Physics 106a – Problem Set 3 – Due Oct 21, 2004

These problems cover the material on analytical mechanics in Hand and Finch Chapter 1 and Section 2.1 of the lecture notes. Pay special note to how, in many of the problems, use of generalized coordinates lets you ignore constraint forces or lets you work with non-inertial coordinates easily. Also, please write down roughly how much time you are spending on each problem; this will help me determine whether I am assigning the right amount of homework. If it makes you feel any better, I already cut this problem set down from 7 to 5 problems by popular demand.

1. Hand and Finch, Problem 1.6. Hints:
   - "invariant" means you get the same equation back after changing the Lagrangian
   - Note especially that F is a function of the q's and t but not of the $\dot{q}$'s
   - Be careful with the total time derivatives; review some of the steps in the derivation of dot cancellation and the generalized equation of motion as a guide.
2. Hand and Finch, Problem 1.15, but skip the paragraph that starts with "Prove that, if the virtual work vanishes ..."
3. Hand and Finch, Problem 1.18. You should assume $\dot{\theta} = 0$; *i.e.*, we are interested in determining the equilibrium θ as a function of ω, we do not want to determine the dynamics of θ. Also, you may stop once you have found an equation relating θ and ω; *i.e.*, you should not try to invert the equation to explicitly give $\theta(\omega)$ – it is doable, but not worth the time.
4. Hand and Finch, Problem 1.21
5. Hand and Finch, Problem 1.5. In addition to what is asked for in the problem, also write down the Lagrangian and obtain the Euler-Lagrange equations for the system (but do not try to solve!). A couple notes:
   - Recall that for a spring with equilibrium length l_0, the force exerted when the length is changed to $l = l_0 + x$ is $F = -k\,(l - l_0) = -kx$, acting opposite to the displacement x. Recall also that the potential energy is $U = \frac{1}{2}\,k\,x^2$.
   - Note that it says to use the principle of virtual work to find the generalized forces. That is, rather than simply plugging into $\mathcal{F}_k = \sum_i \vec{F}_i^{(nc)} \cdot \frac{\partial \vec{r}_i}{\partial q_k}$, you should construct $\delta\vec{r}_i$ in terms of virtual displacements of the generalized coordinates, calculate $\delta W = \sum_i \vec{F}_i^{(nc)} \cdot \delta\vec{r}_i$, and recognize that the coefficients of the different generalized coordinate variations δq_k are the generalized forces. If it is not clear how to calculate the virtual work, review Hand and Finch from Equations 1.14 through Equation 1.17.