Physics 106b/196b – Problem Set 10 – Due Jan 26, 2007

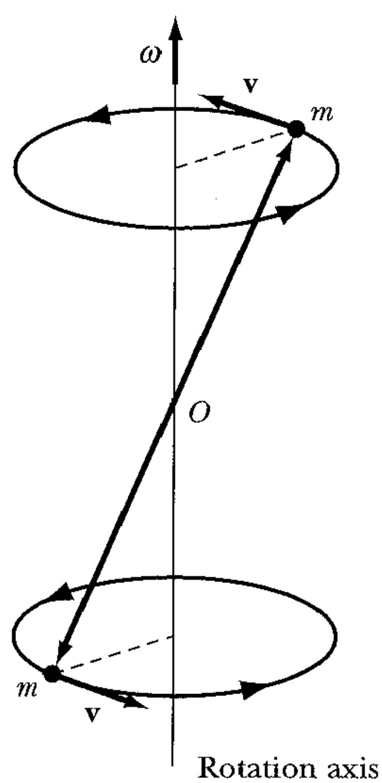
This problem set focuses on dynamics of rigid bodies (Section 5.3 of the lecture notes). Problems 1 and 2 are for 106b students only, 3 and 4 for 106b and 196b students, and 5 and 6 are for 196b students only. Because the material is so nonintuitive, I suggest you start thinking about Problems 2, 4, and 6 early in the week.

1. (106b) Find the inertia tensor, principal axes, and principal moments for an ellipsoid of semiaxes a, b, c and total mass M. Recall that an ellipsoid satisfies the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

   The coordinate system implied by the above definition is obviously the easiest one in which to do the problem. You will need a table of integrals or Mathematica to do the integrals.

2. (106b) A bar of negligible weight and length l has equal mass points m at the two ends. The bar is made to rotate uniformly about an axis passing through the center of the bar and making an angle θ with the bar. (See the figure.) Using Euler's equations with torque, find the components along the principal axes of the bar of the torque forcing the bar to execute this motion. Find the components of the torque in the space frame also (by rotation or from first principles).



3. (106b/196b) Hand and Finch 8-25. You will have to read Hand and Finch Section 8-11. You don't have to do the numerical integration in part (d), but you should derive the relation that, if numerically integrated, would give you the result.

4. (106b/196b) When a coin is dropped on a horizontal table, it usually settles down only after a wobbling motion. In this motion, the center of the coin remains approximately at rest (except for a slow settling due to loss of energy), and the edge of the coin rolls on the surface, while the axis normal to the coin precesses rapidly about the vertical. Suppose that a coin of mass M, radius R, and negligible thickness executes this motion. Assume that the edge of the coin rolls without slipping on the table, that energy is conserved, and that the angle between the normal to the coin and the vertical is θ. The problem is to calculate the angular speed ω_p of the precession in terms of M, R, θ, and g (where g is the acceleration due to gravity). You will find it useful to know that the principal moments of a disk are $\frac{1}{2}\,M\,R^2$ and $\frac{1}{4}\,M\,R^2$.

   (a) The angular velocity of the coin is the sum of two components:
   $$\vec{\omega} = \omega_p\,\hat{z}\,' + \Omega\,\hat{z}$$
   where $\hat{z}\,'$ is the normal to the plane (the obvious inertial coordinate system) and $\hat{z}$ is the normal to the coin (the body's 3-axis). Thus, the coin rotates about its axis of symmetry with angular speed Ω and precesses about the vertical with angular speed ω_p. From the condition that the coin rolls without slipping, show that $\Omega = -\omega_p\,\cos\theta$.

   (b) It is convenient to analyze the motion of the coin in a reference frame (not the same as the body-fixed frame) whose z-axis aligns with the body 3-axis but does not spin with the body. This system rotates about the vertical with angular speed ω_p. Let the $y'z'$ and $y_P\,z_P$ planes coincide at $t = 0$. (This frame is like the F_P frame used in the study of the torque-free top in the lecture notes.) In this (rotating) frame, the coin's 3-axis does not precess, and the angular velocity of the coin is a constant vector. Show that the constant angular velocity vector is $\vec{\omega} = -\hat{y}_P\,\omega_p\,\sin\theta$ where $\hat{y}_P$ is the y-axis of the frame F_P.

   (c) In the rotating reference frame F_P, use the Euler equations with torque to determine the precession speed ω_p in terms of M, R, θ, and g. To do this, you will have to think about how $\vec{L}$ must behave in the F_P frame.

5. (196b) Two parts:

   (a) Suppose one has a Lagrangian L in n generalized coordinates q_k, $k = 1$ to n and that the Lagrangian is cyclic in q_n. Since L is cyclic in q_n, the conjugate momentum $p_n = \frac{\partial L}{\partial \dot{q}_n}$ is constant. By using a Legendre transformation, we create a new function, the Routhian,
   $$R\,(q_1, \ldots, q_{n-1}, \dot{q}_1, \ldots, \dot{q}_{n-1}) \equiv L - p_n\,\dot{q}_n$$
   Why, according to the rules of Legendre transformations, is R no longer dependent on $\dot{q}_n$? Show that the Routhian obeys the Euler-Lagrange equations in the remaining $n-1$ degrees of freedom.

   (b) As an alternative approach to finding the Lagrangian for a heavy symmetric top, use this Routhian technique to eliminate the ϕ and ψ degrees of freedom. Show that this leads to the same equations of motion as our original derivation.

6. (196b) Hand and Finch 8-20.