

# Physics 125b – Problem Set 13 – Due Feb 26, 2008

Version 1 – Feb 21, 2008

This problem set focuses on addition of angular momentum and the definition and application of spherical tensor states and operators; Shankar Chapter 15 and Lecture Notes 15. It is clear that the last two weeks have been very rough going for much of the class. Rotations and angular momentum are, in my experience, the toughest part of quantum mechanics because one has to deal with so many foreign, nonintuitive concepts in quick succession; it is especially tough if your understanding of rotations and tensors from classical mechanics is not strong. This set is intended to give you a bit of a break so you can go back and really learn the material. Please do not waste this opportunity.

Many basic problems in QM can be found in textbooks – there are only so many solvable elementary problems out there. Please refrain from using solutions from other textbooks. Obviously, you will learn more and develop better intuition for QM by solving the problems yourself. We are happy to provide hints to get you through the tricky parts of a problem, but you *must* learn to set up and solve these problems from scratch by yourself.

1. Email to the instructor one question about any of the material since Lecture 31. **Send your email by 9 pm Feb 25 so that answers can be prepared for class on Feb 27. Print out your email and include it with your problem set, including the date information; questions sent after the above deadline will be given only half credit.**
2. Make a diagram that summarizes the logical structure of all the material from Lecture 31 onward (Lecture Notes 13, 14, 15). This should be no more than one page, and should be a diagram that uses arrows to show how various concepts and topics are connected, not just an outline of the lecture notes. Some items that likely should be included (but feel free to include more): Classical Rotation Transformations; Quantum Rotation Transformations; Eigenvalues and Eigenvectors of  $L_z$  and  $L^2$ ; Spherical Harmonics; Raising and Lowering Operators;  $|j, m\rangle$  Basis; Rotational Invariance; Radial Equation; Addition of Angular Momentum; Direct Product Spaces; Direct Sum Spaces; Clebsch-Gordan Coefficients; Classical Cartesian Tensors; Classical Spherical Tensors; QM Cartesian Tensor States; QM Spherical Tensor States; QM Cartesian Tensor Operators; QM Spherical Tensor Operators; Wigner-Eckart Theorem; Accidental Degeneracies.

This exercise will give you a chance to go back over the past 5 weeks of material and try to see the big picture; there are not “wrong” answers, but there are “insufficient” answers.

3. Consider addition of three  $j = 1$  systems:  $1 \otimes 1 \otimes 1$ . First, write the space down as a direct sum of spaces of specific  $j$ . Second, what is the analogy for this in terms of Cartesian tensor states and operators? That is, what kind of tensor state and operator is the left side of the equation equivalent (in an unrigorous sense) to, and similarly for the right side? This is easy if you have understood Lectures 40-42; if not, you will have to go back and learn the material!
4. Shankar 15.3.2
5. Shankar 15.3.3. Without a doubt, you can find this in other textbooks; but you will learn much more if you do it using only Shankar and the lecture notes.