebulae," E. Hubble, Astrophys. J. **64**, 321-369 (1926). In this paper, among other things, Hubble also developed his galaxy classification scheme (of ellipticals, normal and barred spirals, and irregulars) and showed that the averaged large-scale galaxy distribution is spatially isotropic. Öpik [183] had earlier estimated the mass of M31. (E, I)

268. "Die Rotverschiebung von Extragalaktischen Nebeln," F. Zwicky, Helvetica Physica Acta **6**, 110-127 (1933). An English translation should appear in Gen. Relativ. Gravit. in 2008. (E)

269. "The Mass of the Virgo Cluster," S. Smith, Astrophys. J. **83**, 23-30 (1936). (E)

270. "On the Masses of Nebulae and Clusters of Nebulae," F. Zwicky, Astrophys. J. **86**, 217-246 (1937). In this paper Zwicky also proposes the remarkable idea of using gravitational lensing of background objects by foreground clusters of galaxies to estimate cluster masses. (E, I)

271. "The Rotation of the Andromeda Nebula," H. W. Babcock, Lick Obs. Bull. **19**, 41-51 (1939). (E, I)

272. "Some Problems Concerning the Structure and Dynamics of the Galactic System and the Elliptical Nebulae NGC 3115 and 4494," J. H. Oort, Astrophys. J. **91**, 273-306 (1940). (I)

273. "Rotation and Density Distribution of the Andromeda Nebula derived from Observations of the 21-cm Line," H. C. van de Hulst, E. Raimond, and H. van Woerden, Bull. Astron. Inst. Netherlands **14**, 1-16 (1957). (I)

274. "Rotation of the Andromeda Nebula from a Spectroscopic Survey of the Emission Regions," V. C. Rubin and W. K. Ford, Jr., Astrophys. J. **159**, 379-403 (1970). (I)

275. "Radial Velocities and Masses of Double Galaxies," T. Page, Astrophys. J. **116**, 63-80 (1952). (I)

276. "Intergalactic Matter and the Galaxy," F. D. Kahn and L. Woltjer, Astrophys. J. **130**, 705-717 (1959). (I, A)

277. "Photometry of the Outer Corona of M87," G. de Vaucouleurs, Astrophys. Lett. **4**, 17-22 (1969). (E, I)

278. "The Large Optical Extent of M87," H. Arp and F. Bertola, Astrophys. Lett. **4**, 23-25 (1969). (E, I)

279. "A Numerical Study of the Stability of Flattened Galaxies: Or Can Cold Galaxies Survive?" J. P. Ostriker and P. J. E. Peebles, Astrophys. J. **186**, 467-480 (1973). (A)

280. "Dynamic Evidence on Massive Coronas of Galaxies," J. Einasto, A. Kaasik, and E. Saar, Nature **250**, 309-310 (1974). (I)

281. "The Size and Mass of Galaxies, and the Mass of the Universe," J. P. Ostriker, P. J. E. Peebles, and A. Yahil, Astrophys. J. Lett. **193**, L1-L4 (1974). (I)

282. "Masses and Mass-to-Light Ratios of Galaxies," S. M. Faber and J. S. Gallagher, Annu. Rev. Astron. Astrophys. **17**, 135-187 (1979). (I)

283. "Existence and Nature Of Dark Matter in the Universe," V. Trimble, Annu. Rev. Astron. Astrophys. **25**, 425-472 (1987). (I)

284. "Dark Matter in Galaxies," K. M. Ashman, Publ. Astron. Soc. Pac. **104**, 1109-1138 (1992). (I)

285. "Dark Matter: Early Considerations," J. Einasto, in Ref. [81], 241-262. (I)

286. "Internal Motions in Spiral Nebulae," J. H. Jeans, Mon. Not. R. Astron. Soc. **84**, 60-76 (1923). (I, A)

287. "On the Validity of Newton's Law at a Long Distance," A. Finzi, Mon. Not. R. Astron. Soc. **127**, 21-30 (1963). (I)

288. "The Cosmological Constant and Dark Energy," P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559-606 (2003). (E, I)

289. "A Modification of the Newtonian Dynamics as a Possible Alternative to the Hidden Mass Hypothesis," M. Milgrom, Astrophys. J. **270**, 365-370 (1983). (E, I)

290. "Relativistic Gravitation Theory for the MOND Paradigm," J. D. Bekenstein, Phys. Rev. D **70**, 083509 (2004), erratum **71**, 069901 (2005). (A)

291. "Inflation and Accelerated Expansion Tensor-Vector-Scalar Cosmological Solutions," L. M. Diaz-Rivera, L. Samushia, and B. Ratra, Phys. Rev. D **73**, 083503 (2006). (A)

292. "Modified Newtonian Dynamics as an Alternative to Dark Matter," R. H. Sanders and S. S. McGaugh, Annu. Rev. Astron. Astrophys. **40**, 263-317 (2002). (I)

293. "Baryonic Dark Matter," B. Carr, Annu. Rev. Astron. Astrophys. **32**, 531-590 (1994). (I)

294. "Dark Matter Detection in the Light of Recent Experimental Results," C. Muñoz, Int. J. Mod. Phys. A **19**, 3093-3170 (2004). (I, A)

295. "Particle Dark Matter: Evidence, Candidates and Constraints," G. Bertone, D. Hooper, and J. Silk, Phys. Rept. **405**, 279-390 (2005). (I, A)

296. "Dark Matter Candidates," E. A. Baltz, eConf **C040802**, L002 (2004). Available at `www.stanford.slac.edu/econf/c040802/proceedings.htm`. (I)

297. "Formation of Galaxies in a Gravitino-Dominated Universe," J. R. Bond, A. S. Szalay, and M. S. Turner, Phys. Rev. Lett. **48**, 1636-1639 (1982). (A)

298. "Galaxy Formation by Dissipationless Particles Heavier than Neutrinos," G. R. Blumenthal, H. Pagels, and J. R. Primack, Nature **299**, 37-38 (1982). (A)

299. "Large-Scale Background Temperature and Mass Fluctuations due to Scale-Invariant Primeval Perturbations," P. J. E. Peebles, Astrophys. J. Lett. **263**, L1-L5 (1982). (A)

300. "Formation of Galaxies and Large Scale Structure with Cold Dark Matter," G. R. Blumenthal, S. M. Faber, J. R. Primack, and M. J. Rees, Nature **311**, 517-525 (1984). (A)

301. "On the Relation between the Expansion and the Mean Density of the Universe," A. Einstein and W. de Sitter, Proc. Natl. Acad. Sci. (USA) **18**, 213-214 (1932). (E, I)

302. "Tests of Cosmological Models Constrained by Inflation," P. J. E. Peebles, Astrophys. J. **284**, 439-444 (1984). (A)

303. "Cosmology with a Time-Variable Cosmological "Constant"," P. J. E. Peebles and B. Ratra, Astrophys. J. Lett. **325**, L17-L20 (1988). (I, A)

304. "Cosmological Consequences of a Rolling Homogeneous Scalar Field," B. Ratra and P. J. E. Peebles, Phys. Rev. D **37**, 3406-3427 (1988). (A)

305. "Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant," A. G. Riess, et al., Astron. J. **116**, 1009-1038 (1998). (I, A)

306. "Measurements of Ω and Λ from 42 High-Redshift Supernovae," S. Perlmutter, et al., Astrophys. J. **517**, 565-586 (1999). (I, A)

307. "A Quintessential Introduction to Dark Energy," P. J. Steinhardt, Phil. Trans. R. Soc. Lond. A **361**, 2497-2513 (2003). (I, A)

308. "Why is the Universe Accelerating?," S. M. Carroll, in **Measuring and Modeling the Universe**, edited by W. L. Freeman (Cambridge Univ., Cambridge, 2004), 235-255. (I)

309. "Dark Energy: The Cosmological Challenge of the Millenium," T. Padmanabhan, Curr. Sci. **88**, 1057-1069 (2005). (I, A)

310. "Accelerating Universe: Observational Status and Theoretical Implications," L. Perivolaropoulos, in **Recent Advances in Astronomy and Astrophysics**, edited by N. Solomos (AIP, Melville, 2006), 698-712. (I, A)

311. "Dynamics of Dark Energy," E. J. Copeland, M. Sami, and S. Tsujikawa, Int. J. Mod. Phys. D **15**, 1753-1936 (2006). (A)

312. "The Cosmological Constant Problem, an Inspiration for New Physics," S. Nobbenhuis, Utrecht Univ. Ph.D. thesis, gr-qc/0609011 (2006). (A)

313. "Reconstructing Dark Energy," V. Sahni and A. Starobinsky, Int. J. Mod. Phys. D **15**, 2105-2132 (2006). (A)

X. GROWTH OF STRUCTURE

A. Gravitational instability and microphysics in the expanding Universe

1. Gravitational instability theory from Newton onwards

The primary driver for the formation of large-scale structure in the Universe is gravitational instability. The detailed growth of structure depends on the nature of the initial fluctuations, the background cosmology, and the constituents of the mass-energy density, as causal physics influences the rate at which structure may grow on different scales.

Newton, prompted by questions posed to him by Bentley, realized that a gas of randomly-positioned massive particles interacting gravitationally in flat spacetime is unstable, and that as time progresses the mass density distribution grows increasingly more anisotropic and inhomogeneous. Awareness of this instability led Newton to abandon his preference for a finite and bounded Universe of stars for one that is infinite and homogeneous on average (see discussion in Ref. [10]); this was an early discussion of the cosmological principle.

Jeans (1902) [314] studied the stability of a spherical distribution of gravitating gas particles in flat spacetime, motivated by possible relevance to the process of star formation. He discovered that gas pressure prevents gravitational collapse on small spatial scales and gives rise to acoustic oscillations in the mass density inhomogeneity, as pressure gradient and gravitational forces compete. On large scales the gravitational force dominates and mass density inhomogeneities grow exponentially with time. The length scale on which the two forces balance has come to be known as the Jeans length or the acoustic Hubble length c_s/H_0, where c_s is the speed of sound.

On scales smaller than the Jeans length, adiabatic energy density perturbations oscillate as acoustic waves. On scales well below the Jeans length dissipative fluid effects (e.g., viscosity and radiation diffusion) must be accounted for. These effects remove energy from the acoustic waves, thus damping them. In an expanding Universe, damping is effective when the dissipation time scale is shorter than the expansion time scale, and the smallest length scale for which this is the case is called the damping length. This is discussed in more detail below.

2. Structure growth in an expanding Universe

Study of gravitational instability in an evolving spacetime, appropriate for the expanding Universe, began with Lemaître in the early 1930's. He pioneered two approaches, both of which are still in use: a "nonperturbative" approach based on a spherically symmetric solution of the Einstein equations (further developed by Dingle, Tolman, Bondi, and others and discussed in Sec. X.C below); and a "perturbative" approach in which one studies small departures from spatial homogeneity and isotropy evolving in homogeneous and isotropic background spacetimes.

At early times, and up to the present epoch on sufficiently large scales, the growth of structure by gravitational instability is accurately described by linear perturbation theory. The growth of small density and velocity perturbations must take into account the effects of the expansion of the Universe. A fully relativistic theory must be employed to describe the growth of structure, since it is necessary to also describe the evolution of modes with wavelength larger than the Hubble length, although a Newtonian approximation is valid and used on smaller length scales.

Lifshitz (1946) [315] laid the foundations of the general-relativistic perturbative approach to structure formation. He linearized the Einstein and stress-energy conservation equations about a spatially homogeneous and isotropic Robertson-Walker background spacetime metric and decomposed the departures from homogeneity and isotropy into independently evolving spatial harmonics (the so-called scalar, vector, and tensor modes). (Lifshitz treated matter as a fluid which is a good approximation when the underlying particle mean free path is small.) Lifshitz discovered that the vector transverse peculiar velocity (the peculiar velocity is the velocity that remains after subtracting off that due to the Hubble expansion) perturbation decays with time (as a consequence of angular momentum conservation) and that the contemporary Universe could contain a residual tensor gravitational wave background left over from earlier times.

Unlike the exponentially growing energy density irregularity that Jeans found in flat spacetime on large scales, Lifshitz found only a much slower power-law temporal growth, leading him to the incorrect conclusion that "gravitational instability is not the source of condensation of matter into separate nebulae". It was almost two decades before Novikov [316] (see Ref. [317] for an earlier hint) corrected this misunderstanding, noting that even with power-law growth there was more than enough time for inhomogeneities to grow, since they could do so even while they were on scales larger than the Hubble length $r_H = c/H_0$ at early time.

The approach to the theory of linear perturbations initiated by Lifshitz is based on a specific choice of spacetime coordinates called synchronous coordinates. This approach is discussed in detail in Sec. V (also see Sec. II) of Ref. [1], Sec. III of Ref. [2], Ref. [318], and other textbooks listed above. Bardeen [319] (building on earlier work) recast the Lifshitz analysis in a coordinate-independent form, and this approach has also become popular. For reviews of this approach see Ref. [320], as well as textbooks listed above.

A useful formalism for linear growth of density and velocity fields is given by the "Zel'dovich approximation" [321, 322, 323], based on anisotropic collapse and so "pancake" formation (a concept earlier discussed in the context of the initial singularity). This method accurately describes structure formation up to the epoch when nonlinearities become significant. Numerical simulations (see Sec. X.D below) of fully non-linear structure growth often employ the Zel'dovich approximation for setting the initial conditions of density and velocity.

3. Space curvature

The evolution of the background spacetime influences the rate of growth of structure. An early example of this effect is seen in Gamow and Teller's (1939) [324] approximate generalization of Jeans' analysis to the expanding Universe, in particular to a model with open spatial hypersurfaces. At late times the dominant form of energy density in such a model is that due to the curvature of spatial hypersurfaces, because this redshifts away slower than the energy density in nonrelativistic matter. The gravitational instability growth rate is determined by the matter energy density, but the expansion rate becomes dominated by the space curvature and so the Universe expands too fast for inhomogeneities to grow and large-scale structure formation ceases. (A quarter century later, Ref. [325] noted the importance of this effect.) This was the first example of an important and general phenomenon: a dominant spatially-homogeneous contributor to the energy density budget will prevent the growth of irregularity in matter.

4. Dark energy

Matter perturbations also cannot grow when a cosmological constant or nearly homogeneous dark energy dominates. There is strong evidence that dark energy — perhaps in the form of Einstein's cosmological constant — currently contributes ~ 70 % of the mass-energy density of the universe. This dark energy was subdominant until recently, when it started slowing the rate of growth of structure [302], thus its effect on dynamical evolution is milder than that of space curvature.

5. Radiation and its interaction with baryonic matter

Reference [326] showed that a dominant homogeneous radiation background makes the Universe expand too fast to allow matter irregularities to start growing until the model becomes matter dominated (when the radiation redshifts away). Because of this effect, as discussed next, the acoustic Hubble length at the epoch when the densities of matter and radiation are equal is an important scale for structure formation in the expanding Universe. This imprints a feature in the power spectrum of matter fluctuations on the scale of the acoustic Hubble length at matter-radiation equality that can be used to measure the cosmic density of non-relativistic matter. We return to this in Sec. XII; a related CMB anisotropy effect is discussed in the next subsection X.B.

Gamow (1948) [191] noted that at early times in the Big Bang Model radiation (which has large relativistic pressure) dominates over baryonic matter. Also at high temperature radiation and baryonic matter are strongly coupled by Thomson-Compton scattering and so behave like a single fluid. As a result of the large radiation pressure during this early epoch the Jeans or acoustic Hubble length is large and so gravitational growth of inhomogeneity occurs only on large scales, with acoustic oscillations on small scales. Peebles and Yu [262] develop this picture.

As the Universe cooled down below a temperature $T \sim 3000$ K at a redshift $z \sim 10^3$, the radiation and baryons decoupled. Below this temperature proton nuclei can capture and retain free electrons to form electrically neutral hydrogen atoms — this process is called "recombination" — because fewer photons remained in the high energy tail of the distribution with enough energy to disassociate the hydrogen atoms. Peebles and Zel'dovich et al. [327, 328] perform an analysis of cosmological recombination, finding that at the "end" of recombination there were enough charged particles left over for the Universe to remain a good conductor all the way to the present. The finite time required for recombination results in a surface of non-zero thickness within which the decoupling of now-neutral baryons and photons occurs. The mean-free path for photons quickly grew, allowing the photons to travel (almost) freely, thus this "last-scattering surface" is the "initial" source of the observed CMB; it is an electromagnetically opaque "cosmic photosphere". See the textbooks listed above for discussions of recombination.

Decoupling leads to a fairly steep drop in the pressure of the baryon gas, and so a fairly steep decrease in the baryon Jeans length. Peebles [325] was developing this picture as the CMB was being discovered. Peebles and Dicke [329] (also see Ref. [330]) noted that the baryonic Jeans mass after decoupling is of the order of the mass of a typical globular cluster and so proposed that proto-globular-clusters were the first objects to gravitationally condense out of the primordial gas. This model would seem to predict the existence of extra-galactic globular clusters, objects that have not yet been observationally recognized, although there are almost equally low-mass dwarf galaxies, and we now also know that some globular clusters are young and so globular clusters might form in more than one way (for a recent review see Ref. [331]).

On scales smaller than the Jeans mass, dissipative effects become important and the ideal fluid approximation for radiation and baryonic matter is no longer accurate. As the Universe cools down towards recombination and decoupling, the photon mean free path grows and so photons diffuse out of denser regions to less dense regions. As they diffuse the photons drag some of the baryons with them and so damp small-scale inhomogeneities in the photon-baryon fluid. This collisional damping — a consequence of Thomson-Compton scattering — is known as Silk damping in the cosmological context; it was first studied by Michie, Peebles, and Silk [332, 330, 333]. The Silk damping scale is roughly that of a cluster of galaxies.

6. Possible matter constituents

If baryons were the only form of non-relativistic matter the density of matter would be so low that the Universe would remain radiation dominated until after recombination. The expansion rate would be too large for gravitational instability to cause inhomogeneity growth until matter starts to dominate well after last scattering. The short time allowed for the gravitational growth of inhomogeneity from the start of matter domination to today would require a large initial fluctuation amplitude to produce the observed large-scale structure. This scenario is ruled out by measurements of the anisotropy of the CMB which indicate that fluctuations in the baryons at decoupling are too small to have grown by gravitational instability into the structures seen today in the galaxy distribution.

A solution to this puzzle is provided by dark matter, of the same type and quantity needed to explain gravitational interactions on galactic and cluster scales. Including this component of matter the Universe becomes matter dominated at a redshift comparable to, or even larger than, the redshift of last scattering. Because CDM does not directly couple to radiation, inhomogeneities in the distribution of CDM begin to grow as soon as the Universe becomes matter dominated. Growth in structure in the baryons, on scales small compared to the Hubble length, remains suppressed by Thomson-Compton scattering until recombination, after which baryons begin to gravitate toward the potential wells of dark matter and the baryon fluctuation amplitude quickly grows. Thus, the low observed CMB anisotropy is reconciled with observed large-scale structure. (The CMB, while not directly coupled to the CDM, feels the gravitational potential fluctuations of the CDM. Consequently, measurements of the CMB anisotropy probe the CDM distribution.) This is an independent, although model-dependent and indirect, argument for the existence of CDM.

As mentioned above in Sec. VII.C, the Universe also contains low mass neutrinos (precise masses are not yet known). These neutrinos are relativistic and weakly coupled (nearly collisionless) and so have a very long mean free path or free-streaming length. Consequently, they must be described by a distribution function, not a fluid. Since they are relativistic they have a large Jeans mass and gravitational instability is effective at collecting them only on very large scales, i.e., low mass neutrinos suppress power on small and intermediate length scales. This effect makes it possible to observationally probe these particles with cosmological measurements [334, 335].

7. Free streaming

Thus, the properties of dark matter are reflected in the spectrum of density fluctuations because scales smaller than the free-streaming scale of massive particles are damped [336]. For hot dark matter (HDM), e.g., neutrinos, the free-streaming scale is larger than the Hubble length at matter-radiation equality, hence the spectrum retains only large-scale power. In such a "top-down" scenario, superclusters form first, then fragment into smaller structures including clusters of galaxies and individual galaxies, as first discussed by Zel'dovich and collaborators. The top-down model was inspired by experimental suggestions (now known to be incorrect) that massive neutrinos could comprise the nonbaryonic dark matter, and by an early (and now known to be incorrect) interpretation of observational data on superclusters and voids (see Sec. XI.B below) that postulated that these were the basic organizational blocks for large-scale structure. It predicts that smaller scale structure (e.g., galaxies) is younger than larger scale structure (e.g., superclusters), contrary to current observational indications. In fact, these observational constraints on the evolution of structure constrain the amount of HDM neutrino matter-energy density and so neutrino masses [337]. Cosmological observations provide the best (model-dependent) upper limits on neutrino masses.

For CDM, e.g., weakly interacting massive particles (WIMPs), the free-streaming scale is negligible for cosmological purposes. This "bottom-up" or "hierarchical" scenario, pioneered by Peebles and collaborators, begins with the formation of bound objects on small scales that aggregate into larger structures, thus galaxies result from mergers of sub-galaxies, with superclusters being the latest structures to form. This is in better agreement with the observational data. See Sec. IX for more details on this model.

8. Initial density perturbations and the transfer function

The current standard model for structure formation assumes that structure in the Universe arose primarily from gravitational amplification of infinitesimal scalar density perturbations in the early Universe. The processes listed in this section modify these initial inhomogeneities. Reviews are given in Sec. V of Ref. [1] Sec. III of Ref. [2], Ref. [338], Ch. 9 of Ref. [47], Ch. 4 of Ref. [3], Ref. [76], and Part II of Ref. [54].

As discussed in Secs. IX and X.B, observations to date are consistent with primordial fluctuations that are Gaussian random phase. These are the type of fluctuations expected if the seeds for structure formation result from the superposition of quantum mechanical zero-point fluctuations of the scalar field that drove inflation of the early Universe, in the simplest inflation models, as discussed in Sec. VIII above. In the simplest inflation models the fluctuations are adiabatic. Furthermore, observational data are consistent with only adiabatic perturbations, so in what follows we focus on this case (see Ref. [339] for a recent discussion of constraints on isocurvature models).

As discussed in Sec. VIII above and Sec. XII below, current large-scale observational results are reasonably well fit by an $n = 1$ scale-invariant primordial spectrum of perturbations, the kind considered by Harrison [261], Peebles and Yu [262], and Zel'dovich [263], and predicted in some of the simpler inflation models. The effect of causal physics on the later growth of structure, as discussed above, may then be represented by a "transfer function" that describes the relative growth of fluctuations on different wavelength scales. Observations of the anisotropy of the CMB and the clustering of galaxies and clusters at the present epoch probe the shape of the transfer function (as well as the primordial spectrum of perturbations) and thereby constrain structure formation models. Such observations are discussed below in Secs. X.B and XI.

9. Gravitational waves and magnetic fields

As noted in Sec. VIII above, more complicated models of inflation can generate gravitational wave or magnetic field fluctuations that break adiabaticity. A primordial magnetic field might provide a way of explaining the origin of the uniform part of contemporary galactic magnetic fields; there are enough charged particles left over after recombination to ensure that primordial magnetic field lines will be pulled in, and the field amplified, by a collapsing gas cloud. References [340, 341] review primordial gravity waves, and cosmological magnetic fields are reviewed in Refs. [342, 343]. In the next subsection we consider the effects of such fields on the CMB.

314. "The Stability of a Spherical Nebula," J. Jeans, Philo. Trans. R. Soc. London A **199**, 1-53 (1902). (I, A)

315. "On the Gravitational Stability of the Expanding Universe," E. Lifshitz (1946), English translation in J. Phys. **10**, 116-129 (1946). (A)

316. "On the Possibility of Appearance of Large Scale Inhomogeneities in the Expanding Universe," I. D. Novikov (1964), English translation in Sov. Phys. JETP **19**, 467-469 (1964). (A)

317. "Jeans' Formula for Gravitational Instability," W. B. Bonnor, Mon. Not. R. Astron. Soc. **117**, 104-116 (1957). (A)

318. "Expressions for Linearized Perturbations in Ideal-Fluid Cosmological Models," B. Ratra, Phys. Rev. D **38**, 2399-2414 (1988). (A)

319. "Gauge-Invariant Cosmological Perturbations," J. M. Bardeen, Phys. Rev. D **22**, 1882-1905 (1980). (A)

320. "Theory of Cosmological Perturbations," V. F. Mukhanov, H. A. Feldman, and R. H. Brandenberger, Phys. Rept. **215**, 203-333 (1992). (A)

321. "Gravitational Instability: An Approximate Theory for Large Density Perturbations," Ya. B. Zel'dovich, Astron. Astrophys. **5**, 84-89 (1970). (A)

322. "The Large-Scale Structure of the Universe: Turbulence, Intermittency, Structures in a Self-Gravitating Medium," S. F. Shandarin and Ya. B. Zel'dovich, Rev. Mod. Phys. **61**, 185-220 (1989). (I)

323. "Approximation Methods for Non-Linear Gravitational Clustering," V. Sahni and P. Coles, Phys. Rept. **262**, 1-135 (1995). (I, A)

324. "On the Origin of Great Nebulae," G. Gamow and E. Teller, Phys. Rev. **55**, 654-657 (1939). (I)

325. "The Black-Body Radiation Content of the Universe and the Formation of Galaxies," P. J. E. Peebles, Astrophys. J. **142**, 1317-1326 (1965). (I, A)

326. "Gravitational Instability of a Two-Component Fluid: Matter and Radiation," M. Guyot and Ya. B. Zel'dovich, Astron. Astrophys. **9**, 227-231 (1970). (A)

327. “Recombination of the Primeval Plasma,” P. J. E. Peebles, Astrophys. J. **153**, 1-11 (1968). (A)

328. “Recombination of Hydrogen in the Hot Model of the Universe,” Ya. B. Zel’dovich, V. G. Kurt, and R. A. Sunyaev (1968), English translation in Sov. Phys. JETP **28**, 146-150 (1969). (A)

329. “Origin of the Globular Star Clusters,” P. J. E. Peebles and R. H. Dicke, Astrophys. J. **154**, 891-908 (1968). (I, A)

330. “Primeval Galaxies,” P. J. E. Peebles, unpublished manuscript based on a paper presented at the 4^{th} Texas Conference on Relativistic Astrophysics (May 1967). (I, A)

331. “Extragalactic Globular Clusters and Galaxy Formation,” J. P. Brodie and J. Strader, Annu. Rev. Astron. Astrophys. **44**, 193-267 (2006). (I)

332. “On the Growth of Condensations in the Expanding Universe,” R. W. Michie, Kitt Peak Natl. Obs. Contr. **440**, 1-16 (1969). This is a version of a manuscript submitted to the Astrophysical Journal on September 1, 1967, and only minimally revised (in response to the referee’s suggestions) before the author died. (A)

333. “Cosmic Black-Body Radiation and Galaxy Formation,” J. Silk, Astrophys. J. **151**, 459-471 (1968). (A)

334. “Neutrino Masses from Cosmological Probes,” Ø. Elgarøy and O. Lahav, New J. Phys. **7**, 61 (2005). (I, A)

335. “Massive Neutrinos and Cosmology,” J. Lesgourgues and S. Pastor, Phys. Rept. **429**, 307-379 (2006). (I, A)

336. “Massive Neutrinos and the Large-Scale Structure of the Universe,” J. R. Bond, G. Efstathiou, and J. Silk, Phys. Rev. Lett. **45**, 1980-1984 (1980). (A)

337. “Neutrino Mass Limit from Galaxy Cluster Number Density Evolution,” T. Kahniashvili, E. von Toerne, N. Arhipova, and B. Ratra, Phys. Rev. D **71**, 125009 (2005). (A)

338. “Cosmological Perturbations,” G. Efstathiou, in **Physics of the Early Universe**, edited by J. A. Peacock, A. F. Heavens, and A. T. Davies (Adam Hilger, Bristol, 1990), 361-464. (I, A)

339. “Constraining Isocurvature Initial Conditions with WMAP 3-Year Data,” R. Bean, J. Dunkley, and E. Pierpaoli, Phys. Rev. D **74**, 063503 (2006). (A)

340. “Gravitational Wave Experiments and Early Universe Cosmology,” M. Maggiore, Phys. Rept. **331**, 283-367 (2000). (A)

341. "TASI Lectures on Gravitational Wave from the Early Universe," A. Buonanno, in **Particle Physics and Cosmology: The Quest for Physics Beyond the Standard Model(s)**, edited by H. E. Haber and E. Nelson (World Scientific, Singapore, 2004), 855-892. (I, A)

342. "Galactic and Extragalactic Magnetic Fields," L. M. Widrow, Rev. Mod. Phys. **74**, 775-823 (2002). (I, A)

343. "The Magnetized Universe," M. Giovannini, Int. J. Mod. Phys. D **13**, 391-502 (2004). (I, A)

B. CMB anisotropies

As a result of the gravitational growth of inhomogeneities in the matter distribution, when the photons decouple from the baryons at last scattering at a redshift $z \sim 10^3$ (see Sec. X.A above) the photon temperature distribution is spatially anisotropic. Also, in the presence of a CMB temperature quadrupole anisotropy, Thomson-Compton scattering of CMB photons off electrons prior to decoupling generates a linear polarization anisotropy of the CMB. After decoupling the CMB photons propagate almost freely, influenced only by gravitational perturbations and late-time reionization. Measurements of the temperature anisotropy and polarization anisotropy provide important constraints on many parameters of models of structure formation. This area of research has seen spectacular growth in the last decade or so, following the COBE discovery of the CMB temperature anisotropy. It has been the subject of a recent Resource Letter [344] and other recent reviews, Ref. [345], Sec. IV.B.11 of Ref. [288], and Refs. [346, 347, 348]. Here we focus only on a few recent developments.

The three-year WMAP observations of CMB temperature anisotropies [349] are state-of-the-art data. On all but the very largest angular scales, the WMAP data are consistent with the assumption that the CMB temperature anisotropy is well-described by a spatial Gaussian random process [350], consistent with earlier indications [351, 352]. The few largest-scale angular modes exhibit a lack of power compared to what is expected in a spatially-flat CDM model dominated by a cosmological constant, [353], resulting in some debate about the assumptions of large-scale Gaussianity and spatial isotropy. This feature was also seen in the COBE data [354]. The estimated large-angular-scale CMB temperature anisotropy power depends on the model used to remove foreground Galactic emission contamination. Much work has been devoted to understanding foreground emission on all scales, e.g., Refs. [355, 356, 357], and the current consensus is that foregrounds are not the cause of the large-angular-scale WMAP effects.

The CMB temperature anisotropy is conventionally expressed as an expansion in spherical harmonic multipoles on the sky, and for a Gaussian random process the multipole (or angular) power spectrum completely characterizes the CMB temperature anisotropy. The observed CMB anisotropy is reasonably well fit by assuming only adiabatic fluctuations with a scale-invariant power spectrum. These observational results are consistent with the predictions of the simplest inflation models, where quantum-mechanical fluctuations in a weakly-coupled scalar field are the adiabatic, Gaussian seeds for the observed CMB anisotropy and large-scale structure.

Smaller-scale inhomogeneities in the coupled baryon-radiation fluid oscillate (see Sec. X.A above), and at decoupling some of these modes will be at a maximum or at a minimum, giving rise to acoustic peaks and valleys in the CMB anisotropy angular spectrum. The relevant length scale is the acoustic Hubble length at the epoch of recombination; this may be predicted by linear physics and so provides a standard ruler on the sky. Through the angular diameter distance relation, the multipole numbers ℓ of oscillatory features in the temperature anisotropy spectrum C_ℓ reflect space curvature ($\Omega_{\rm K}$) and the expansion history (which depends on $\Omega_{\rm M}$ and Ω_Λ) of the Universe. The angular scales of the peaks are sensitive to the value of the matter density parameter in an open Universe, but not in a spatially-flat ($\Omega_{\rm K} = 0$) Universe dominated by a cosmological constant, where the first peak is at a multipole index $\ell \sim 220$. This provides a good way to measure the curvature of spatial hypersurfaces. See Refs. [358, 359, 360] for early discussions of the CMB temperature anisotropy in an open Universe, and Refs [361, 362, 363, 364] for the case of scalar field dark energy in a spatially-flat Universe. CMB temperature anisotropy data on the position of the first peak is consistent with flat spatial hypersurfaces, see, e.g., Refs. [365, 366, 367]. Model-based CMB data analysis is used to constrain more cosmological parameters, see, e.g., Refs. [368, 369, 370]. For example, the relative amplitudes of peaks in this spectrum are sensitive to the mass densities of the different possible constituents of matter (e.g., CDM, baryons, and neutrinos, $\Omega_{\rm CDM}, \Omega_{\rm B}$, and Ω_ν).

The CMB polarization anisotropy was first detected from the ground by the DASI experiment at the South Pole [371]. The three-year WMAP observations are the current state of the art [372]. (For a recent review of polarization measurements see Ref. [373].) The polarization anisotropy peaks at a larger angular scale than the temperature anisotropy, indicating that there are inhomogeneities on scales larger than the acoustic Hubble length at recombination, consistent with what is expected in the inflation scenario. The polarization anisotropy signal is interpreted as the signature of reionization of the Universe. The ability of WMAP to measure polarization anisotropies allows this experiment to probe the early epochs of non-linear structure formation, through sensitivity to the reionization optical depth τ.

Primordial gravitational waves or a primordial magnetic field can also generate CMB anisotropies. Of particular current interest are their contributions to various CMB polarization anisotropies. (Because polarization is caused by quadrupole fluctuations, these anisotropies constrain properties of the primordial fluctuations, such as the ratio of tensor to scalar fluctuations, r.) The effects of gravity waves on the CMB are discussed in the more recent textbooks listed in Secs. III.A.1 and III.A.5 above and in Ref. [347]. The magnetic field case is reviewed in Refs. [374, 375]; recent topics of interest may be traced from Refs. [376, 377, 378, 379].

We continue discussion of the CMB anisotropies and cosmological parameters in Sec. XII.

344. "Resource Letter: TACMB-1: The Theory of Anisotropies in the Cosmic Mi-

crowave Background," M. White and J. D. Cohn, Am. J. Phys. **70**, 106-118 (2002). (I)

345. "Cosmic Microwave Background Anisotropies," W. Hu and S. Dodelson, Annu. Rev. Astron. Astrophys. **40**, 171-216 (2002). (I)

346. "The Physics of CMBR Anisotropies," K. Subramanian, Curr. Sci. **88**, 1068-1087 (2005). (I, A)

347. "Theoretical Tools for CMB Physics," M. Giovannini, Int. J. Mod. Phys. D **14**, 363-510 (2005). (A)

348. "Cosmic Microwave Background Anisotropies," A. Challinor, in **The Physics of the Early Universe**, edited by E. Papantonopoulos (Springer, Berlin, 2005), 71-103. (A)

349. "Three-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Temperature Analysis," G. Hinshaw, et al., Astrophys. J. Suppl. **170**, 288-334 (2007). (A)

350. "First-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Tests of Gaussianity," E. Komatsu, et al., Astrophys. J. Suppl. **148**, 119-134 (2003). (A)

351. "Gaussianity of Degree-Scale Cosmic Microwave Background Anisotropy Observations," C.-G. Park, C. Park, B. Ratra, and M. Tegmark, Astrophys. J. **556**, 582-589 (2001). (A)

352. "Tests for Gaussianity of the MAXIMA-1 Cosmic Microwave Background Map," J. H. P. Wu, et al., Phys. Rev. Lett. **87**, 251303 (2001). (A)

353. "First-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Preliminary Maps and Basic Results," C. L. Bennett, et al., Astrophys. J. Suppl. **148**, 1-27 (2003). (I, A)

354. "COBE-DMR-Normalized Open Cold Dark Matter Cosmogonies," K. M. Górski, et al., Astrophys. J. Suppl. **114**, 1-36 (1998). (I, A)

355. "Galactic Foreground Constraints from the Python V Cosmic Microwave Background Anisotropy Data," P. Mukherjee, et al., Astrophys. J. **592**, 692-698 (2003). (A)

356. "First-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Foreground Emission," C. L. Bennett, et al., Astrophys. J. Suppl. **148**, 97-117 (2003). (A)

357. "High Resolution Foreground Cleaned CMB Map from WMAP," M. Tegmark, A. de Oliveira-Costa, and A. J. S. Hamilton, Phys. Rev. D **68**, 123523 (2003). (A)

358. "Perturbation of the Cosmic Microwave Background Radiation and Structure Formations," N. Sugiyama and N. Gouda, Prog. Theor. Phys. **88**, 803-849 (1992). (A)

359. "Small-Scale Cosmic Microwave Background Anisotropy as Probe of the Geometry of the Universe," M. Kamionkowski, D. N. Spergel, and N. Sugiyama, Astrophys. J. Lett. **426**, L57-L60 (1994). (A)

360. "Cosmic Background Radiation Anisotropy in an Open Inflation, Cold Dark Matter Cosmogony," M. Kamionkowski, B. Ratra, D. N. Spergel, and N. Sugiyama, Astrophys. J. Lett. **434**, L1-L4 (1994). (A)

361. "Exhaustive Study of Cosmic Microwave Background Anisotropies in Quintessential Scenarios," P. Brax, J. Martin, and A. Riazuelo, Phys. Rev. D **62**, 103505 (2000). (A)

362. "Constraints on Flat Cosmologies with Tracking Quintessence from Cosmic Microwave Background Observations," C. Baccigalupi, et al., Phys. Rev. D **65**, 063520 (2002). (A)

363. "Cosmic Microwave Background and Supernova Constraints on Quintessence: Concordance Regions and Target Models," R. R. Caldwell and M. Doran, Phys. Rev. D **69**, 103517 (2004). (A)

364. "COBE-DMR-Normalized Dark Energy Cosmogonies," P. Mukherjee, et al., Astrophys. J. **598**, 767-778 (2003). (A)

365. "Binned Cosmic Microwave Background Anisotropy Power Spectrum: Peak Location," S. Podariu, et al., Astrophys. J. **559**, 9-22 (2001). (I)

366. "Acoustic Peaks and Dips in the Cosmic Microwave Background Power Spectrum: Observational Data and Cosmological Constraints," R. Durrer, B. Novosyadlyj, and S. Apunevych, Astrophys. J. **583**, 33-48 (2003). (A)

367. "First-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Interpretation of the TT and TE Angular Power Spectrum Peaks," L. A. Page, et al., Astrophys. J. Suppl. **148**, 233-241 (2003). (I, A)

368. "Cosmological Parameters from CMB and Other Data: A Monte Carlo Approach," A. Lewis and S. Bridle, Phys. Rev. D **66**, 103511 (2002). (A)

369. "CMB Anisotropy Constraints on Flat-Λ and Open CDM Cosmogonies from DMR, UCSB South Pole, Python, ARGO, MAX, White Dish, OVRO, and SuZIE Data," P. Mukherjee, et al., Int. J. Mod. Phys. A **18**, 4933-4954 (2003). (A)

370. "Three-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Implications for Cosmology," D. N. Spergel, et al., Astrophys. J. Suppl. **170**, 377-408 (2007). (A)

371. “Detection of Polarization in the Cosmic Microwave Background using DASI,” J. M. Kovac, et al., Nature **420**, 772-787 (2002). (A)

372. “Three Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Polarization Analysis,” L. Page et al., Astrophys. J. Suppl. **170**, 335-376 (2007). (A)

373. “The CMB Polarization: Status and Prospects,” A. Balbi, P. Natoli, and N. Vittorio, astro-ph/0606511. (I)

374. “Magnetized CMB Anisotropies,” M. Giovannini, Class. Quantum Grav. **23**, R1-R44 (2006). (A)

375. “Primordial Magnetic Fields and CMB Anisotropies,” K. Subramanian, Astron. Nachr. **327**, 403-409 (2006). (A)

376. “CMB Anisotropies from Primordial Inhomogeneous Magnetic Fields,” A. Lewis, Phys. Rev. D **70**, 043011 (2004). (A)

377. “Effects of Cosmological Magnetic Helicity on the Cosmic Microwave Background,” T. Kahniashvili and B. Ratra, Phys. Rev. D **71**, 103006 (2005). (A)

378. “Non-Gaussianity from Cosmic Magnetic Fields,” I. Brown and R. Crittenden, Phys. Rev. D **72**, 063002 (2005). (A)

379. “CMB Anisotropies due to Cosmological Magnetosonic Waves,” T. Kahniashvili and B. Ratra, Phys. Rev. D **75**, 023002 (2007). (A)

C. Galaxy formation and the end of the dark age

The emission of the first light in the Universe, seen today as the CMB, is followed by a “dark age” before the first stars and quasars form. Reference [380] reviews formation of the first stars. Eventually, high energy photons from stars and quasars reionize intergalactic gas throughout the Universe (see Refs. [381, 382, 383, 167] for reviews). Observations of polarization of microwave background photons by WMAP [372] suggest that reionization occurs at redshift $z \approx 11$. However, strong absorption of Lyman-α photons by intergalactic neutral hydrogen [384], seen in spectra of quasars at redshift $z \approx 6$ [385, 386] indicates that reionization was not complete until somewhat later. This is an area of ongoing research (see, e.g., Refs. [387, 388, 389]).

Current models for galaxy formation follow the picture suggested by Refs. [390, 391, 392, 393, 394] in which dark matter halos form by collisionless collapse, after which baryons fall into these potential wells, are heated to virial temperature, then cool and condense at the centers of the halos to form galaxies as we know them. In short, baryons fall into the gravitational potentials of “halos” of dark matter, at the same time that those halos grow in size, hierarchically aggregating small clumps into larger ones. The baryons cool by emitting radiation and shed angular momentum, leading to concentrations of star formation and accretion onto supermassive black holes within the dark matter halos.

In addition to the perturbative approach to structure formation discussed in Sec. X.A, Lemaître also pioneered a "nonperturbative" approach based on a spherically symmetric solution of the Einstein equations. This spherical accretion model [395] describes the salient features of the growth of mass concentrations. See Ref. [396], Sec. 22 of Ref. [4], and Ref. [323] for reviews of such models.

A phenomenological description for the statistics of non-linear collapse of structure, i.e., the formation of gravitationally bound objects, is given by the Press-Schechter formulae [397, 398]. Attempts to firm up the theoretical basis of such formulae form the "excursion set" formalism which treats the formation of a gravitationally bound halo as the result of a random walk [399, 400, 401]. For a review see Ref. [402]. These methods provide probability distributions for the number of bound objects as a function of mass threshold, and can be generalized to develop a complementary description of the evolution of voids [403]. A more rigorous approach assumes structure forms at high peaks in the smoothed density field [404, 405, 323]. Recent reviews of galaxy formation include Refs. [406, 407]. The next subsection, X.D, describes numerical methods for studying structure formation.

Apparent confirmation of the hierarchical picture of structure formation includes the striking images of galaxies apparently in the process of assembly obtained by the HST in the celebrated "Hubble Deep Fields" [408]. The detailed properties of galaxies and their evolution are outside the scope of this review. Recent reviews of the observational situation are Refs. [409, 410]. Texts covering this topic include Refs. [23, 24].

While the current best model of structure formation, in which CDM dominates the matter density, works quite well on large scales, current observations indicate some possible problems with the CDM model on smaller scales; see Ref. [411], Sec. IV.A.2 of Ref. [288], and Ref. [412] for reviews. Simulations of structure formation indicate that CDM model halos may have cores that are cuspier [413, 414] and central densities that are higher [415, 416] than are observed in galaxies. Another concern is that CDM models predict a larger than observed number of low-mass satellites of massive galaxies [417, 418]. These issues have led to consideration of models with reduced small-scale power. However, it seems difficult to reconcile suppression of small-scale power with the observed small-scale clustering in the neutral hydrogen at redshifts near 3.

The relationship between the distributions of galaxies (light) and matter is commonly referred to as "biasing." The currently-favored dark energy dominated CDM model does not require significant bias between galaxies and matter; in the best-fit model the ratio of galaxy to matter clustering is close to unity for ordinary galaxies [419].

380. "The First Stars," V. Bromm and R. B. Larson, Annu. Rev. Astron. Astrophys. **42**, 79-118 (2004). (I)

381. "Observational Constraints on Cosmic Reionization," X. Fan, C. L. Carilli, and B. Keating, Annu. Rev. Astron. Astrophys. **44**, 415-462 (2006). (I)

382. “Physics of Cosmic Reionization,” T. R. Choudhury and A. Ferrara, astro-ph/0603149. (I, A)

383. “First Light,” A. Loeb, astro-ph/0603360. (I, A)

384. “On the Density of Neutral Hydrogen in Intergalactic Space,” J. E. Gunn and B. A. Peterson, Astrophys. J. **142**, 1633-1636 (1965). (A)

385. “Evidence for Reionization at $z \sim 6$: Detection of a Gunn-Peterson Trough in a $z = 6.28$ Quasar,” R. H. Becker, et al., Astron. J. **122**, 2850-2857 (2001). (A)

386. “Evolution of the Ionizing Background and the Epoch of Reionization from the Spectra of $z \sim 6$ Quasars,” X. Fan, et al., Astron. J. **123**, 1247-1257 (2002). (A)

387. “Updating Reionization Scenarios After Recent Data,” T. R. Choudhury and A. Ferrara, Mon. Not. Roy. Astron. Soc. **371**, L55-L59 (2006). (A)

388. “Cosmic Reionization Redux,” N. Y. Gnedin and X. Fan, Astrophys. J. **648**, 1-6 (2006). (A)

389. “Implications of the WMAP 3 Year Data for the Sources of Reionization,” M. A. Alvarez, P. R. Shapiro, K. Ahn, and I. I. Iliev, Astrophys. J. Lett. **644**, L101-L104 (2006). (A)

390. “On the Fragmentation of Gas Clouds Into Galaxies and Stars.,” F. Hoyle, Astrophys. J. **118**, 513-528 (1953). (A)

391. “On the Fragmentation of Cosmic Gas Clouds. I — The Formation of Galaxies and the First Generation of Stars,” J. Silk, Astrophys. J. **211**, 638-648 (1977). (A)

392. “The Physics of Dissipational Galaxy Formation,” J. Binney, Astrophys. J. **215**, 483-491 (1977). (A)

393. “Cooling, Dynamics and Fragmentation of Massive Gas Clouds — Clues to the Masses and Radii of Galaxies and Clusters,” M. J. Rees and J. P. Ostriker, Mon. Not. R. Astron. Soc. **179**, 541-559 (1977). (A)

394. “Core Condensation in Heavy Halos — A Two-Stage Theory for Galaxy Formation and Clustering,” S. D. M. White and M. J. Rees, Mon. Not. R. Astron. Soc. **183**, 341-358 (1978). (A)

395. “On the Infall of Matter Into Clusters of Galaxies and Some Effects on Their Evolution,” J. E. Gunn and J. R. Gott, III, Astrophys. J. **176**, 1-19 (1972). (A)

396. “Recent Theories of Galaxy Formation,” J. R. Gott, III, Annu. Rev. Astron. Astrophys. **15**, 235-266 (1977). (I)

397. "Formation of Galaxies and Clusters of Galaxies by Self-Similar Gravitational Condensation," W. H. Press and P. Schechter, Astrophys. J. **187**, 425-438 (1974). (A)

398. "Large-Scale Bias and the Peak Background Split," R. K. Sheth and G. Tormen, Mon. Not. R. Astron. Soc. **308**, 119-126 (1999). (A)

399. "An Analytic Model for the Spatial Clustering of Dark Matter Haloes," H. J. Mo and S. D. M. White, Mon. Not. R. Astron. Soc. **282**, 347-361 (1996). (A)

400. "Formation and Evolution of Galaxies," S. D. M. White, in Ref. [75], 349-430. (I, A)

401. "Ellipsoidal Collapse and an Improved Model for the Number and Spatial Distribution of Dark Matter Haloes," R. K. Sheth, H. J. Mo, and G. Tormen, Mon. Not. R. Astron. Soc. **323**, 1-12 (2001). (A)

402. "Halo Model of Large Scale Structure," A. Cooray and R. K. Sheth, Phys. Rept. **372**, 1-129 (2002). (I, A)

403. "A Hierarchy of Voids: Much Ado About Nothing," R. K. Sheth and R. van de Weygaert, Mon. Not. R. Astron. Soc. **350**, 517-538 (2004). (A)

404. "On the Spatial Correlations of Abell Clusters," N. Kaiser, Astrophys. J. Lett. **284**, L9-12 (1984). (A)

405. "The Statistics of Peaks of Gaussian Random Fields," J. M. Bardeen, J. R. Bond, N. Kaiser, and A. S. Szalay, Astrophys. J. **304**, 15-61 (1986). A comprehensive discussion and a survey of earlier work. (I, A)

406. "Understanding Galaxy Formation and Evolution," V. Avila-Reese, in **Solar, Stellar and Galactic Connections between Particle Physics and Astrophysics**, edited by A. Carramiñana, F. S. Guzmán, and T. Matos (Springer, Berlin, 2007), 115-165 . (I, A)

407. "A Primer on Hierarchical Galaxy Formation: The Semi-Analytical Approach," C. M. Baugh, Rept. Prog. Phys. **69**, 3101-3156 (2006). (I, A)

408. "The Hubble Deep Fields," H. C. Ferguson, M. Dickinson, and R. Williams, Annu. Rev. Astron. Astrophys. **38**, 667-715 (2000). (I)

409. "Galaxy Formation," E. Gawiser, in **New Horizons in Astronomy: Frank N. Bash Symposium 2005**, edited by S. Kannappan, et al. (Astron. Soc. Pac., San Francisco, 2006), 177-190. (A)

410. "Observations of the High Redshift Universe," R. S. Ellis, astro-ph/0701024. (I, A)

411. "The Cold Dark Matter Crisis on Galactic and Subgalactic Scales," A. Tasitsiomi, Int. J. Mod. Phys. D **12**, 1157-1205 (2003). (I)

412. “Precision Cosmology,” J. R. Primack, New Astron. Rev. **49**, 25-34 (2005). (I)

413. “A Universal Density Profile from Hierarchical Clustering,” J. F. Navarro, C. S. Frenk, and S. D. M. White, Astrophys. J. **490**, 493-508 (1997). (A)

414. “High-Resolution Rotation Curves of Low Surface Brightness Galaxies,” R. A. Swaters, B. F. Madore, and M. Trewhella, Astrophys. J. Lett. **531**, L107-L110 (2000). (A)

415. “Dark Matter Substructure within Galactic Halos,” B. Moore, et al., Astrophys. J. Lett. **524**, L19-L22 (1999). (A)

416. “Constraints on Dark Matter Physics from Dwarf Galaxies through Galaxy Cluster Haloes,” C. Firmani, et al., Mon. Not. R. Astron. Soc. **321**, 713-722 (2001). (A)

417. “Cold Collapse and the Core Catastrophe,” B. Moore, et al., Mon. Not. R. Astron. Soc. **310**, 1147-1152 (1999). (A)

418. “Where Are the Missing Galactic Satellites?,” A. Klypin, A. V. Kravtsov, O. Valenzuela, and F. Prada, Astrophys. J. **522**, 82-92 (1999). (A)

419. “Cosmological Parameters from SDSS and WMAP,” M. Tegmark, et al., Phys. Rev. D **69**, 103501 (2004). (A)

D. Simulations of structure formation

Cosmological simulations using increasingly sophisticated numerical methods provide a testbed for models of structure formation. Reference [420] reviews methods and results.

Computer simulations of structure formation in the Universe began with purely gravitational codes that directly compute the forces between a finite number of particles (“Particle-Particle” or PP codes) that sample the matter distribution. Early results used direct N-body calculations [421]. Binning the particles on a grid and computing the forces using the Fast Fourier Transform (the “Particle-Mesh” or PM method) is computationally more efficient, allowing simulation of larger volumes of space, but has force resolution of order the grid spacing. A compromise is the P^3M method, which uses PM for large scale forces supplemented by direct PP calculations on small scales, as used for the important suite of CDM simulations by Davis et al. [422]. For details on these methods see Ref. [423].

The force resolution of PM codes and the force resolution and speed of P^3M codes may be increased by employing multiple grid levels [424, 425, 426, 427]. Adaptive mesh refinement (AMR; [428]) does this dynamically to increase force resolution in the PM gravity solver [429, 430].

Another approach to achieving both speed and good force resolution in gravitational N-body simulation is use of the hierarchical tree algorithm [431]. Large cosmological simulations have used a parallelized version of this method [432]. Significant increase in speed was found with the Tree Particle-Mesh algorithm [433]. GOTPM [434], a parallelized hybrid PM+tree scheme, has been used for simulations involving up to 8.6×10^9 particles. PMFAST [435] is a recent parallelized multi-level PM code.

Incorporation of hydrodynamics and radiative transfer in cosmological simulations has made it possible to study not only the gravitational formation of dark matter halos, but also the properties of baryonic matter, and thus the formation of galaxies associated with those halos. Methods for solving the fluid equations include smooth-particle hydrodynamics (SPH; see Ref. [436] for a review), which is an inherently Lagrangian approach, and Eulerian grid methods. Cosmological SPH simulations were pioneered in Refs. [437, 438]. To date, the cosmological simulation with the largest number of particles (10^{10}) employs SPH and a tree algorithm (GADGET; [439]). Grid-based codes used for cosmological simulation include that described in Refs. [440, 441].

To date, no code has sufficient dynamic range to compute both the large scale cosmological evolution on scales of many hundreds of megaparsecs and the formation of stars from baryons, but physical heuristics have been successfully incorporated into some codes to model the conversion of baryons to stars (see, e.g., Ref. [440]).

Another approach to modeling the properties of the galaxies associated with dark matter halos is to use the history of halo mergers together with semi-analytic modeling of galaxy properties [442, 443, 444, 445]. When normalized to the observed luminosity function of galaxies and Tully-Fisher relation for spiral galaxies, these semi-analytic models (SAMs) reproduce many of the observed features of the galaxy distribution. A common approach is to use SAMs to "paint on" the properties of galaxies that would reside in the dark matter halos found in purely gravitational simulations. See Refs. [406, 407] for recent reviews. Related to the SAMs approach are halo occupation models [446, 447] that parameterize the statistical relationship between the masses of dark matter halos and the number of galaxies resident in each halo.

420. "Simulations of Structure Formation in the Universe." E. Bertschinger, Annu. Rev. Astron. Astrophys. **36**, 599-654 (1998) (I)

421. "N-Body Simulations of Galaxy Clustering. I. Initial Conditions and Galaxy Collapse Times," S. J. Aarseth, E. L. Turner, and J. R. Gott, Astrophys. J. **228**, 664-683 (1979). (A)

422. "The Evolution of Large-Scale Structure in a Universe Dominated by Cold Dark Matter," M. Davis, G. Efstathiou, C. S. Frenk, and S. D. M. White, Astrophys. J. **292**, 371-394 (1985). (A)

423. **Computer Simulation Using Particles**, R. W. Hockney and J. W. Eastwood (Adam Hilger, Bristol, 1988). (I, A)

424. "A New Hierarchical Particle-Mesh Code for Very Large Scale Cosmological N-Body Simulations," J. V. Villumsen. Astrophys. J. Suppl. **71**, 407-431 (1989). (A)

425. "Mesh-Refined P^3M: A Fast Adaptive N-Body Algorithm," H. M. P. Couchman. Astrophys. J. Lett. **368**, L23-L26 (1991). (A)

426. "Cosmological N-Body Simulations," E. Bertschinger and J. M. Gelb, Comp. Phys. **5**, 164-175, 178, 179 (1991). (A)

427. "Building a Cosmological Hydrodynamic Code: Consistency Condition, Moving Mesh Gravity, and SLH-P^3M," N. Y. Gnedin and E. Bertschinger, Astrophys. J. **470**, 115-130 (1996). (A)

428. "Local Adaptive Mesh Refinement for Shock Hydrodynamics," M. J. Berger and B. Collela, J. Comp. Phys. **82**, 64-84 (1989). (A)

429. "Adaptive Refinement Tree: A New High-Resolution N-Body Code for Cosmological Simulations," A. V. Kravtsov, A. A. Klypin, and A. M. Khokhlov, Astrophys. J. Suppl. **111**, 73-94 (1997). (A)

430. "Cosmological Adaptive Mesh Refinement," M. L. Norman and G. L. Bryan, in **Numerical Astrophysics**, edited by S. M. Miyama, K. Tomisaka, and T. Hanawa (Kluwer, Boston, 1999), 19-29. (A)

431. "A Hierarchical $O(N\log N)$ Force-Calculation Algorithm," J. Barnes and P. Hut, Nature **324**, 446-449 (1986). (A)

432. "Large-Scale Structure after COBE: Peculiar Velocities and Correlations of Cold Dark Matter Halos," W. H. Zurek, P. J. Quinn, J. K. Salmon, and M. S. Warren, Astrophys. J. **431**, 559-568 (1994). (A)

433. "The Tree Particle-Mesh N-Body Gravity Solver," P. Bode, J. P. Ostriker, and G. Xu, Astrophys. J. Suppl. **128**, 561-569 (2000). (A)

434. "GOTPM: a Parallel Hybrid Particle-Mesh Treecode," J. Dubinski, J. Kim, C. Park, and R. Humble, New Astron. **9**, 111-126 (2004). (A)

435. "Towards Optimal Parallel PM N-Body Codes: PMFAST," H. Merz, U.-L. Pen, and H. Trac, New Astron. **10**, 393-407 (2005). (A)

436. "Smoothed Particle Hydrodynamics," J. J. Monaghan, Annu. Rev. Astron. Astrophys. **30**, 543-574 (1992). (I, A)

437. "Beyond N-body: 3D Cosmological Gas Dynamics," A. E. Evrard, Mon. Not. R. Astron. Soc. **235**, 911-934 (1988). (A)

438. "TREESPH - A Unification of SPH with the Hierarchical Tree Method," L. Hernquist and N. Katz, Astrophys. J. Suppl. **70**, 419-446 (1989). (A)

439. "GADGET: a Code for Collisionless and Gasdynamical Cosmological Simulations," V. Springel, N. Yoshida, and S. D. M. White, New Astron. **6**, 79-117 (2001). (A)

440. “A Hydrodynamic Approach to Cosmology: Methodology,” R. Cen, Astrophys. J. Suppl. **78**, 341-364 (1992). (A)

441. “A Cosmological Hydrodynamic Code Based on the Total Variation Diminishing Scheme,” D. Ryu, J. P. Ostriker, H. Kang, and R. Cen, Astrophys. J. **414**, 1-19 (1993). (A)

442. “Merger Rates in Hierarchical Models of Galaxy Formation,” C. Lacey and S. Cole, Mon. Not. R. Astron. Soc. **262**, 627-649 (1993). (A)

443. “The Formation and Evolution of Galaxies Within Merging Dark Matter Haloes,” G. Kauffmann, S. D. M. White, and B. Guiderdoni, Mon. Not. R. Astron. Soc. **264**, 201-218 (1993). (A)

444. “A Recipe for Galaxy Formation,” S. Cole, et al., Mon. Not. R. Astron. Soc. **271**, 781-806 (1994). (A)

445. “Semi-Analytic Modelling of Galaxy Formation: The Local Universe,” R. S. Somerville and J. R. Primack, Mon. Not. R. Astron. Soc. **310**, 1087-1110 (1999). (A)

446. “The Halo Occupation Distribution: Toward an Empirical Determination of the Relation between Galaxies and Mass,” A. A. Berlind and D. H. Weinberg, Astrophys. J. **575**, 587-616 (2002). (A)

447. “The Dark Side of the Halo Occupation Distribution,” A. V. Kravtsov, et al., Astrophys. J. **609**, 35-49 (2004). (A)

XI. MAPPING THE UNIVERSE

The observed features of the large-scale distribution of matter include clusters, superclusters, filaments, and voids. By mapping the distribution of galaxies in the Universe, both in two dimensions as projected on the sky and in three dimensions using spectroscopic redshifts, astronomers seek to quantify these inhomogeneities in order to test models for the formation of structure in the Universe. Not only the spatial distribution of galaxies, but also the distribution of clusters of galaxies, quasars, and absorption line systems provide constraints on these models. Peculiar velocities of galaxies, which reflect inhomogeneities in the mass distribution, provide further constraints. Here we briefly review important milestones and surveys relevant for testing cosmological models.

A. Galaxy photometric surveys

Studies of the global spacetime of the Universe assume the “cosmological principle” [170] which is the supposition that the Universe is statistically homogeneous when viewed on sufficiently large scales. The angular distribution of radio galaxies provides a good test of this approach to homogeneity, because radio-bright galaxies and quasars may be seen in flux-limited samples to nearly a Hubble distance, c/H_0. Indeed, the $\sim 31,000$ brightest radio galaxies observed at a wavelength of 6 cm [448] are distributed nearly isotropically, and similar results are found in the FIRST radio survey [449]. (For a review of other evidence for large-scale spatial isotropy see Sec. 3 of Ref. [4].) In contrast, the Universe is clearly inhomogeneous on the more modest scales probed by optically-selected samples of bright galaxies, For example, significant clustering is observed among the roughly $30,000$ galaxies in the Zwicky et al. catalog [450].

Maps of the distribution of nebulae revealed anisotropy in the sky before astronomers came to agree that many of these nebulae were distant galaxies [451]. The Shapley and Ames catalog of galaxies [452] clearly showed the nearby Virgo cluster of galaxies. Surveys of selected areas on the sky using photographic plates to detect distant galaxies clearly revealed anisotropy of the galaxy distribution and were used to quantify this anisotropy [453]. De Vaucouleurs [454] recognized in this anisotropy the projected distribution of the local supercluster of galaxies.

Rubin [455] used two-point correlations of galaxy counts from Harvard College Observatory plates to detect fluctuations on the scale of clusters of galaxies. The Shane and Wirtanen Lick Survey of galaxies [456] used counts of galaxies found on large-format photographic plates taken at Lick Observatory to make the first large-scale map of the angular distribution of galaxies suitable for statistical analysis. Early analysis of these data included methods such as counts-in-cells and the two-point correlation function [457, 458]. The sky map of the Lick counts produced by Seldner et al. [459] visually demonstrated the rich structure in the galaxy distribution. Peebles and collaborators used these data for a lot of their extensive work on galaxy clustering [460]; for a review see Sec. III of Ref. [1] .

The first Palomar Observatory Sky Survey (POSS) yielded two important catalogs: Abell's catalog of clusters [461] and Zwicky et al.'s [450] catalog of clusters and galaxies identified by eye from the photographic plates. Abell [462] found evidence for angular "superclustering" (clustering of galaxy clusters) that was confirmed statistically by Hauser and Peebles [463]. Photographic plates taken at the UK Schmidt telescope were digitized using the Automatic Plate Measuring (APM) machine to produce the APM catalog of roughly two million galaxies. Calibration with CCD photometry made the APM catalog invaluable for accurate study of the angular correlation function of galaxies on large scales [464]. Perhaps the last large-area galaxy photometric survey to employ photographic plates is the Digitized Palomar Observatory Sky Survey (DPOSS) [465].

The largest imaging survey that employs a camera with arrays of charge-coupled devices (CCDs) is the Sloan Digital Sky Survey (SDSS; [466]). The imaging portion of this survey includes five-color digital photometry of 8000 deg^2 of sky, with 215 million detected objects. Imaging for the SDSS is obtained using a special-purpose 2.5 m telescope with a three-degree field of view [467].

Important complements to optical surveys include large-area catalogs of galaxies selected in the infrared and ultraviolet. Nearly all-sky source catalogs were produced from infrared photometry obtained with the Infrared Astronomical Satellite (IRAS; [468]) and the ground-based Two Micron All Sky Survey (2MASS; [469]). The ongoing

Galaxy Evolution Explorer satellite (GALEX; [470]) is obtaining ultraviolet imaging over the whole sky.

448. "The 87GB Catalog of Radio Sources Covering $0° < \delta < +75°$ at 4.85 GHz," P. C. Gregory and J. J. Condon, Astrophys. J. Suppl. **75**, 1011-1291 (1991). (A)

449. "The FIRST Survey: Faint Images of the Radio Sky at Twenty Centimeters," R. H. Becker, R. L. White, and D. J. Helfand, Astrophys. J. **450**, 559-577 (1995). (A)

450. **Catalogue of Galaxies and of Clusters of Galaxies**, F. Zwicky, E. Herzog, and P. Wild (California Institute of Technology, Pasadena, 1961-1968). (A)

451. "On the Structure of the Universe," C. V. L. Charlier, Publ. Astron. Soc. Pac. **37**, 115-135 (1925). (A)

452. "A Survey of the External Galaxies Brighter than the Thirteenth Magnitude," H. Shapley and A. Ames, Ann. Harvard Coll. Obs. **88**, No. 2 (Astron. Obs. Harvard Coll., Cambridge, 1932). (A)

453. "Non-Random Distribution of Extragalactic Nebulae," A. G. Mowbray, Publ. Astron. Soc. Pac. **50**, 275-285 (1938). (A)

454. "Evidence for a Local Supergalaxy," G. de Vaucouleurs, Astron. J. **58**, 30-32 (1953). (A)

455. "Fluctuations in the Space Distribution of the Galaxies," V. C. Rubin, Proc. Natl. Acad. Sci. (USA) **40**, 541-549 (1954). (A)

456. "The Distribution of Extragalactic Nebulae," C. D. Shane and C. A. Wirtanen, Astron. J. **59**, 285-304 (1954). (A)

457. "The Analysis of Counts of the Extragalactic Nebulae in Terms of a Fluctuating Density Field. II," D. N. Limber, Astrophys. J. **119**, 655-681 (1954). (A)

458. "The Correlation Function for the Distribution of Galaxies," H. Totsuji and T. Kihara, Publ. Astron. Soc. Japan **21**, 221-229 (1969). (A)

459. "New Reduction of the Lick Catalog of Galaxies," M. Seldner, B. Siebers, E. J. Groth, and P. J. E. Peebles, Astron. J. **82**, 249-256, 313, 314 (1977). (A)

460. "Statistical Analysis of Catalogs of Extragalactic Objects. VII. Two- and Three-Point Correlation Functions for the High-Resolution Shane-Wirtanen Catalog of Galaxies," E. J. Groth and P. J. E. Peebles, Astrophys. J. **217**, 385-405 (1977). (A)

461. "The Distribution of Rich Clusters of Galaxies," G. O. Abell, Astrophys. J. Suppl. **3**, 211-289 (1958). (A)

462. “Evidence Regarding Second-Order Clustering of Galaxies and Interactions between Clusters of Galaxies,” G. O. Abell, Astron. J. **66**, 607-613 (1961). (A)

463. “Statistical Analysis of Catalogs of Extragalactic Objects. II. The Abell Catalog of Rich Clusters,” M. G. Hauser and P. J. E. Peebles, Astrophys. J. **185**, 757-785 (1973). (A)

464. “Galaxy Correlations on Large Scales,” S. J. Maddox, G. Efstathiou, W. J. Sutherland, and J. Loveday, Mon. Not. R. Astron. Soc. **242**, 43P-47P (1990). (A)

465. “The Digitized Second Palomar Observatory Sky Survey (DPOSS). II. Photometric Calibration,” R. R. Gal, et al., Astron. J. **128**, 3082-3091 (2004). (A)

466. “Sloan Digital Sky Survey: Early Data Release,” C. Stoughton, et al., Astron. J. **123**, 485-548 (2002). (A)

467. “The 2.5 m Telescope of the Sloan Digital Sky Survey,” J. E. Gunn, et al., Astron. J. **131**, 2332-2359 (2006). (A)

468. “Infrared Astronomical Satellite (IRAS) Catalogs and Atlases. Vol. 1: Explanatory Supplement,” C. A. Beichman, et al., NASA RP-1190, Vol. 1 (1988). (A)

469. “2MASS Extended Source Catalog: Overview and Algorithms,” T. H. Jarrett, et al., Astron. J. **119**, 2498-2531 (2000). (A)

470. “The Galaxy Evolution Explorer: A Space Ultraviolet Survey Mission,” D. C. Martin, et al., Astrophys. J. Lett. **619**, L1-L6 (2005). (A)

B. Galaxy spectroscopic surveys

Systematic surveys of galaxies using spectroscopic redshifts to infer their distances began with observations of galaxies selected from the Shapley-Ames catalog [471, 472]. Important early mapping efforts include identification of superclusters and voids in the distribution of galaxies and Abell clusters by Jôeveer et al. [473], the Gregory and Thompson study [474] of the Coma/Abell1367 supercluster and its environs that identified voids, and the Kirshner et al. [475] study of the correlation function of galaxies and discovery of the giant void in Boötes. Early targeted surveys include the Giovanelli and Haynes [476] survey of the Perseus-Pisces supercluster.

Redshift surveys of large areas of the sky began with the first Center for Astrophysics redshift survey (CfA1; [477]), which includes redshifts for 2401 galaxies brighter than apparent magnitude $m_B = 14.5$ over a wide area toward the North Galactic Pole. CfA2 [478] followed over roughly the same area, extending to apparent magnitude $m_B = 15.5$. At this depth, the rich pattern of voids, clusters, and superclusters were strikingly obvious [479]. Reference [480] reviews the status of galaxy redshift surveys ca. 1991.

Both CfA redshift surveys used the Zwicky catalog of galaxies to select targets for spectroscopy. The Southern Sky Redshift Survey (SSRS; [481]) covers a large area of the southern hemisphere (contiguous with CfA2 in the northern galactic cap) to similar depth, using the ESO/Uppsala Survey to select galaxy targets and a spectrograph similar to that employed for the CfA surveys. The Optical Redshift Survey (ORS) supplemented existing redshift catalogs with 1300 new spectroscopic redshifts to create a nearly all-sky survey [482].

Deep "pencil-beam" surveys of narrow patches on the sky revealed apparently-periodic structure in the galaxy distribution [483].

The Las Campanas Redshift Survey (LCRS; [484]), the first large-area survey to use fiber optics, covered over 700 deg^2 near the South Galactic Pole. This survey was important because it showed that structures such as voids and superclusters found in shallower surveys are ubiquitous but the structures seen by LCRS were no larger than those found earlier. The Century Survey [485] and the ESO Deep Slice survey [486] were likewise useful for statistically confirming this emerging picture of large-scale structure.

Sparse surveys of galaxies to efficiently study statistical properties of the galaxy distribution include the Stromlo-APM redshift survey [487] based on 1/20 sampling of the APM galaxy catalog and the Durham/UK Schmidt redshift survey [488].

While optically-selected surveys are relatively blind to structure behind the Milky Way, redshift surveys based on objects detected in the infrared provide nearly all-sky coverage. A sequence of surveys of objects detected by IRAS were carried out, flux-limited to 2 Jy [489], 1.2 Jy [490], and 0.6 Jy [491]. The 6dF Galaxy Survey [492] will measure redshifts of $150,000$ galaxies photometrically identified by 2MASS [469].

The 2-degree Field Galaxy Redshift Survey (2dFGRS) of $250,000$ galaxies [493] was selected from the APM galaxy catalog and observed using the two-degree field multi-fiber spectrograph at the Anglo-Australian 4 m telescope. The survey is complete to apparent magnitude $m_J = 19.45$ and covers about 1500 deg^2.

The spectroscopic component of the SDSS [466] includes medium-resolution spectroscopy of $675,000$ galaxies and $96,000$ quasars identified from SDSS photometry. These spectra are obtained with dual fiber-optic CCD spectrographs on the same 2.5 m telescope. The main galaxy redshift survey is complete to $m_r = 17.77$ and is complemented by a deeper survey of luminous red galaxies. The ongoing extension of this survey (SDSS-II) will expand the spectroscopic samples to more than $900,000$ galaxies and $128,000$ quasars.

Spectroscopic surveys that trace structure in the galaxy distribution at much larger redshift include the DEEP2 survey [494] and others [495] employing the Keck Observatory, and the VIMOS VLT Deep survey [496].

471. "Redshifts and Magnitudes of Extragalactic Nebulae.," M. L. Humason, N. U. Mayall, and A. R. Sandage, Astron. J. **61**, 97-162 (1956). (A)

472. "Optical Redshifts for 719 Bright Galaxies," A. Sandage, Astron. J. **83**, 904-937 (1978). (A)

473. “Spatial Distribution of Galaxies and of Clusters of Galaxies in the Southern Galactic Hemisphere,” M. Jôeveer, J. Einasto, and E. Tago, Mon. Not. R. Astron. Soc. **185**, 357-369 (1978). (A)

474. “The Coma/A1367 Supercluster and its Environs,” S. A. Gregory and L. A. Thompson, Astrophys. J. **222**, 784-799 (1978). (A)

475. “A Million Cubic Megaparsec Void in Boötes,” R. P. Kirshner, A. Oemler, Jr., P. L. Schechter, and S. A. Shectman, Astrophys. J. Lett. **248**, L57-L60 (1981). (A)

476. “A 21 cm Survey of the Pisces-Perseus Supercluster. I. The Declination Zone +27.5 to +33.5 Degrees,” R. Giovanelli and M. P. Haynes, Astron. J. **90**, 2445-2473 (1985). (A)

477. “A Survey of Galaxy Redshifts. IV. The Data,” J. P. Huchra, M. Davis, D. Latham, and J. Tonry, Astrophys. J. Suppl. **52**, 89-119 (1983). (A)

478. “The Updated Zwicky Catalog (UZC),” E. E. Falco, et al., Publ. Astron. Soc. Pac. **111**, 438-452 (1999). (A)

479. “A Slice of the Universe,” V. de Lapparent, M. J. Geller, and J. P. Huchra, Astrophys. J. Lett. **302**, L1-L5 (1986). (A)

480. “Redshift Surveys Of Galaxies,” R. Giovanelli and M. P. Haynes, Annu. Rev. Astron. Astrophys. **29**, 499-541 (1991). (I)

481. “The Southern Sky Redshift Survey,” L. N. da Costa, et al., Astron. J. **116**, 1-7 (1998). (A)

482. “The Optical Redshift Survey: Sample Selection and the Galaxy Distribution,” B. X. Santiago, et al., Astrophys. J. **446**, 457-471 (1995). (A)

483. “Large-Scale Distribution of Galaxies at the Galactic Poles,” T. J. Broadhurst, R. S. Ellis, D. C. Koo, and A. S. Szalay, Nature **343**, 726-728 (1990). (A)

484. “The Las Campanas Redshift Survey,” S. A. Shectman, et al., Astrophys. J. **470**, 172-188 (1996). (A)

485. “The Century Survey: A Deeper Slice of the Universe,” M. J. Geller, et al., Astron. J. **114**, 2205-2211, 2836 (1997). (A)

486. “The ESO Slice Project (ESP) Galaxy Redshift Survey. III. The Sample,” G. Vettolani, et al., Astron. Astrophys. Suppl. **130**, 323-332 (1998). (A)

487. “The Stromlo-APM Redshift Survey. IV. The Redshift Catalog,” J. Loveday, B. A. Peterson, S. J. Maddox, and G. Efstathiou, Astrophys. J. Suppl. **107**, 201-214 (1996). (A)

488. "The Durham/UKST Galaxy Redshift Survey — V. The Catalogue," A. Ratcliffe, et al., Mon. Not. R. Astron. Soc. **300**, 417-462 (1998). (A)

489. "A Redshift Survey of IRAS Galaxies. VII. The Infrared and Redshift Data for the 1.936 Jansky Sample," M. A. Strauss, et al., Astrophys. J. Suppl. **83**, 29-63 (1992). (A)

490. "The IRAS 1.2 Jy Survey: Redshift Data," K. B. Fisher, et al., Astrophys. J. Suppl. **100**, 69-103 (1995). (A)

491. "The PSCz Catalogue," W. Saunders, et al., Mon. Not. R. Astron. Soc. **317**, 55-63 (2000). (A)

492. "The 6dF Galaxy Survey: Samples, Observational Techniques and the First Data Release," D. H. Jones, et al., Mon. Not. R. Astron. Soc. **355**, 747-763 (2004). (A)

493. "The 2dF Galaxy Redshift Survey: Spectra and Redshifts," M. Colless, et al., Mon. Not. R. Astron. Soc. **328**, 1039-1063 (2001). (A)

494. "The DEEP2 Galaxy Redshift Survey: Clustering of Galaxies in Early Data," A. L. Coil, et al., Astrophys. J. **609**, 525-538 (2004). (A)

495. "A Survey of Star-forming Galaxies in the $1.4 < z < 2.5$ Redshift Desert: Overview," C. C. Steidel, et al., Astrophys. J. **604**, 534-550 (2004). (A)

496. "The VIMOS VLT deep survey. The Evolution of Galaxy Clustering to $z \simeq 2$ from First Epoch Observations," O. Le Fèvre, et al., Astron. Astrophys. **439**, 877-885 (2005). (A)

C. Cluster surveys

Mapping of the Universe using galaxy clusters as tracers began with study of the Abell catalog [461, 497]. Studies of the angular clustering of Abell clusters includes Ref. [463]. Several redshift surveys of Abell clusters have been conducted, including those described in Refs. [498, 499]. Important cluster samples have also been identified from digitized photographic plates from the UK Schmidt telescope, followed up by redshift surveys of cluster galaxies [500, 501]. More distant samples of clusters have been identified using deep CCD photometry, see, e.g., Refs. [502, 503]. In X-ray bandpasses, cluster samples useful for studying large-scale structure have been identified using data from ROSAT [504, 505]. The SDSS is producing large catalogs of galaxy clusters using a variety of selection methods [506]. Use of the Sunyaev-Zel'dovich effect (the microwave decrement caused by Thomson-Compton scattering of the CMB photons by the intracluster gas) holds great promise to identify new deep samples of galaxy clusters [507]. General reviews of clusters of galaxies include Refs. [508, 509, 510].

497. “A Catalog of Rich Clusters of Galaxies,” G. O. Abell, H. G. Corwin, and R. P. Olowin, Astrophys. J. Suppl. **70**, 1-138 (1989). (A)

498. “The Distribution of Nearby Rich Clusters of Galaxies,” M. Postman, J. P. Huchra, and M. J. Geller, Astrophys. J. **384**, 404-422 (1992). (A)

499. “The ESO Nearby Abell Cluster Survey. I. Description of the Dataset and Definition of Physical Systems,” P. Katgert, et al., Astron. Astrophys. **310**, 8-30 (1996). (A)

500. “The Edinburgh-Durham Southern Galaxy Catalogue — IV. The Cluster Catalogue,” S. L. Lumsden, R. C. Nichol, C. A. Collins, and L. Guzzo, Mon. Not. R. Astron. Soc. **258**, 1-22 (1992). (A)

501. “Spatial Correlations in a Redshift Survey of APM Galaxy Clusters,” G. B. Dalton, G. Efstathiou, S. J. Maddox, and W. J. Sutherland, Astrophys. J. Lett. **390**, L1-L4 (1992). (A)

502. “The Palomar Distant Clusters Survey. I. The Cluster Catalog,” M. Postman, et al., Astron. J. **111**, 615-641, 988-993 (1996). (A)

503. “The Red-Sequence Cluster Survey. I. The Survey and Cluster Catalogs for Patches RCS 0926+37 and RCS 1327+29,” M. D. Gladders and H. K. C. Yee, Astrophys. J. Suppl. **157**, 1-29 (2005). (A)

504. “The Large-Scale Distribution of X-ray Clusters of Galaxies,” A. K. Romer, et al., Nature **372**, 75-77 (1994). (A)

505. “The ROSAT-ESO Flux Limited X-ray (REFLEX) Galaxy Cluster Survey. V. The Cluster Catalogue,” H. Böhringer, et al., Astron. Astrophys. **425**, 367-383 (2004). (A)

506. “A Merged Catalog of Clusters of Galaxies from Early Sloan Digital Sky Survey Data,” N. A. Bahcall, et al., Astrophys. J. Suppl. **148**, 243-274 (2003). (A)

507. “Cosmology with the Sunyaev-Zel’dovich Effect,” J. E. Carlstrom, G. P. Holder, and E. D. Reese, Annu. Rev. Astron. Astrophys. **40**, 643-680 (2002). (I)

508. “The Evolution of X-ray Clusters of Galaxies,” P. Rosati, S. Borgani, and C. Norman, Annu. Rev. Astron. Astrophys. **40**, 539-577 (2002). (I)

509. “Tracing Cosmic Evolution with Clusters of Galaxies,” G. M. Voit, Rev. Mod. Phys. **77**, 207-258 (2005). (I)

510. “Cosmology with Clusters of Galaxies,” S. Borgani, astro-ph/0605575. (I, A)

D. Quasar surveys

The advent of multi-object wide-field spectrographs has made possible collection of very large samples of spectroscopically-confirmed quasars, as observed by the 2dF QSO Redshift Survey [511] and the SDSS [512]. For a ca. 1990 review of the field see Ref. [513]. While quasars themselves are too sparsely distributed to provide good maps of the large-scale distribution of matter, their clustering in redshift space has been measured [514] and generally found to be similar to that of galaxies [515]. Similar results obtain from clustering analyses of active galactic nuclei in the nearby universe [516], although this clustering depends in detail on the type of AGN [517].

The distribution of absorption lines from gas, particularly from the Lyman-α "forest" of neutral hydrogen clouds along the line of sight toward bright quasars [518, 519] provides an important statistical probe of the distribution of matter (see, e.g., Ref. [520]) on small scales and at large redshift.

511. "The 2dF QSO Redshift Survey — XII. The Spectroscopic Catalogue and Luminosity Function," S. M. Croom, et al., Mon. Not. R. Astron. Soc. **349**, 1397-1418 (2004). (A)

512. "The Sloan Digital Sky Survey Quasar Catalog. III. Third Data Release," D. P. Schneider, et al., Astron. J. **130**, 367-380 (2005). (A)

513. "The Space Distribution of Quasars," F. D. A. Hartwick and D. Schade, Annu. Rev. Astron. Astrophys. **28**, 437-489 (1990). (I)

514. "The Three-Dimensional Distribution of Quasars in the CTIO Surveys," P. S. Osmer, Astrophys. J. **247**, 762-773 (1981). (A)

515. "The 2dF QSO Redshift Survey — XI. The QSO Power Spectrum," P. J. Outram, et al., Mon. Not. R. Astron. Soc. **342**, 483-495 (2003). (A)

516. "The Clustering of Active Galactic Nuclei in the Sloan Digital Sky Survey," D. A. Wake, et al., Astrophys. J. Lett. **610**, L85-L88 (2004). (A)

517. "The Spatial Clustering of Low-Luminosity Active Galactic Nuclei," A. Constantin and M. S. Vogeley, Astrophys. J. **650**, 727-748 (2006). (A)

518. "The Absorption-Line Spectrum of 4C 05.34," R. Lynds, Astrophys. J. Lett. **164**, L73-L78, plate L3 (1971). (A)

519. "The Lyman Alpha Forest in the Spectra of QSOs," M. Rauch, Annu. Rev. Astron. Astrophys. **36**, 267-316 (1998). (A)

520. "The Linear Theory Power Spectrum from the Lyα Forest in the Sloan Digital Sky Survey," P. McDonald, et al., Astrophys. J. **635**, 761-783 (2005). (A)

E. Peculiar velocity surveys

When measured over sufficiently large scales, the peculiar motions of galaxies or clusters simply depend on the potential field generated by the mass distribution (see, e.g., Refs. [1, 521, 4]). Techniques for measuring distances to other galaxies are critically reviewed in Refs. [20, 522, 523, 22]. Together with the galaxy or cluster redshifts, these measurements yield maps of the line-of-sight component of the peculiar velocity. From such data it is possible to reconstruct a map of the matter density field (e.g., [524, 525]) or to trace the galaxy orbits back in time (e.g., [526, 527]). Analyses of correlations of the density and velocity fields also yield constraints on the cosmic matter density, e.g., [528].

Rubin et al. [529] were the first to find evidence for bulk flows from galaxy peculiar velocities. Dressler et al. [530] found evidence for a bulk flow toward a large mass concentration, dubbed the "Great Attractor." Lauer and Postman [531] found surprising evidence for motion of the Local Group on a larger scale. However, analysis of subsequent peculiar velocity surveys indicates that the inferred bulk flow results, including those of Lauer and Postman, are consistent within the uncertainties [532]. The status of this field ca. 1999 is surveyed in Ref. [533], recent results include Ref. [534], and reviews of this topic are in Refs. [525, 523]. Comparison of peculiar velocity surveys with the peculiar velocity of our Galaxy implied by the CMB dipole indicates that a significant component of our motion must arise from density inhomogeneities that lie at rather large distance, beyond $60h^{-1}$ Mpc [534].

521. "Evidence for Local Anisotropy of the Hubble Flow," M. Davis and P. J. E. Peebles, Annu. Rev. Astron. Astrophys. **21**, 109-130 (1983). (I)

522. "A Critical Review of Selected Techniques for Measuring Extragalactic Distances," G. H. Jacoby, et al., Publ. Astron. Soc. Pac. **104**, 599-662 (1992). (A)

523. "The Density and Peculiar Velocity Fields of Nearby Galaxies," M. A. Strauss and J. A. Willick, Phys. Rept. **261**, 271-431 (1995). (I)

524. "Recovering the Full Velocity and Density Fields from Large-Scale Redshift-Distance Samples," E. Bertschinger and A. Dekel, Astrophys. J. Lett. **336**, L5-L8 (1989). (A)

525. "Dynamics of Cosmic Flows," A. Dekel, Annu. Rev. Astron. Astrophys. **32**, 371-418 (1994). (I)

526. "The Gravitational Instability Picture and the Formation of the Local Group," P. J. E. Peebles, Astrophys. J. **362**, 1-13 (1990). (A)

527. "Using Perturbative Least Action to Recover Cosmological Initial Conditions," D. M. Goldberg and D. N. Spergel, Astrophys. J. **544**, 21-29 (2000). (A)

528. "Maximum Likelihood Comparisons of Tully-Fisher and Redshift Data: Constraints on Ω and Biasing," J. A. Willick, M. A. Strauss, A. Dekel, and T. Kolatt, Astrophys. J. **486**, 629-664 (1997). (A)

529. "Motion of the Galaxy and the Local Group Determined from the Velocity Anisotropy of Distant Sc I Galaxies. II — The Analysis for the Motion," V. C. Rubin, N. Thonnard, W. K. Ford Jr., and M. S. Roberts, Astron. J. **81**, 719-737 (1976). (A)

530. "Spectroscopy and Photometry of Elliptical Galaxies: A Large-Scale Streaming Motion in the Local Universe," A. Dressler, et al., Astrophys. J. Lett. **313**, L37-L42 (1987). (A)

531. "The Motion of the Local Group with Respect to the 15,000 kilometer per second Abell Cluster Inertial Frame," T. R. Lauer and M. Postman, Astrophys. J. **425**, 418-438 (1994). (A)

532. "Are Recent Peculiar Velocity Surveys Consistent?," M. J. Hudson, M. Colless, A. Dressler, and R. Giovanelli, in Ref. [533], 159-166 (2000). (A)

533. **Cosmic Flows Workshop**, edited by S. Courteau and J. Willick (Astron. Soc. Pac., San Francisco, 2000). (A)

534. "Streaming Motions of Galaxy Clusters Within 12 000 km s^{-1} — V. The Peculiar Velocity Field," M. J. Hudson, R. J. Smith, J. R. Lucey, and E. Branchini, Mon. Not. R. Astron. Soc. **352**, 61-75 (2004). (A)

F. Statistics of large-scale structure

The clustering pattern of extragalactic objects reflects both the initial conditions for structure formation and the complex astrophysics of formation and evolution of these objects. In the standard picture described above, linear perturbation theory accurately describes the early evolution of structure, thus measurement of clustering on very large scales, where the clustering remains weak, closely reflects the initial conditions. On these scales the density field is very nearly Gaussian random phase, therefore the two-point correlation function of the galaxy number density field (also called the autocorrelation or covariance function) or its Fourier transform, the power spectrum, provides a complete statistical description. (Temperature anisotropies of the CMB discussed in Sec. X.B arise from density fluctuations at redshift $z \sim 10^3$ that evolve in the fully linear regime.) On the scales of galaxies and clusters of galaxies, gravitational evolution is highly non-linear and the apparent clustering depends strongly on the detailed relationship between mass and light in galaxies. In between the linear and non-linear regimes lies the "quasi-linear" regime in which clustering growth proceeds most rapidly. A wide range of statistical methods have been developed to quantify this complex behavior. Statistical properties of the galaxy distribution and details of estimating most of the relevant statistics are described in depth by Refs. [1, 535, 536]. Methods of using galaxy redshift surveys to constrain cosmology are reviewed by Refs. [537, 538]. Constraints on cosmological parameters from such measurements are discussed below in Sec. XII.

The simplest set of statistical measures are the n-point correlation functions, which describe the joint probability in excess of random of finding n galaxies at specified relative separation. Early applications of correlation functions to galaxy data include Refs. [457, 458, 460]. The n-point functions may be estimated by directly examining the positions of n-tuples of galaxies or by using moments of galaxy counts in cells of varying size. Tests of scaling relations among the n-point functions are discussed in detail by Ref. [536].

Power spectrum analyses of large galaxy redshift surveys [539, 540, 541] yield useful constraints on cosmological models. Closely related to power spectrum analyses are estimates of cosmological parameters using orthogonal functions [542, 543]. Reference [544] discusses the merits of different methods of power spectrum estimation. Reference [545] describes a measurement of the galaxy bispectrum.

A number of statistics have been developed to quantify the geometry and topology of large-scale structure. The topological genus of isodensity contours characterizes the connectivity of large-scale structure [546]. Measurements of the genus are consistent with random phase initial conditions (as predicted by inflation) on large scales [547], with departures from Gaussianity on smaller scales where nonlinear gravitational evolution and biasing of galaxies are evident [548, 549]. Similar techniques are used to check on the Gaussianity of the CMB anisotropy, [351, 352, 350], as well as identify foreground emission signals in CMB anisotropy data [550].

The void probability function, which characterizes the frequency of completely empty regions of space [551], has been estimated from galaxy redshift surveys [552, 553]. Catalogs of voids have been constructed with objective void finding algorithms [554, 555].

Early investigations of the pattern of galaxy clustering dating back to Charlier [451] suggested a clustering hierarchy. The fractal model of clustering introduced by Mandelbrot (see [556] and references therein) further motivated investigation of the possibility of scale-invariant clustering of galaxies. Results of such analyses of galaxy survey data were controversial (compare, e.g., [557] with [558] and [559] and references therein). While fractal behavior is seen on small scales, there is fairly strong evidence for an approach to homogeneity in galaxy redshift and photometric surveys on very large scales. Thus, a simple scale-invariant fractal description seems to be ruled out. A multi-fractal description of clustering continues to provide a useful complement to other statistical descriptors [560]. Consideration of modified forms of the fractal picture are of interest for providing slight non-Gaussianity on very large scales that might be needed to explain the very largest structures in the Universe.

Anisotropy of galaxy clustering in redshift space results from bulk flows on large scales that amplify clustering along the line of sight to the observer and from motions of galaxies in virialized systems such as clusters that elongate those structures along the line of sight [561]. Reference [562] provides an extensive review and Ref. [563] describes recent methods for estimating cosmological parameters from redshift-space distortions of the correlation function or power spectrum.

The dependence of clustering statistics on properties of galaxies provides important clues to their history of formation and reflects the complex relationship between the distributions of mass and luminous matter. The amplitude of galaxy clustering is seen to vary with galaxy morphology (e.g., [564, 565]) and with luminosity (e.g., [566, 567]). In recent analyses of the SDSS and 2dFGRS, these and similar trends with color, surface brightness, and spectral type are seen [568, 569].

Spectroscopy obtained with 8-10 m class telescopes has recently made it possible to accurately study structure in the galaxy distribution at higher redshift [494, 570, 496].

535. **Statistics of the Galaxy Distribution**, V. J. Martínez and E. Saar (Chapman & Hall/CRC, Boca Raton, 2002). (I, A)

536. "Large-scale Structure of the Universe and Cosmological Perturbation Theory," F. Bernardeau, S. Colombi, E. Gaztañaga, and R. Scoccimarro, Phys. Rept. **367**, 1-248 (2002). (A)

537. "Measuring our Universe from Galaxy Redshift Surveys," O. Lahav and Y. Suto, Living Rev. Relativity **7**, 8 (2004). (I, A)

538. "Cosmological Constraints from Galaxy Clustering," W. J. Percival, astro-ph/0601538. (I, A)

539. "Large-Scale Clustering of Galaxies in the CfA Redshift Survey," M. S. Vogeley, C. Park, M. J. Geller, and J. P. Huchra, Astrophys. J. Lett. **391**, L5-L8 (1992). (A)

540. "The Power Spectrum of IRAS Galaxies," K. B. Fisher, et al., Astrophys. J. **402**, 42-57 (1993). (A)

541. "The Three-Dimensional Power Spectrum of Galaxies from the Sloan Digital Sky Survey," M. Tegmark, et al., Astrophys. J. **606**, 702-740 (2004). (A)

542. "Eigenmode Analysis of Galaxy Redshift Surveys. I. Theory and Methods," M. S. Vogeley and A. S. Szalay, Astrophys. J. **465**, 34-53 (1996). (A)

543. "Cosmological Parameters from Eigenmode Analysis of Sloan Digital Sky Survey Galaxy Redshifts," A. C. Pope, et al., Astrophys. J. **607**, 655-660 (2004). (A)

544. "Measuring the Galaxy Power Spectrum with Future Redshift Surveys," M. Tegmark, et al., Astrophys. J. **499**, 555-576 (1998). (A)

545. "The 2dF Galaxy Redshift Survey: The Bias of Galaxies and the Density of the Universe," L. Verde, et al., Mon. Not. R. Astron. Soc. **335**, 432-440 (2002). (A)

546. "A Quantitative Approach to the Topology of Large-Scale Structure," J. R. Gott, III, D. H. Weinberg, and A. L. Melott, Astrophys. J. **319**, 1-8 (1987). (A)

547. "The Topology of Large-Scale Structure. III. Analysis of Observations," J. R. Gott, III, et al., Astrophys. J. **340**, 625-646 (1989). (A)

548. "Topological Analysis of the CfA Redshift Survey," M. S. Vogeley, et al., Astrophys. J. **420**, 525-544 (1994). (A)

549. "Topology of Structure in the Sloan Digital Sky Survey: Model Testing," J. R. Gott, III, et al., Astrophys. J., submitted, astro-ph/0610762 (A)

550. "Effect of Foreground Contamination on the Cosmic Microwave Background Anisotropy Observations Measured by MAP," C.-G. Park, C. Park, and B. Ratra, Astrophys. J. **568**, 9-19 (2002). (A)

551. "The Hierarchy of Correlation Functions and its Relation to Other Measures of Galaxy Clustering," S. D. M. White, Mon. Not. R. Astron. Soc. **186**, 145-154 (1979). (A)

552. "Void Probabilities in the Galaxy Distribution. Scaling and Luminosity Segregation," S. Maurogordato and M. Lachièze-Rey, Astrophys. J. **320**, 13-25 (1987). (A)

553. "Voids in the Two-Degree Field Galaxy Redshift Survey," F. Hoyle and M. S. Vogeley, Astrophys. J. **607**, 751-764 (2004). (A)

554. "Automated Detection of Voids in Redshift Surveys," H. El-Ad, T. Piran, and L. N. da Costa, Astrophys. J. Lett. **462**, L13-L16 (1996). (A)

555. "Voids in the Point Source Catalogue Survey and the Updated Zwicky Catalog," F. Hoyle and M. S. Vogeley, Astrophys. J. **566**, 641-651 (2002). (A)

556. **The Fractal Geometry of Nature**, B. B. Mandelbrot (Freeman, San Francisco, 1982). (I)

557. "Scale Invariance of Galaxy Clustering," F. Sylos Labini, M. Montouri, and L. Pietronero, Phys. Rept. **293**, 61-226 (1998). (A)

558. "Approaching a Homogeneous Galaxy Distribution: Results from the Stromlo-APM Redshift Survey," S. Hatton, Mon. Not. R. Astron. Soc. **310**, 1128-1136 (1999). (A)

559. "Does the Galaxy Correlation Length Increase with the Sample Depth?," V. J. Martínez, B.Łópez-Martí, and M.-J. Pons-Bordería, Astrophys. J. Lett. **554**, L5-L8 (2001). (A)

560. "Scaling Laws in the Distribution of Galaxies," B. J. T. Jones, V. J. Martínez, E. Saar, and V. Trimble, Rev. Mod. Phys. **76**, 1211-1266 (2005). (A)

561. "Clustering in Real Space and in Redshift Space," N. Kaiser, Mon. Not. R. Astron. Soc. **227**, 1-21 (1987). (A)

562. "Linear Redshift Distortions: A Review," A. J. S. Hamilton, in **The Evolving Universe**, edited by D. Hamilton (Kluwer Academic, Dordrecht, 1998), 185-275. (A)

563. "Redshift-Space Distortions with the Halo Occupation Distribution - I. Numerical Simulations," J. L. Tinker, D. H. Weinberg, and Z. Zheng, Mon. Not. R. Astron. Soc. **368**, 85-108 (2006). (A)

564. "Galaxy Correlations as a Function of Morphological Type," M. Davis and M. J. Geller, Astrophys. J. **208**, 13-19 (1976). (A)

565. "Redshift-Space Distortions and the Real-Space Clustering of Different Galaxy Types," L. Guzzo, et al., Astrophys. J. **489**, 37-48 (1997). (A)

566. "Evidence for Biasing in the CfA Survey," A. J. S. Hamilton, Astrophys. J. Lett. **331**, L59-L62 (1988). (A)

567. "Power Spectrum, Correlation Function, and Tests for Luminosity Bias in the CfA Redshift Survey," C. Park, M. S. Vogeley, M. J. Geller, and J. P. Huchra, Astrophys. J. **431**, 569-585 (1994). (A)

568. "The 2dF Galaxy Redshift Survey: The Dependence of Galaxy Clustering on Luminosity and Spectral Type," P. Norberg, et al., Mon. Not. R. Astron. Soc. **332**, 827-838 (2002). (A)

569. "The Luminosity and Color Dependence of the Galaxy Correlation Function," I. Zehavi, et al., Astrophys. J. **630**, 1-27 (2005). (A)

570. "The Spatial Clustering of Star-forming Galaxies at Redshifts $1.4 < z < 3.5$," K. L. Adelberger, et al., Astrophys. J. **619**, 697-713 (2005). (A)

XII. MEASURING COSMOLOGICAL PARAMETERS

A. The case for a flat, accelerating Universe

As mentioned in Sec. IX, observations of Type Ia supernovae (SNeIa) provide strong evidence that the expansion of the Universe is accelerating. Type Ia supernovae have the useful property that their peak intrinsic luminosities are correlated with how fast they dim, which allows them to be turned into standard candles. At redshifts approaching unity, observations indicate that they are dimmer (and so farther away) than would be predicted in an unaccelerating Universe [305, 306]. In the context of general relativity this acceleration is attributed to dark energy that varies slowly with time and space, if at all. A mass-energy component that maintains constant (or nearly constant) density has negative pressure. Since pressure contributes to the active gravitational mass density, large enough negative pressure can overwhelm the attraction caused by the usual (including dark) matter mass density and result in accelerated expansion. For a careful review of the early supernova tests see Ref. [186] and for discussions of the cosmological implications of this test see Refs. [288, 310]. Current supernova data show that models with vanishing cosmological constant are more than four standard deviations away from the best fit.

The supernova test assumes general relativity and probes the geometry of spacetime. The result is confirmed by a test using the CMB anisotropy that must in addition assume the CDM model for structure formation discussed in Sec. IX (see Sec. X.C for apparent problems with this model). As discussed in Sec. X.B, CMB anisotropy data on the position of the first peak in the angular power spectrum is consistent with the curvature of spatial hypersurfaces being small. Many independent lines of evidence indicate that the mass density of nonrelativistic matter (CDM and baryons) — a number also based on the CDM structure formation model — is about 25 or 30 % of the critical Einstein-de Sitter density (see Sec. IX). Since the contemporary mass density of radiation and other relativistic matter is small, this requires that a cosmological constant or dark energy contribute 70 or 75 % of the current mass budget of the Universe. For reviews of the CMB data constraints see Refs. [288, 311, 370].

B. Observational constraints on cosmological parameters

The model suggested by the SNIa and CMB data, spatially flat and with contemporary mass-energy budget split between a cosmological constant or dark energy ($\sim$ 70 %), dark matter ($\sim$ 25 %), and baryonic matter ($\sim$ 5 %), is broadly consistent with the results of a large number of other cosmological tests. In this subsection we present a very brief discussion of some of these tests and the constraints they impose on the parameters of this "standard" cosmological model. Two nice reviews of the cosmological tests are Sec. 13 of Ref. [4] and Sandage's in Ref. [73]. (Reference [571] provides a concise summary of various geometrical measures used in these tests.) Section IV of Ref. [288] reviews more recent developments and observational constraints. Here we summarize some of these as well as the significant progress of the last four years. Numerical values for cosmological parameters are listed in Ref. [572], although in some cases there is still significant ongoing debate.

There have been many — around 500 — measurements of the Hubble constant H_0, [573], the current expansion rate. Since there is debate about the error estimates of some of these measurements, a median statistics meta-analysis estimate of H_0 is probably the most robust estimate [574]. At two standard deviations this gives $H_0 = 100h$ km s^{-1} Mpc^{-1} $= 68 \pm 7$ km s^{-1} Mpc^{-1} $= (14 \pm 1$ Gyr$)^{-1}$, [575], where the first equation defines h. It is significant that this result agrees with the estimate from the HST Key Project [576], the HST estimate of Sandage and collaborators [577], and the WMAP three-year data estimate (which assumes the CDM structure formation model) [370].

A measurement of the redshift dependence of the Hubble parameter can be used to constrain cosmological parameters [578, 579]. For applications of this test using preliminary data see Refs. [580, 581].

Expansion time tests are reviewed in Sec. IV.B.3 of Ref. [288]. A recent development is the WMAP CMB anisotropy data estimate of the age of the Universe, $t_0 = 13.7 \pm 0.3$ Gyr at two standard deviations [370], which assumes the CDM structure formation model. This WMAP t_0 estimate is consistent with t_0 estimated from globular cluster observations [582, 583, 584] and from white dwarf star measurements

[585]. The above values of H_0 and t_0 are consistent with a spatially-flat, dark energy dominated Universe.

As discussed in Sec. VII.C and Refs. [288, 204, 206], ^{4}He and ^{7}Li abundance measurements favor a higher baryon density than the D abundance measurements and the WMAP CMB anisotropy data. (This difference is under active debate.) However, it is remarkable that high-redshift ($z \sim 10^3$) CMB data and low-redshift ($z \lesssim$ few) abundance measurements indicate a very similar baryon density. A summary range of the baryonic density parameter from nucleosynthesis is $\Omega_B = (0.0205 \pm 0.0035)h^{-2}$ at two standard deviations [204].

As mentioned above, Type Ia supernovae apparent magnitudes as a function of redshift may be used to constrain the cosmological model. See Sec. IV.B.4 of Ref. [288] for a summary of this test. References [305, 306] provided initial constraints on a cosmological constant from this test, and Refs. [586, 587] generalize the method to constrain scalar field dark energy. Developments may be traced back from Refs. [588, 589, 590, 591, 592, 593, 594]. Proposed satellite experiments are under active discussion and should result in tight constraints on dark energy and its evolution. See Refs. [595, 596, 597] for developments in this area.

The angular size of objects (e.g., quasars, compact radio sources, radio galaxies) as a function of redshift provides another cosmological test. These observations are not as numerous as the supernovae, so this test is much less constraining, but the results are consistent with those from the SNIa apparent magnitude test. Developments may be traced back through Refs. [598, 599]. Reference [600] describes a way of combining the apparent magnitude and angular size data to more tightly constrain cosmological parameters.

"Strong" gravitational lensing, by a foreground galaxy or cluster of galaxies, produces multiple images of a background radio source. The statistics of strong lensing may be used to constrain the cosmological model. References [601, 602] note that for low nonrelativistic matter density the predicted lensing rate is significantly larger in a cosmological constant dominated spatially-flat model than in an open model. The scalar field dark energy case is discussed in Ref. [603] and lies between these two limits. For reviews of the test see Sec. IV.B.6 of Ref. [288] and Ref. [604]. Recent developments may be traced back from Ref. [605]. Cosmological constraints from the CLASS gravitational lens statistics data are discussed in Refs. [606, 607, 608]. These constraints are consistent with those derived from the supernova apparent magnitude data, but are not as restrictive.

Galaxy motions respond to fluctuations in the gravitational potential, thus peculiar velocities of galaxies may be used to estimate the nonrelativistic matter density parameter Ω_{M} (as discussed in Secs. IX and XI.E above and in Ref. [609] and Sec. IV.B.7 of Ref. [288]) by comparing the pattern of flows with maps of the galaxy distribution. Note that peculiar velocities are not sensitive to a homogeneously distributed mass-energy component. For a summary of recent results from the literature see Ref. [610]. Measurements of the anisotropy of the redshift-space galaxy distribution that is produced by peculiar velocities also yield estimates of the matter density [561, 562]. Most methods measure this anisotropy in the galaxy autocorrelation or power spectrum (see, e.g., Ref. [563]). Recent analyses include Refs. [611, 612] from the 2dFGRS and 2QZ surveys. Also of interest are clustering analyses of the SDSS that explicitly take into account this redshift-space anisotropy either by using the predicted distortions when constructing eigenmodes [543] or by constructing modes that are sensitive to radial vs. angular fluctuations [541].

A median statistics analysis of many such measurements, as well as other types of data, indicates that the nonrelativistic matter density parameter lies in the range $0.2 \lesssim \Omega_0 \lesssim 0.35$ at two standard deviations [613]. This is consistent with estimates based on other data, e.g., the WMAP CMB data result in a very similar range [370].

"Weak" gravitational lensing (which mildly distorts the images of background objects), in combination with other data, should soon provide tight constraints on the nonrelativistic matter density parameter. For reviews of weak lensing see Refs. [614, 615, 616]; see Refs. [617, 618, 619] for recent developments. (Weak gravitational lensing also provides evidence for dark matter; see, e.g., Refs. [620, 621].)

Rich clusters of galaxies are thought to have originated from large enough volumes to have fairly sampled both the baryons and the dark matter. In conjunction with the nucleosynthesis estimate of the baryonic mass density parameter, the rich cluster estimate of the ratio of baryonic and nonrelativistic (including baryonic) matter — the cluster baryon fraction — provides an estimate of the nonrelativistic matter density parameter [622, 623]. (A promising method for measuring the cluster baryonic gas mass fraction uses the Sunyaev-Zel'dovich effect [507].) Estimates of $\Omega_{\rm M}$ from this test are in the range listed above.

An extension of this cluster test makes use of measurements of the rich cluster baryon mass fraction as a function of redshift. For relaxed rich clusters (not those in the process of collapsing) the baryon fraction should be independent of redshift. The cluster baryon fraction depends on the angular diameter distance [624, 625], so the correct cosmological model places clusters at the right angular diameter distances to ensure that the cluster baryon mass fraction is independent of redshift. This test provides a fairly restrictive constraint on $\Omega_{\rm M}$, consistent with the range above; developments may be traced back through Refs. [626, 627, 628, 629]. When combined with complementary cosmological data, especially the restrictive SNIa data, the cluster baryon mass fraction versus redshift data provide tight constraints on the cosmological model, favoring a cosmological constant but not yet ruling out slowly varying dark energy [630, 631, 632].

The number density of rich clusters of galaxies as a function of cluster mass, both at the present epoch and as a function of redshift, constrains the amplitude of mass fluctuations and the nonrelativistic matter density parameter (see Sec. IV.B.9 of Ref. [288] and references therein). Current cluster data favor a matter density parameter in the range discussed above [508, 509, 633, 510].

The rate at which large-scale structure forms could eventually provide another direct test of the cosmological model. The constant cosmological constant model is discussed in Ref. [302] and some of the more recent textbooks above. The scalar field dark energy model is not as tractable; developments may be traced from Refs. [634, 635, 636].

Measurements of CMB temperature and polarization anisotropies (see Sec. X.B above and Sec. IV.B.11 of Ref. [288]) provide some of the strongest constraints on several cosmological model parameters. These constraints depend on the assumed structure formation model. Current constraints are usually based on the CDM model (or some variant of it). As discussed in Sec. X.B, the three-year WMAP data [349] provide state-of-the-art constraints [370].

Data on the large-scale power spectrum (or correlation function) of galaxies complement the CMB measurements by connecting the inhomogeneities observed at redshift $z \sim 10^3$ in the CMB to fluctuations in galaxy density close to $z = 0$, and by relating fluctuations in gravitating matter to fluctuations in luminous matter (which is an additional complication). For a recent discussion of the galaxy power spectrum see Ref. [637], from which earlier developments may be traced. It is a remarkable success of the current cosmological model that it succeeds in providing a reasonable fit to both sets of data. The combination of WMAP data with clustering measurements from SDSS or the 2dFGRS reduces several of the parameter uncertainties. For recent examples of such analyses see Refs. [419, 638].

The peak of the galaxy power spectrum reflects the Hubble length at matter-radiation equality and so constrains $\Omega_{\rm M} h$. The overall shape of the spectrum is sensitive to the densities of the different matter components (e.g., neutrinos would cause damping on small scales) and the density of dark energy. The same physics that leads to acoustic peaks in the CMB anisotropy causes oscillations in the galaxy power spectrum — or a single peak in the correlation function. Reference [639] reports a three standard deviation detection of this "baryon acoustic oscillation" peak at $\sim 100h^{-1}$ Mpc in the correlation function of luminous red galaxies (LRG's) measured in the SDSS. The resulting measurement of $\Omega_{\rm M}$ is independent of and consistent with other low redshift measurements and with the high redshift WMAP result. This is remarkable given the widely different redshifts probed (LRG's probe $z = 0.35$) and notable because possible systematics are different. For discussions of the efficacy of future measurements of the baryon acoustic oscillation peak to constrain dark energy see Refs. [640, 641, 642]. For constraints from a joint analysis of these data with supernovae and CMB anisotropy data see Ref. [643].

Reference [644] includes a nice description of how the large-scale galaxy power spectrum provides independent measurement of $\Omega_{\rm M}$ and $\Omega_{\rm B}$, which breaks several parameter degeneracies and thereby decreases uncertainties on $\Omega_{\rm M}, h$ and t_0. A combined WMAP+SDSS analysis reduces uncertainties on the matter density, neutrino density, and tensor-to-scalar ratio by roughly a factor of two. See Ref. [645] for an analysis of the 2dFGRS large-scale structure data in conjunction with CMB measurements.

Measurements of the clustering of Lyman-α forest clouds complement larger-scale constraints, such as those from the CMB and large-scale structure, by probing the power spectrum of fluctuations on smaller scales [520]. Combining observations of 3000 SDSS Lyman-α forest cloud spectra with other data, Ref. [646] constrains possible variation with scale of the spectral index of the primordial power spectrum and finds that Lyman-α cloud clustering may indicate a slightly higher power spectrum normalization, σ_8 (the fractional mass density inhomogeneity smoothed over $8h^{-1}$ Mpc), than do the WMAP data alone, or the WMAP data combined with large-scale structure measurements.

The presence of dark energy or non-zero spatial curvature causes time evolution of gravitational potentials as CMB photons traverse the Universe from their "emission" at $z \sim 10^3$ to today. The resulting net red or blue shifts of photons cause extra CMB anisotropy, known as the Integrated Sachs-Wolfe (ISW) contribution. This contribution has been detected by cross-correlation of CMB anisotropy and large-scale structure data. The resulting constraints on dark energy are consistent with the model discussed above (see Refs. [647, 648] and references cited therein). In principle, measurements of the ISW effect at different redshifts can constrain the dark energy model. Reference [649] discusses recent developments and the potential of future ISW measurements.

571. "Distance Measures in Cosmology," D. W. Hogg, astro-ph/9905116. (I)

572. "21. The Cosmological Parameters," O. Lahav and A. R. Liddle, J. Phys. G **33**, 224-232 (2006). The latest version is available at `http://pdg.lbl.gov/`. (I)

573. "Estimates of the Hubble Constant," J. Huchra, is a list of measurements of H_0 available at `http://cfa-www.harvard.edu/∼huchra/`. (E)

574. "Median Statistics, H_0, and the Accelerating Universe," J. R. Gott, III, M. S. Vogeley, S. Podariu, and B. Ratra, Astrophys. J. **549**, 1-17 (2001). (I)

575. "Non-Gaussian Error Distribution of Hubble Constant Measurements," G. Chen, J. R. Gott, III, and B. Ratra, Publ. Astron. Soc. Pac. **115**, 1269-1279 (2003). (I)

576. "Final Results from the Hubble Space Telescope Key Project to Measure the Hubble Constant," W. L. Freedman, et al., Astrophys. J. **553**, 47-72 (2001). (I, A)

577. "The Hubble Constant: A Summary of the Hubble Space Telescope Program for the Luminosity Calibration of Type Ia Supernovae by Means of Cepheids," A. Sandage, et al., Astrophys. J. **653**, 843-860 (2006). (I, A)

578. "Constraining Cosmological Parameters Based on Relative Galaxy Ages," R. Jimenez and A. Loeb, Astrophys. J. **573**, 37-42 (2002). (A)

579. "Constraints on the Redshift Dependence of the Dark Energy Potential," J. Simon, L. Verde, and R. Jimenez, Phys. Rev. D **71**, 123001 (2005). (A)

580. "Cosmological Constraints from Hubble Parameter versus Redshift Data," L. Samushia and B. Ratra, Astrophys. J. Lett. **650**, L5-L8 (2006). (A)

581. "The Weak Energy Condition and the Expansion History of the Universe," A. A. Sen and R. J. Scherrer, astro-ph/0703416. (A)

582. "Age Estimates of Globular Clusters in the Milky Way: Constraints on Cosmology," L. M. Krauss and B. Chaboyer, Science **299**, 65-69 (2003). (I)

583. “Distances and Ages of NGC 6397, NGC 6752 and 47 Tuc,” R. G. Gratton, et al., Astron. Astrophys. **408**, 529-543 (2003). (A)

584. “The Bottleneck of CNO Burning and the Age of Globular Clusters,” G. Imbriani, et al., Astron. Astrophys. **420**, 625-629 (2004). (A)

585. “Hubble Space Telescope Observations of the White Dwarf Cooling Sequence of M4,” B. M. S. Hansen, et al., Astrophys. J. Suppl. **155**, 551-576 (2004). (A)

586. “Supernova Ia Constraints on a Time-Variable Cosmological “Constant”,” S. Podariu and B. Ratra, Astrophys. J. **532**, 109-117 (2000). (A)

587. “New Constraints from High Redshift Supernovae and Lensing Statistics upon Scalar Field Cosmologies,” I. Waga and J. A. Frieman, Phys. Rev. D **62**, 043521 (2000). (A)

588. “Uncorrelated Measurements of the Cosmic Expansion History and Dark Energy from Supernovae,” Y. Wang and M. Tegmark, Phys. Rev. D **71**, 103513 (2005). (A)

589. “Hubble Space Telescope and Ground-Based Observations of Type Ia Supernovae at Redshift 0.5: Cosmological Implications,” A. Clocchiatti, et al., Astrophys. J. **642**, 1-21 (2006). (I, A)

590. “The Supernova Legacy Survey: Measurement of $\Omega_{\rm M}$, Ω_Λ and w from the First Year Data Set,” P. Astier, et al., Astron. Astrophys. **447**, 31-48 (2006). (I, A)

591. “New Hubble Space Telescope Discoveries of Type Ia Supernovae at $z \geq 1$: Narrowing Constraints on the Early Behavior of Dark Energy,” A. G. Riess, et al., Astrophys. J. **659**, 98-121 (2007). (A)

592. “Comparison of the Legacy and Gold SnIa Dataset Constraints on Dark Energy Models,” S. Nesseris and L. Perivolaropoulos, Phys. Rev. D **72**, 123519 (2005). (A)

593. “The Vanishing Phantom Menace,” H. K. Jassal, J. S. Bagla, and T. Padmanabhan, astro-ph/0601389. (A)

594. “Accelerating Cosmologies Tested by Distance Measures,” V. Barger, Y. Gao, and D. Marfatia, Phys. Lett. B **648**, 127-132 (2007). (A)

595. “Cosmological Model Parameter Determination from Satellite-Acquired Supernova Apparent Magnitude versus Redshift Data,” S. Podariu, P. Nugent, and B. Ratra, Astrophys. J. **553**, 39-46 (2001). (I, A)

596. “SNAP,” S. Perlmutter, et al., at `http://snap.lbl.gov/`. (E, I)

597. “DUNE: The Dark Universe Explorer,” A. Réfrégier, et al., Proc. SPIE **6265**, 62651Y (2006). (A)

598. "Cosmological Constraints from Compact Radio Source Angular Size versus Redshift Data," G. Chen and B. Ratra, Astrophys. J. **582**, 586-589 (2003). (A)

599. "Radio Galaxy Redshift-Angular Size Data Constraints on Dark Energy," S. Podariu, R. A. Daly, M. P. Mory, and B. Ratra, Astrophys. J. **584**, 577-579 (2003). (A)

600. "A Nearly Model-Independent Characterization of Dark Energy Properties as a Function of Redshift," R. A. Daly and S. G. Djorgovski, astro-ph/0609791. (A)

601. "A Possible Test for the Cosmological Constant with Gravitational Lenses," M. Fukugita, T. Futamase, and M. Kasai, Mon. Not. R. Astron. Soc. **246**, 24P-27P (1990). (I, A)

602. "Gravitational Lensing Limits on the Cosmological Constant in a Flat Universe," E. L. Turner, Astrophys. J. Lett. **365**, L43-L46 (1990). (I, A)

603. "Gravitational Lensing Effects in a Time-Variable Cosmological 'Constant' Cosmology," B. Ratra and A. Quillen, Mon. Not. R. Astron. Soc. **259**, 738-742 (1992). (I, A)

604. "Strong Gravitational Lensing," C. S. Kochanek, in Ref. [83], 91-267. (I, A)

605. "Effects of Early Dark Energy on Strong Cluster Lensing," C. Fedeli and M. Bartelmann, Astron. Astrophys. **461**, 49-57 (2007). (A)

606. "Constraints on Cosmological Parameters from the Analysis of the Cosmic Lens All Sky Survey Radio-Selected Gravitational Lens Statistics," K.-H. Chae, et al., Phys. Rev. Lett. **89**, 151301 (2002). (A)

607. "Constraints on Scalar-Field Dark Energy from the Cosmic Lens All-Sky Survey Gravitational Lens Statistics," K.-H. Chae, G. Chen, B. Ratra, and D.-W. Lee, Astrophys. J. Lett. **607**, L71-L74 (2004). (I, A)

608. "Constraints on the Cardassian Expansion from the Cosmic Lens All-Sky Survey Gravitational Lens Statistics," J. S. Alcaniz, A. Dev, and D. Jain, Astrophys. J. **627**, 26-31 (2005). (I, A)

609. "The Mass of the Universe," P. J. E. Peebles, in Ref. [76], 435-465. (I, A)

610. "Cosmological Parameters from the Comparison of the 2MASS Gravity Field with Peculiar Velocity Surveys," R. W. Pike and M. J. Hudson, Astrophys. J. **635**, 11-21 (2005). (A)

611. "The 2dF Galaxy Redshift Survey: Correlation Functions, Peculiar Velocities and the Matter Density of the Universe," E. Hawkins, et al., Mon. Not. R. Astron. Soc. **346**, 78-96 (2003). (A)

612. “The 2dF QSO Redshift Survey–XV. Correlation Analysis of Redshift-Space Distortions,” J. da Ângela, et al., Mon. Not. R. Astron. Soc. **360**, 1040-1054 (2005). (A)

613. “Median Statistics and the Mass Density of the Universe,” G. Chen and B. Ratra, Publ. Astron. Soc. Pac. **115**, 1143-1149 (2003). (I)

614. “Weak Gravitational Lensing by Large-Scale Structure,” A. Réfrégier, Annu. Rev. Astron. Astrophys. **41**, 645-668 (2003). (I)

615. “Weak Gravitational Lensing,” P. Schneider, in Ref. [83], 269-451. (I, A)

616. “Cosmology with Weak Lensing Surveys,” D. Munshi, P. Valageas, L. van Waerbeke, and A. Heavens, astro-ph/0612667 (2006). (A)

617. “Tracking Quintessence by Cosmic Shear: Constraints from VIRMOS-Descart and CFHTLS and Future Prospects,” C. Schimd, et al., Astron. Astrophys. **463**, 405-421 (2007). (A)

618. “GaBoDS: The Garching-Bonn Deep Survey VI. Cosmic Shear Analysis,” M. Hetterscheidt, et al., astro-ph/0606571. (A)

619. “Cosmological Constraints from COMBO-17 using 3D Weak Lensing,” T. D. Kitching, et al., Mon. Not. R. Astron. Soc. **376**, 771-778 (2007). (A)

620. “A Direct Empirical Proof of the Existence of Dark Matter,” D. Clowe, et al., Astrophys. J. Lett. **648**, L109-L113 (2006). (A)

621. “Dark Matter Maps Reveal Cosmic Scaffolding,” R. Massey, et al., Nature **445**, 286-290 (2007). (A)

622. “The Baryon Content of Galaxy Clusters: A Challenge to Cosmological Orthodoxy,” S. D. M. White, J. F. Navarro, A. E. Evrard, and C. S. Frenk, Nature **366**, 429-433 (1993). (I, A)

623. “The Cosmic Baryon Budget,” M. Fukugita, C. J. Hogan, and P. J. E. Peebles, Astrophys. J. **503**, 518-530 (1998). (I, A)

624. “A New Method to Estimate Cosmological Parameters Using the Baryon Fraction of Clusters of Galaxies,” S. Sasaki, Publ. Astron. Soc. Japan **48**, L119-L122 (1996). (A)

625. “Measuring the Universal Deceleration Using Angular Diameter Distances to Clusters of Galaxies,” U.-L. Pen, New Astron. **2**, 309-317 (1997). (A)

626. “Constraints on Dark Energy from Chandra Observations of the Largest Relaxed Galaxy Clusters,” S. W. Allen, et al., Mon. Not. R. Astron. Soc. **353**, 457-467 (2004). (I, A)

627. “Constraints on Scalar-Field Dark Energy from Galaxy Cluster Gas Mass Fraction versus Redshift Data,” G. Chen and B. Ratra, Astrophys. J. Lett. **612**, L1-L4 (2004). (I, A)

628. “Effects of Cooling and Star Formation on the Baryon Fractions in Clusters,” A. V. Kravtsov, D. Nagai, and A. A. Vikhlinin, Astrophys. J. **625**, 588-598 (2005). (A)

629. “Constraints on Holographic Dark Energy from X-Ray Gas Mass Fraction of Galaxy Clusters,” Z. Chang, F.-Q. Wu, and X. Zhang, Phys. Lett. B **633**, 14-18 (2006). (A)

630. “Constraining Dark Energy with X-Ray Galaxy Clusters, Supernovae and the Cosmic Microwave Background,” D. Rapetti, S. W. Allen, and J. Weller, Mon. Not. R. Astron. Soc. **360**, 555-564 (2005). (I, A)

631. “Complementary Constraints on Brane Cosmology,” J. S. Alcaniz and Z.-H. Zhu, Phys. Rev. D **71**, 083513 (2005). (A)

632. “Supernova Ia and Galaxy Cluster Gas Mass Fraction Constraints on Dark Energy,” K. M. Wilson, G. Chen, and B. Ratra, Mod. Phys. Lett. A **21**, 2197-2204 (2006). (A)

633. “Evolution of the Cluster Mass and Correlation Functions in a ΛCDM Cosmology,” J. D. Younger, N. A. Bahcall, and P. Bode, Astrophys. J. **622**, 1-6 (2005). (A)

634. “Modeling Dynamical Dark Energy,” R. Mainini, A. V. Macció, S. A. Bonometto, and A. Klypin, Astrophys. J. **599**, 24-30 (2003). (A)

635. “On the Spherical Collapse Model in Dark Energy Cosmologies,” D. F. Mota and C. van de Bruck, Astron. Astrophys. **421**, 71-81 (2004). (A)

636. “Early Structure Formation in Quintessence Models and its Implications for Cosmic Reionization from First Stars,” U. Maio, et al., Mon. Not. R. Astron. Soc. **373**, 869-878 (2006). (A)

637. “The Shape of the Sloan Digital Sky Survey Data Release 5 Galaxy Power Spectrum,” W. J. Percival, et al., Astrophys. J. **657**, 645-663 (2007). (A)

638. “Impact of Three Years of Data from the Wilkinson Microwave Anisotropy Probe on Cosmological Models with Dynamical Dark Energy,” M. Doran, G. Robbers, and C. Wetterich, Phys. Rev. D **75**, 023003 (2007). (A)

639. “Detection of the Baryon Acoustic Peak in the Large-Scale Correlation Function of SDSS Luminous Red Galaxies,” D. J. Eisenstein, et al., Astrophys. J. **633**, 560-574 (2005). (A)

640. “Dark Energy Constraints from Baryon Acoustic Oscillations,” Y. Wang, Astrophys. J. **647**, 1-7 (2006). (A)

641. “Dark Energy and Curvature from a Future Baryon Acoustic Oscillation Survey Using the Lyα Forest,” P. McDonald and D. J. Eisenstein, astro-ph/0607122. (A)

642. “Baryon Acoustic Oscillations and Dynamical Dark Energy,” M. Doran, S. Stern, and E. Thommes, J. Cosmol. Astropart. Phys. **0704**, 015 (2007). (A)

643. “Robust Dark Energy Constraints from Supernovae, Galaxy Clustering, and 3 yr Wilkinson Microwave Anisotropy Probe Observations,” Y. Wang and P. Mukherjee, Astrophys. J. **650**, 1-6 (2006). (A)

644. “Cosmological Constraints from the SDSS Luminous Red Galaxies,” M. Tegmark, et al., Phys. Rev. D **74**, 123507 (2006). (A)

645. “Cosmological Parameters from CMB Measurements and the Final 2dFGRS Power Spectrum,” A. G. Sánchez, et al., Mon. Not. R. Astron. Soc. **366**, 189-207 (2006). (A)

646. “Cosmological Parameters from Combining the Lyman-α Forest with CMB, Galaxy Clustering and SN Constraints,” U. Seljak, A. Slosar, and P. McDonald, J. Cosmol. Astropart. Phys. **0610**, 014 (2006). (A)

647. “A Detection of the Integrated Sachs-Wolfe Effect,” S. P. Boughn and R. G. Crittenden, New Astron. Rev. **49**, 75-78 (2005). (A)

648. “New Light on Dark Cosmos,” E. Gaztañaga, M. Manera, and T. Multamäki, Mon. Not. R. Astron. Soc. **365**, 171-177 (2006). (A)

649. “Integrated Sachs-Wolfe Effect in the Era of Precision Cosmology,” L. Pogosian, New Astron. Rev. **50**, 932-937 (2006). (A)

C. Cosmic complementarity: combining measurements

The plethora of observational constraints on cosmological parameters has spawned interest in statistical methods for combining these constraints. References [368, 650, 419] discuss statistical methods employed in some of the recent analyses described above. Use of such advanced statistical techniques is important because of the growing number of parameters in current models and possible degeneracies between them in fitting the observational data. Developments may be traced back through Refs. [651, 652, 653, 654, 655, 656].

To describe large-scale features of the Universe (including CMB anisotropy measured by WMAP and some smaller-angular-scale experiments, large-scale structure in the galaxy distribution, and the SNIa luminosity-distance relation) the simplest version of the “power-law-spectrum spatially-flat ΛCDM model” requires fitting no fewer than six parameters [370]: nonrelativistic matter density parameter $\Omega_{\rm M}$, baryon density parameter $\Omega_{\rm B}$, Hubble constant H_0, amplitude of fluctuations σ_8, optical depth to reionization τ, and scalar perturbation index n. This model assumes that the primordial fluctuations are Gaussian random phase and adiabatic. As suggested by its name, this model further assumes that the primordial fluctuation spectrum is a power law (running power-spectral index independent of scale $dn/d\ln k = 0$), the Universe is flat ($\Omega_{\rm K} = 0$), the bulk of the matter density is CDM ($\Omega_{\rm CDM} = \Omega_{\rm M} - \Omega_{\rm B}$) with no contribution from hot dark matter (neutrino density $\Omega_\nu = 0$), and that dark energy in the form of a cosmological constant comprises the balance of the mass-energy density ($\Omega_\Lambda = 1 - \Omega_{\rm M}$). Of course, constraints on this model assume the validity of the CDM structure formation model.

Combinations of observations provide improved parameter constraints, typically by breaking parameter degeneracies. For example, the constraints from WMAP alone are relatively weak for H_0, Ω_Λ and $\Omega_{\rm K}$. Other measurements such as from SNeIa or galaxy clusters are needed to break the degeneracy between $\Omega_{\rm K}$ and Ω_Λ, which lies approximately along $\Omega_{\rm K} \approx -0.3 + 0.4\Omega_\Lambda$. The degeneracy between $\Omega_{\rm M}$ and σ_8 is broken by including weak lensing and cluster measurements. The degeneracy between $\Omega_{\rm M}$ and H_0 can be removed, of course, by including a constraint on H_0. As a result, including H_0 data restricts the geometry to be very close to flat. A caveat regarding this last conclusion is that it assumes that the dark energy density does not evolve.

CDM-model-dependent clustering constraints on baryon density ($\Omega_{\rm B} = (0.0222 \pm 0.0014)h^{-2}$ from WMAP and SDSS data combined at 95 % confidence, [644]) are now better than those from light element abundance data (because of the tension between the ^{4}He and ^{7}Li data and the D data). It is important that the galaxy observations complement the CMB data in such a way as to lessen reliance on the assumptions stated above for the "power-law flat ΛCDM model." If the SDSS LRG $P(k)$ measurement is combined with WMAP data, then several of the prior assumptions used in the WMAP-alone analysis ($\Omega_{\rm K} = 0, \Omega_\nu = 0$, no running of the spectral index n of scalar fluctuations, no inflationary gravity waves, no dark energy temporal evolution) are not important. A major reason for this is the sensitivity of the SDSS LRG $P(k)$ to the baryon acoustic scale, which sets a "standard ruler" at low redshift.

The SNIa observations are a powerful complement to CMB anisotropy measurements because the degeneracy in $\Omega_{\rm M}$ vs. Ω_Λ for SNIa measurements is almost orthogonal to that of the CMB. Without any assumption about the value of the Hubble constant but assuming that the dark energy does not evolve, combining SNIa and CMB anisotropy data clearly favors nearly flat cosmologies. On the other hand, assuming the Universe is spatially flat, combined SNIa and cluster baryon fraction data favors dark energy that does not evolve — a cosmological constant — see Refs. [630, 631, 632].

650. "First-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Parameter Estimation Methodology," L. Verde, et al., Astrophys. J. Suppl. **148**, 195-211 (2003). (A)

651. "Exploring the Properties of Dark Energy Using Type Ia Supernovae and Other

Datasets," U. Alam, V. Sahni, and A. A. Starobinsky, J. Cosmol. Astropart. Phys. **0702**, 011 (2007). (A)

652. "Reconstructing Generalized Ghost Condensate Model with Dynamical Dark Energy Parametrizations and Observational Datasets," J. Zhang, X. Zhang, and H. Liu, astro-ph/0612642. (A)

653. "Probing for Dynamics of Dark Energy and Curvature of Universe with Latest Cosmological Observations," G. Zhao, et al., Phys. Lett. B **648**, 8-13 (2007). (A)

654. "Scrutinizing Exotic Cosmological Models Using ESSENCE Supernova Data Combined with Other Cosmological Probes," T. M. Davis, et al., astro-ph/0701510. (A)

655. "Constraints on Dark Energy from Supernovae, γ-Ray Bursts, Acoustic Oscillations, Nucleosynthesis and Large Scale Structure and the Hubble Constant," E. L. Wright, astro-ph/0701584. (A)

656. "The ΛCDM Model on the Lead — a Bayesian Cosmological Models Comparison," A. Kurek and M. Szydłowski, astro-ph/0702484. (A)

XIII. OPEN QUESTIONS AND MISSING LINKS

We conclude this review by emphasizing that cosmology is by no means "solved." Here we list some outstanding questions, which we do not prioritize, although the first two questions are certainly paramount (What is most of the Universe made of?). It may interest the reader to compare this discussion of outstanding problems in cosmology to those discussed in 1996 (see Ref. [657]). Recent discussions of key questions, with regard to funding for answering such questions, may be found in Refs. [658, 659].

What is "dark energy"?

As discussed in Secs. IX and XII, there is strong evidence that the dominant component of mass-energy is in the form of something like Einstein's cosmological constant. In detail, does the dark energy vary with space or time? Data so far are consistent with a cosmological constant with no spatial or temporal evolution, but the constraints do not strongly exclude other possibilities. This uncertainty is complemented by the relatively weak direct evidence for a spatially-flat universe; as Refs. [660, 644, 661] point out, it is incorrect to assume $\Omega_{\rm K} = 0$ when constraining the dark energy time dependence, because observational evidence for spatial flatness assumes that the dark energy does not evolve.

More precisely, dark energy is often described by the XCDM parameterization, where it is assumed to be a fluid with pressure $p_{\rm X} = \omega_{\rm X}\rho_{\rm X}$, where $\rho_{\rm X}$ is the energy density and $\omega_{\rm X}$ is time-independent and negative but not necessarily -1 as in the ΛCDM model. This is an inaccurate parameterization of dark energy; see Ref. [662]

for a discussion of the scalar field case. In addition, dark energy and dark matter are coupled in some models now under discussion, so this also needs to be accounted for when comparing data and models; see Refs. [663, 664, 665, 666] for recent discussions.

On the astronomy side, the evidence is not iron-clad; for example, inference of the presence of dark energy from CMB anisotropy data relies on the CDM structure formation model and the SNIa redshift-magnitude results require extraordinary nearly "standard candle"-like behavior of the objects. Thus, work remains to be done to measure (or reject) dark energy spatial or temporal variation and to shore up the observational methods already in use.

With tighter observational constraints on "dark energy," one might hope to be guided to a more fundamental model for this construct. At present, dark energy (as well as dark matter) appears to be somewhat disconnected from the rest of physics.

What is dark matter?

Astronomical observations currently constrain most of the gravitating matter to be cold (small primeval free-streaming velocity) and weakly interacting. Direct detection would be more satisfying and this probably falls to laboratory physicists to pursue. The Large Hadron Collider (LHC) may produce evidence for the supersymmetric sector that provides some of the most-discussed current options for the culprit. As mentioned in the previous question, some models allow for coupling between the dark matter and dark energy. On the astronomy side, observations may provide further clues and, perhaps already do: there are suggestions of problems with "pure" CDM from the properties of dwarf galaxies and galactic nuclear density profiles. Better understanding of the complex astrophysics that connect luminous (or, at least, directly detectable) matter to dark matter will improve such constraints.

What are the masses of the neutrinos?

In contrast to various proposed candidates for the more dominant "cold" component of dark matter, we know that neutrinos exist. While there are indications from underground experiments of non-zero neutrino mass [667] and the cosmological tests discussed above yield upper bounds on the sum of masses of all light neutrino species, there has yet to be a detection of the effect of neutrinos on structure formation. A highly model-dependent analysis of Lyman-α forest clustering [646] results in an upper bound of $\sum m_\nu < 0.17$ eV (95 % confidence; the sum is over light neutrino species).

Are constraints on baryon density consistent?

Using the standard theory for nucleosynthesis to constrain the baryon density from observations of light element abundances, measurements of ^{4}He and ^{7}Li imply a higher baryon density than do D measurements, see Secs. VII.C and XII.B and Refs. [204, 206]. Constraints on the baryon density from WMAP CMB anisotropy data are consistent with that from the D abundance measurements. It is possible that more and better data will resolve this discrepancy. On the other hand, this might be an indication of new physics beyond the standard model.

When and how was the baryon excess generated?

Matter is far more common than anti-matter. It is not yet clear how this came to be. One much-discussed option is that grand unification at a relatively high temperature is responsible for the excess. An alternate possibility is that the matter excess was generated at much lower temperature during the electroweak phase transition.

What is the topology of space?

The observational constraints we have reviewed are local; they do not constrain the global topology of space. On the largest observable scales, CMB anisotropy data may be used to constrain models for the topology of space (see, e.g., Ref. [668] and references therein). Current data do not indicate a real need for going beyond the simplest spatially-flat Euclidean space with trivial topology.

What are the initial seeds for structure formation?

The exact nature of the primordial fluctuations is still uncertain. The currently-favored explanation posits an inflationary epoch that precedes the conventional Big Bang era (see Sec. VII). The simplest inflation models produce nearly scale-invariant adiabatic perturbations. A key constraint on inflation models is the slope of the primordial spectrum; WMAP data [370] suggest a deviation from the scale-invariant $n = 1$ value, but this is not yet well measured. At present, the most promising method for observationally probing this early epoch is through detection of (the scale-invariant spectrum of) inflationary gravity waves predicted in a number of inflation models. Detection of these waves or their effects (e.g., the ratio of tensor to scalar fluctuations via CMB anisotropies), would constrain models for inflation; however, non-detection would not rule out inflation.

Another critical area for studying the initial fluctuations regards the possibility of non-Gaussian perturbations or isocurvature (rather than adiabatic) perturbations. The evidence indicates that these are sub-dominant, but that does not exclude a non-vanishing and interesting contribution.

Some models of inflation also predict primordial magnetic field fluctuations. These can have effects in the low-redshift Universe, including on the CMB anisotropy. Observational detection of some of these effects will place interesting constraints on inflation.

Did the early Universe inflate and reheat?

Probably (although we would not be astonished if the answer turned out to be no). And with tighter observational constraints on the fossil fluctuations generated by quantum mechanics during inflation one might hope to be guided to a more fundamental model of inflation. At present, inflation is more of a phenomenological construct; an observationally-consistent, more fundamental model of inflation could guide the development of very high energy physics. This would be a major development. Another pressing need is to have a more precise model for the end of inflation, when the Universe reheats and matter and radiation are generated. It is possible that the matter excess is generated during this reheating transition.

When, how, and what were the first structures formed?

Discovery of evidence for the epoch of reionization, from observations of absorption line systems toward high-redshift quasars and the polarization anisotropy of the CMB, has prompted intense interest, both theoretical and observational, in studying formation of the first objects. See Sec. X.C above.

How do baryons light up galaxies and what is their connection to mass?

Carrying on from the previous question, the details of the process of turning this most familiar component of mass-energy into stars and related parts of galaxies remains poorly understood. Or so it seems when compared with the much easier task of predicting how collisionless dark matter clusters in a Universe dominated by dark matter and dark energy. Important problems include the effects of "feedback" from star formation and active galactic nuclei, cosmic reionization, radiative transfer, and the effect of baryons on halo profiles. High-resolution hydrodynamic simulations are getting better, but even Moore's law won't help much in the very near future (see comment in Ref. [549]). Solving these problems is critical, not only for understanding galaxy formation, but also for using galaxies — the "atoms of cosmology" — as a probe of the properties of dark matter and dark energy.

Clues to the relationship between mass and light and, therefore, strong constraints on models of galaxy formation, include the detailed dependence of galaxy properties on environment. Outstanding puzzles include the observation that, while galaxy morphology and luminosity strongly vary with environment, the properties of early-type (elliptical and S0) galaxies (particularly their colors) are remarkably insensitive to environment [669].

How do galaxies and black holes co-evolve?

It is now clear that nearly every sufficiently massive galaxy harbors a supermassive black hole in its core. The masses of the central supermassive black holes are found to correlate strongly with properties of the host galaxy, including bulge velocity dispersion [670, 671]. Thus, galaxy formation and the formation and feeding of black holes are intimately related (see, e.g., Refs. [672, 673, 674]).

Does the Gaussian, adiabatic CDM structure formation model have a real flaw?

This model works quite well on large scales. However, on small scales it appears to have too much power at low redshift (excessively cuspy halo cores, excessively large galactic central densities, and too many low-mass satellites of massive galaxies). Modifications of the power spectrum to alleviate this excess small-scale power cause too little power at high redshift and thus delay formation of clusters, galaxies, and Lyman-α clouds. Definitive resolution of this issue will require more and better observational data as well as improved theoretical modeling. If the CDM structure formation model is found to be inadequate, this might have significant implications for a number of cosmological tests that assume the validity of this model.

Is the low quadrupole moment of the CMB anisotropy a problem for flat ΛCDM?

The small amplitude of the quadrupole moment observed by COBE persists in the WMAP observations even after many rounds of reanalysis of possible foreground contributions (see Ref. [675] and references therein). Although one cannot, by definition, rule out the possibility that it is simply a statistical fluke (with significance of about 95 % in flat ΛCDM), this anomaly inspires searches for alternative models, including multiply-connected Universes (see above).

Are the largest structures observed now a problem for flat ΛCDM?

The largest superclusters (e.g., the "Sloan Great Wall" [676]) seen in galaxy redshift surveys are not reproduced by simulations of the concordance flat ΛCDM cosmology (Ref. [677]). Perhaps we need larger simulations (see discussion in Ref. [549]) or better understanding of how galaxies trace mass.

Why do we live just now?

Lastly, because we see the Universe from only one place, at only one time, we must wrestle with questions related to whether or not we (or at least our location) is special.

Peebles [678] notes the remarkable coincidences that we observe the Universe when (1) it has just begun making a transition from being dominated by matter to being dominated by dark energy, (2) the Milky Way is just running out of gas for forming stars and planetary systems, and (3) galaxies have just become useful tracers of mass. While anthropic arguments have been put forward to answer the question of why we appear to live at a special time in the history of the Universe, a physically-motivated answer might be more productive and satisfying. Understanding of the details of structure formation, including conversion of baryons to stars (mentioned above), and constraints on possible evolution of the components of mass-energy in the Universe may provide clues.

Progress in cosmology is likely to come from more and higher-quality observational and simulation data as well as from new ideas. A number of ground-based, space-based, and numerical experiments continue to collect data and new near-future particle physics, cosmology, astronomy, and numerical experiments are eagerly anticipated. It is less straightforward to predict when a significant new idea might emerge.

657. **Critical Dialogues in Cosmology**, edited by N. Turok (World Scientific, Singapore, 1997). (I, A)

658. **Astronomy and Astrophysics in the New Millenium**, National Research Council (U.S.) (National Academy, Washington, 2001). (E, I)

659. **Connecting Quarks with the Cosmos: Eleven Science Questions for the New Century**, National Research Council (U.S.) (National Academy, Washington, 2002). (E, I)

660. "A Century of Cosmology," E. L. Wright, astro-ph/0603750. (I)

661. “Observational Constraints on Dark Energy and Cosmic Curvature,” Y. Wang and P. Mukherjee, astro-ph/0703780. (A)

662. “Joining Conditions for Cosmological Perturbations at an Equation of State Transitions,” B. Ratra, Phys. Rev. D **43**, 3802-3812 (1991). (I, A)

663. “Consequences of Dark Matter-Dark Energy Interaction on Cosmological Parameters Derived from Type Ia Supernova Data,” L. Amendola, G. C. Campos, and R. Rosenfeld, Phys. Rev. D **75**, 083506 (2007). (A)

664. “Cosmologies with Dynamical and Coupled Dark Energy vs. CMB Data,” S. A. Bonometto, L. Casarini, L. P. L. Colombo, and R. Mainini, astro-ph/0612672. (A)

665. “Probing the Coupling between Dark Components of the Universe,” Z.-K. Guo, N. Ohta, and S. Tsujikawa, astro-ph/0702015. (A)

666. “$\Lambda\alpha$DM: Observational Constraints on a Simple Unified Dark Matter Cosmological Model,” A. Balbi, M. Bruni, and C. Quercellini, astro-ph/0702423. (A)

667. “First Results from KamLAND: Evidence for Reactor Antineutrino Disappearance,” K. Eguchi, et al., Phys. Rev. Lett. **90**, 021802 (2003). (A)

668. “Extending the WMAP Bound on the Size of the Universe,” J. S. Key, N. J. Cornish, D. N. Spergel, and G. D. Starkman, Phys. Rev. D **75**, 084034 (2007). (A)

669. “Environmental Dependence of Properties of Galaxies in the Sloan Digital Sky Survey,” C. Park, et al., Astrophys. J. **658**, 898-916 (2007). (A)

670. “A Fundamental Relation between Supermassive Black Holes and Their Host Galaxies,” L. Ferrarese, and D. Merritt, Astrophys. J. Lett. **539**, L9-L12 (2000). (A)

671. “A Relationship between Nuclear Black Hole Mass and Galaxy Velocity Dispersion,” K. Gebhardt, et al., Astrophys. J. Lett. **539**, L13-L16 (2000). (A)

672. “Quasars and Galaxy Formation,” J. Silk and M. J. Rees, Astron. Astrophys. **331**, L1-L4 (1998). (A)

673. “A Unified Model for the Evolution of Galaxies and Quasars,” G. Kauffmann and M. Haehnelt, Mon. Not. R. Astron. Soc. **311**, 576-588 (2000). (A)

674. “Self-regulated Black Hole Accretion, the M-σ Relation and the Growth of Bulges in Galaxies,” M. C. Begelman and B. B. Nath, Mon. Not. R. Astron. Soc. **361**, 1387-1392 (2005). (A)

675. "Cleaned 3 Year Wilkinson Microwave Anisotropy Probe Cosmic Microwave Background Map: Magnitude of the Quadrupole and Alignment of Large-Scale Modes," C.-G. Park, C. Park, and J. R. Gott, III, Astrophys. J. **660**, 959-975 (2007). (A)

676. "A Map of the Universe," J. R. Gott, III, et al., Astrophys. J. **624**, 463-484 (2005). (A)

677. "Luminous Superclusters: Remnants from Inflation?," J. Einasto, et al., Astron. Astrophys. **459**, L1-L4 (2006). (A)

678. "Open Problems in Cosmology," P. J. E. Peebles, Nucl. Phys. Proc. Suppl. **138**, 5-9 (2005). (I, A)

ACKNOWLEDGMENTS

We are indebted to L. Page, J. Peebles, and L. Weaver for detailed comments on drafts of this review. We acknowledge the advice and help of T. Bolton, R. Cen, G. Horton-Smith, T. Kahniashvili, D. Lambert, I. Litvinyuk, L. Page, J. Peebles, M. Strauss, R. Sunyaev, and L. Weaver. B. R. acknowledges support of DOE grant DE-FG03-99EP41093. M. S. V. acknowledges support of NASA grant NAG-12243 and NSF grant AST-0507463 and the hospitality of the Department of Astrophysical Sciences at Princeton University during sabbatical leave.