Physics 106b/196b – Midterm Exam – Due Feb 9, 2007

Instructions

Material: All lectures from start of term through Jan 25. Sections 5 and 6 of lecture notes excluding Sections 5.1.5 (Lie Groups) and 6.1.3 (Mathematical description of Lorentz transformations). Only Ph196 students are responsible for Sections 5.1.5 and 6.1.3. *There will be no material on classical field theory on the exam.* Review the material ahead of time, consult me, the TAs, your fellow students, or other texts if there is material you are having trouble with.

Logistics: The exam consists of this page plus 2 pages of exam questions. Do not look at the exam until you are ready to start it. Please use a blue book if possible (makes grading easier), but there will be no penalty if you don't have one. Problems 1 and 2 are for 106 students only, 3, 4, and 5 for 106 and 196 students, and 6 and 7 for 196 students only.

Time: 4 hrs, fixed time. You may take as many breaks as you like, but they may add up to no more than 30 minutes (2 x 15 minutes, 3 x 10 minutes, etc.).

Reference policy: Hand and Finch, official class lecture notes and errata, problem sets and solutions, your own lecture notes or other notes you have taken to help yourself understand the material. No other textbooks, no web searches, no interaction with your fellow students. Calculators and symbolic manipulation programs are neither needed or allowed.

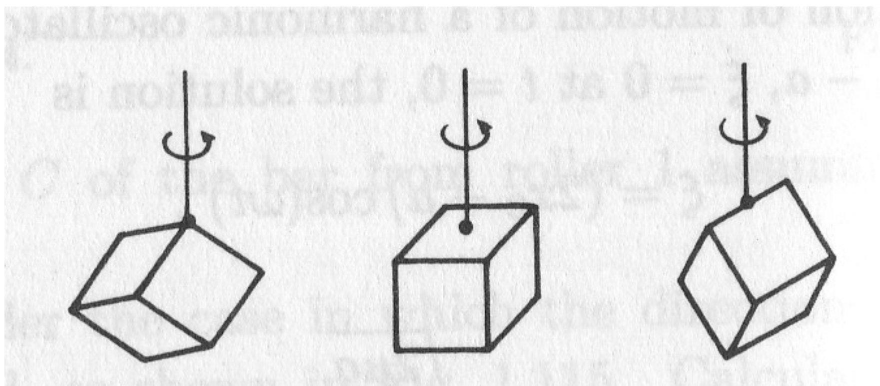
Due date: Friday, Feb 9, 5 pm, my office (311 Downs). 5 pm means 5 pm. Late exams will require extenuating circumstances; otherwise, no credit will be given.

Grading: Each problem is 20 points out of 100. The exam is 1/3 of the class grade.

Suggestions on taking the exam:

- Go through and figure out roughly how to do each problem first; make sure you've got the physical concept straight before you start writing down formulae.
- Don't fixate on a particular problem. They are not all of equal difficulty. Come back to ones you are having difficulty with.
- Don't get buried in algebra (this really should not be an issue on this exam). Get each problem to the point where you think you will get most of the points, then come back and worry about the algebra.

1. (106) A torsion pendulum consists of a vertical wire attached to a mass that may rotate about the vertical. Consider three torsion pendulums that consist of identical wires from which identical homogeneous solid cubes are hung. One cube is hung from a corner, one from midway along an edge, and one from the middle of a face, as shown in the figure. What are the ratios of the periods of the three pendula?



2. (106) A pendulum is located at latitude λ. The bob of the pendulum is started so as to follow a circular path at constant, small deflection angle θ from the vertical. By using the equation for the apparent force in the Earth's frame, show that the vertical component of the Coriolis forces averages to zero and is also negligible in magnitude, and that the Coriolis force gives a contribution to the angular velocity of approximately

$$\Omega = -\omega_E \sin \lambda$$

where ω_E is the Earth's angular velocity .

3. (106/196) A monochromatic light signal with frequency ν propagates in a direction that makes an angle of 60° with the $+x$-axis in the reference frame F of its source. In a frame $\widetilde{F}$ whose axes are parallel to those of F, the source is moving at speed 4/5 ($c = 1$) along the $+x$-axis toward the origin of the $\widetilde{F}$ frame. What are the frequency and apparent direction of the wave as measured in the $\widetilde{F}$ frame?

4. (106/196) Suppose the axis of a symmetrical top ends at a contact point that slides without friction on a horizontal table. Find the Lagrangian and Lagrange equations of motion. What are the differences between the Lagrangian and equations of motion between this case and that of the fixed top?

5. (106/196) In the lecture notes, we considered a "sleeping" top, which is a symmetrical top with one point fixed and initial condition $\theta = \dot{\theta} = 0$. We showed that, if the angular velocity of the top exceeds the critical value $\omega_0 = (2/I_3)\sqrt{M\,g\,l\,I_{1d}}$, then the vertical position is stable. If the sleeping top receives a small horizontal kick, what is the frequency of small oscillations of the symmetry axis of the top about the vertical? Hint: be careful how you do the inevitable Taylor expansion, you can get lost in trigonometric functions. Simplify as much as possible before Taylor-expanding.

6. (196) The theory of rocket motion developed last term must be adjusted for relativity. The key is that, while mass may no longer be conserved – *i.e.*, one may convert some of the mass of the rocket to energy – it is certainly true that four-momentum is conserved. Show that,

assuming a constant mass loss rate $\alpha = -dm/dt$, the equation of motion for the rocket in its starting inertial frame $\widetilde{F}$ is

$$m \frac{d\widetilde{v}}{dm} + a\left(1 - \widetilde{v}^2\right) = 0$$

where $\widetilde{v}$ is the speed of the rocket as measured in $\widetilde{F}$, a is the speed of the exhaust gases relative to the rocket, and we take $c = 1$. (Yes, the signs are correct because $dm < 0$.) Hints:

- Consider the problem from an instantaneous rest frame of the rocket and find a differential equation there, then transform the differential equation into the inertial starting frame of the rocket.
- Assume that mass is converted to energy at a rate κ, use it in the equations of conservation of four-momentum, and then eliminate κ in the end.

7. (196) Consider a thin homogeneous plate with principal moments of inertia, I_1, $I_2 > I_1$, and $I_3 = I_1 + I_2$. Let F denote the rotating system aligned with the principal axes and F' the inertial system. The two systems have their origins at the plate's center of mass. At $t = 0$, the plate is set rotating in a force-free manner with an angular velocity Ω about an axis inclined at an angle α from the plane of the plate and perpendicular to the x_2 axis. If $I_1/I_2 = \cos 2\alpha$, show that at time t the angular velocity about the x_2 axis is

$$\omega_2(t) = \Omega \cos\alpha \tanh\left(\Omega\, t \sin\alpha\right)$$

Find the time dependence of ω_1 and ω_3 also. To do this problem, you will need the integral

$$\int \frac{du}{1 - u^2} = \tanh^{-1} u$$

and, in case you don't remember them, note also the following useful trigonometric identities:

$$\cos^2 u + \sin^2 u = 1$$
$$\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$$
$$\cosh^2 u - \sinh^2 u = 1$$

Hint: Euler's equations are coupled differential equations. Find a way to decouple them by obtaining relations among the components of $\vec{\omega}$.