

# Physics 106a – Problem Set 4 – Due Oct 28, 2004

Version 2

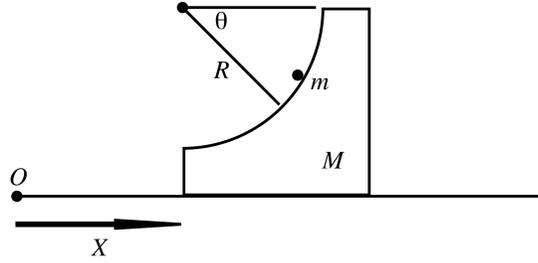
October 26, 2004

These problems cover the material on analytical mechanics in Hand and Finch Chapters 1 and 2 and Section 2.1 and 2.2 of the lecture notes – Lagrangian mechanics with generalized coordinates, variational calculus and variation dynamics with constraints applied via Lagrange multipliers. Please again write down the rough amount of time you are spending on each problem.

When we say “solve using unconstrained generalized coordinates”, that means you should find the set of generalized coordinates such that the constraints no longer appear (*e.g.*, for a point on a sphere, this would be spherical coordinates with motion allowed only in  $\theta$  and  $\phi$ ). In other cases, you will be asked to use cartesian coordinates and incorporate constraints via Lagrange multipliers.

Clarifications since v. 1:

- Explicitly indicate that calculus of variations should be used to do problem 1.
  - Clarify what is meant by “uncoupling” in Problem 3.
1. Use the calculus of variations to show that the geodesic (shortest path length between two points) on the cylindrical surface of a right circular cylinder is a helix (in cylindrical coordinates, a helix has the equation  $\phi = \alpha z$  where  $\alpha$  is a constant that sets the pitch angle of the helix). Do this problem in two ways:
    - By working in cylindrical coordinates (where the constraint that the particle radius  $\rho$  is constant is trivially incorporated).
    - By working in cartesian coordinates and using Lagrange multipliers.
  2. Hand and Finch, Problem 2.12 (b). Of course, don’t do the “compare with part a” part of the question.
  3. A particle of mass  $m$  slides without friction down a smooth circular wedge of mass  $M$  as shown in the figure. The wedge rests on a smooth horizontal table and may move without friction. Find the equations of motion for  $m$  and  $M$  using the unconstrained generalized coordinates  $X$  and  $\theta$  as indicated in the figure and by using cartesian coordinates  $X$ ,  $x$ , and  $y$  (relative to the origin  $O$ ) and employing a Lagrange multiplier. Convert the equations of motion obtained via the Lagrange multiplier technique in  $X$ ,  $x$ , and  $y$  to equations in  $X$  and  $\theta$  and show they match the equations obtained directly using unconstrained generalized coordinates. Rewrite the equations of motion in uncoupled form (*i.e.*, one equation for  $\ddot{X}$  that does not depend on  $\dot{\theta}$  and one equation for  $\ddot{\theta}$  that does not depend on  $\ddot{X}$  – but  $\ddot{X}$  may still depend on  $\theta$  and  $\dot{\theta}$  and  $\ddot{\theta}$  may still depend on  $X$  and  $\dot{X}$ ). Find the Lagrange multiplier constraint force (getting rid of any dependence on  $\ddot{X}$  and/or  $\ddot{\theta}$ ) and explain the physical significance of the terms.



4. Consider the bead on a rotating hoop shown in Figure 1.11 of Hand and Finch (problem 1.23). First, find the equations of motion using unconstrained generalized coordinates (use spherical coordinates). Second, find the equations of motion in cartesian coordinates using Lagrange multipliers instead. So, for the first part of the problem, you will begin by assuming  $R$  is fixed and  $\phi = \Omega t$ , leaving only  $\theta$  as a free coordinate. In the second part, you will assume  $x$ ,  $y$ , and  $z$  are all free, but you will include via Lagrange multipliers the constraints that the bead is forced to stay on the hoop and that the hoop rotates with angular velocity  $\Omega$ . (Note that the latter constraint contains time dependence and so is rheonomic). Once you have the equations of motion in the latter case, assume the system is in equilibrium ( $\ddot{z} = 0$ ,  $z = z_0$  a constant,  $x(t)$  and  $y(t)$  in terms of  $z_0$ ,  $R$ ,  $\Omega$  and  $t$ ), find the values of the Lagrange multipliers, and determine the constraint forces along each direction from the Lagrange multipliers using Hand and Finch equation 2.43.