Physics 106a – Midterm Exam – Due Nov 4, 2004

Instructions

Material: All lectures through Oct 21: Lecture notes through Section 2.2 (including the two reference appendices), Hand and Finch Chapters 1 and 2. Review the material ahead of time, consult myself, the TAs, your fellow students, or other texts if there is material you are having trouble with.

Time: 2 hrs. Fixed time, one contiguous block. Do not look at the exam questions ahead of time.

Reference policy: Hand and Finch, official class lecture notes, problem sets and solutions, your own lecture notes or other notes you have taken to help yourself understand the material. No other textbooks, no web searches, no interaction with your fellow students after you have looked at the exam.

Question policy: Obviously, it will be difficult to ask questions if you are on a fixed-time policy. I have solved all the problems fully and the TAs have looked at the questions, so we have done our best to ensure there are no ambiguities. You are in the same boat as your fellow students, so if everyone has trouble with a particular problem, the grades will be curved accordingly.

Due date: Thursday, Nov 4, 5 pm, my office (311 Downs). 5 pm means 5 pm. We will be very strict about extensions, but extensions for health reasons or family emergency will be granted. You must request the extension ahead of time. Unless you have been granted an extension, **no late exams will be accepted!**

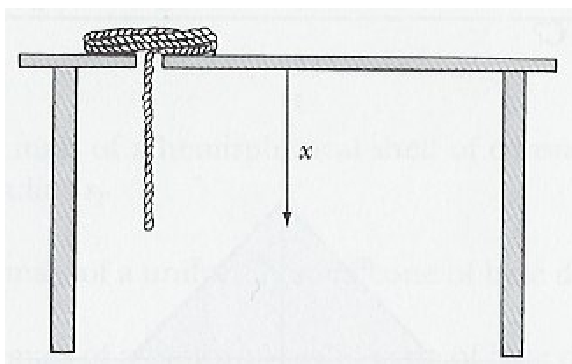
Grading weight: The exam is 30% of the class grade.

Suggestions on taking the exam:

- Go through and figure out roughly how to do each problem first; make sure you've got the physical concept straight before you start writing down formulae.
- Don't fixate on a particular problem. Come back to ones you are having difficulty with.
- Don't get buried in algebra. Get each problem to the point where you think you will get most of the points, then come back and worry about the algebra.

1. (10 points) This problem helps explain the mass distribution in globular clusters, which are nearly spherically symmetric clusters of stars. Suppose a very large number N of point objects are all moving under their mutual gravitational attraction. All objects have equal masses m and equal kinetic energies E and therefore equal speeds v. Each moves in a circular orbit around the common center-of-mass of the system. N is large enough so that the mass density $\rho(r)$ can be considered continuous. Find $\rho(r)$. (Hint: find the force acting on a particle at radius r).

2. (10 points) A smooth rope is placed in a pile above a hole in a table (see figure). One end of the rope falls through the hole at $t = 0$, pulling steadily on the remainder of the rope. (Initial conditions: $x(t = 0) = 0^+$, $\dot{x}(t = 0) = 0$, so an infinitesimal amount of the rope is pushed through the hole at $t = 0$ to start the rope moving.) Find the velocity and acceleration of the rope as a function of the distance to the end of the rope x. Ignore all friction. The total length of the rope is L. Solve by any method you like.



3. (30 points, 10 points each part) Problems in systems of particles.

   a) A toy rocket consists of a plastic bottle partly filled with water containing also air at a high pressure p. The water is ejected through a small nozzle of area A. Calculate the exhaust velocity v by assuming that frictional losses of energy are negligible, so that the kinetic energy of the escaping water is equal to the work done by the gas pressure in pushing it out. Show that the thrust (as defined in the section of the lecture notes on rocket motion) of this rocket engine is $2\,p\,A$. Note: recall that pressure = force / area.

   b) A cloud-chamber picture shows the track of an incident particle which makes a collision and is scattered through an angle ψ_1. The track of the target particle makes an angle ψ_2 with the direction of the incident particle. Assuming the collision was elastic and that the target particle was initially at rest, find the ratio m_1/m_2 of the two masses. Set up the problem but do not do the algebra – *i.e.*, find a set of N equations in N unknowns and stop there. (Hint: don't make this problem harder than it is, you don't need any information beyond what's given.)

   c) A billiard ball sliding on a frictionless table strikes an identical stationary ball. The balls leave the collision at angles $\pm\psi$ with the original direction of motion. Show that after the collision, the total rotational energy of the balls is equal to $1 - \frac{1}{2}\cos^{-2}\psi$ of the initial kinetic energy.

4. (50 points) Consider two particles of masses m_1 and m_2. Let m_1 be confined to move on a circle of radius a in the $z = 0$ plane, centered at $x = y = 0$. Let m_2 be confined to move on

a circle of radius b in the $z = c$ plane, centered at $x = y = 0$. A massless spring of spring constant k is attached between the two particles.

a) (15 points) Determine a set of unconstrained generalized coordinates, write down the Lagrangian in that system, and use the Euler-Lagrange equation to find the equations of motion in the generalized coordinates.

b) (15 points) Repeat, but use cylindrical coordinates (ρ, ϕ, z) and incorporate the constraints via Lagrange multipliers. Find the equations of motion, apply the constraints, and find the Lagrange multipliers (in terms of coordinates and velocities – no accelerations). Calculate the constraint forces from the Lagrange multipliers (again, in terms of coordinates and velocities). Show the equations of motion for the unconstrained coordinates derived in this manner are equivalent to those derived in part (a). Do not worry about the overall sign on the relation between the Lagrange multipliers and the constraint forces; neither Hand and Finch nor the lecture notes have been particular clear on the sign.

c) (15 points) Finally, find the equations of motion using elementary mechanics and Newton's second law; *i.e.*, the techniques of Section 1.1 of the lecture notes. Show again that the equations of motion for the unconstrained coordinates are equivalent to those found above. Find the constraint forces explicitly (in terms of coordinates and velocities) and show they are the same as those found in part (b), up to an overall sign.

d) (5 points) Using the potential energy of the system, determine if there are any equilibria and what kind (stable, unstable, saddle point).

Note: you may make use of the symmetries of the problem to reduce the amount of explicit work you do. Be sure to explain what you are doing, though.