Universal Extra Dimensions

- Add compact dimension(s) of radius R
  - ~ ant crawling on tube
- Kaluza-Klein tower of partners to SM particles due to curled-up extra dimensions of radius R
  - n = quantum number for extra dimension
  - $m_n^2 \sim n^2/R^2$
  - momentum conservation in extra dimension → exact conservation of KK particles (KK parity)
  - KK parity $P_{KK} = (-1)^n$ implies lightest KK partner ($n = 1$) is stable
- Wide variety of more complicated theories, hopefully will discuss later in term

Why SUSY?

- Supersymmetry is the maximal extension of the Poincare group
  - (proper) Poincare group = all combinations of translations (generated by linear mom.), rotations (generated by ang. mom.), and boosts (generated by K operators)
  - Define two new operators (Q, Q-bar) that are the "square-root" of linear mom. with algebra as given

$$\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\sigma^m_{\alpha\dot\beta} P_m$$

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0$$

$$[P_m, Q_\alpha] = [P_m, \bar{Q}_{\dot\alpha}] = 0$$

$$[P_m, P_n] = 0,$$

where the Greek indices denote the two-component Weyl spinors (the dotted indices running over the conjugate components), the Latin indices denote the Lorentz four-vectors, P is the momentum operator, Q is the supersymmetry generator, and $\sigma^m_{\alpha\dot\beta}$ are the Pauli matrices.

Mandic thesis, Sec 2.2.1

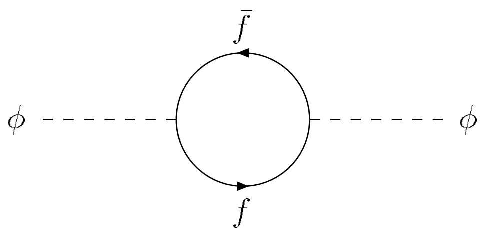
  - Note that Q and Q-bar are fermionic: they anticommute with themselves (Grassman variables)
  - This is, apparently, the maximal extension of the Poincare group (Haag et al, Nucl Phys B 88: 257 (1975)): very special!

Why SUSY?

- Group structure
  - P and Q define a Lie algebra (a sub-algebra of the full SUSY Lie algebra)
  - The corresponding Lie group consists of elements $G(x,\theta,\bar{\theta}) = e^{i(-x^m P_m + \theta Q + \bar{\theta}\bar{Q})}$
- Given the Lie algebra/group, what are the representations?
  - Infinitesimal transformations generated by Q ops mix bosonic (commuting) and fermionic (anticommuting) fields
    → All fields must be parts of supermultiplets that incorporate bosonic and fermionic pieces
  - End up with
    - chiral superfields: (scalar, spin-1/2 fermion, auxiliary vector field). The spin-1/2 fermion is our SM quarks and leptons. The scalars are the "s" partners: squarks and sleptons. The auxiliary vector field is removed by the E-L eqns and has no physical manifestation.
    - vector superfields: (auxiliary scalar field, spin-1/2 fermion, spin-1 boson). The spin-1 boson is our SM gauge bosons. The spin-1/2 fermions are the "ino" partners: gauginos and higgsinos. The auxiliary scalar field is again removed by the E-L eqns and has no physical manifestation.

Why SUSY?

- It turns out that supersymmetry solves the hierarchy problem
  - The standard model has a single scalar, the Higgs boson
  - Radiative corrections to scalar mass is quadratic in the loop cutoff scale:

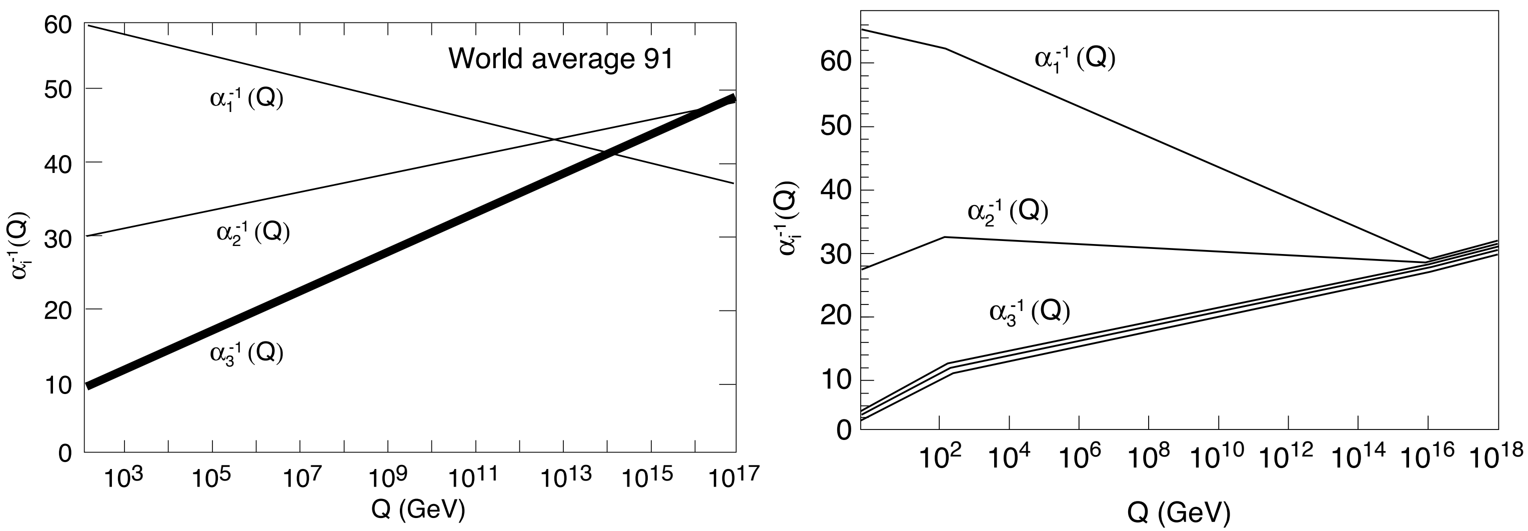


$$\begin{aligned}\pi_\phi^2(p=0) &\sim \int^\Lambda d^4k\, i\lambda_e \frac{i(k+m)}{k^2-m^2} i\lambda_e \frac{i(k+m)}{k^2-m^2} \\ &\sim -\lambda_e^2 \int^\Lambda d^4k \frac{k^2+m^2}{(k^2-m^2)^2} \\ &\sim -\lambda_e^2 \left[\frac{\Lambda^2}{m^2} + 2m^2 \log\frac{\Lambda}{m}\right]\end{aligned}$$

  - Radiative corrections to fermion and gauge boson masses are only logarithmic in cutoff scale
  - Only log divergences can be renormalized away with a single correction; quadratic divergences require renormalization at every order. This is undesirable.
  - SUSY introduces opposite spin superpartners; they give companion loop corrections, but with opposite sign: cutoff scale goes from ? down to SUSY-breaking scale (~TeV)
  - Light Higgs mass from top-quark and LEP electroweak measurements implies SUSY breaking scale can't be much higher than 1 TeV

Why SUSY?

- It turns out that SUSY improves the prospects for gauge coupling unification
  - Extra particles change the running of the couplings. From Bertone, Hooper, Silk (2004) review article:



SUSY Particle Spectrum

- SM fermions → SM fermions + SUSY sfermions
  - left-handed quarks and leptons in EW SU(2) isospin doublets
  - right-handed quarks and leptons in EW SU(2) isospin singlets
  - each particle gets a scalar partner (one each for left-handed and right - handed components)
- SM gauge bosons → SM gauge bosons + SUSY gauginos
  - Hypercharge B and EW SU(2) $W \rightarrow W^{+/-}$, weak Z^0, photon
  - gluons (8)
  - pick up 1 bino, 3 winos, and 8 gluinos

SUSY Particle Spectrum

- Higgs → Higgs doublet + higgsinos
  - single EW SU(2) doublet complex Higgs required in SM
    - 4 dof, 3 are Goldstone bosons that get eaten by W to give $W^{+/-}$ and Z^0 mass
    - remaining single real scalar Higgs h
  - SUSY requires two EW SU(2) doublet complex Higgs
    - One gives up-quark masses, the other gives down-quark masses
    - 8 dof, get same 3 Goldstone bosons to give W and Z mass
    - 5 dof: light and heavy scalars h and H, pseudoscalar A, and charged scalars H^+ and H^-
  - 8 dof → 2 neutral higgsinos, 2 charged higgsinos
    - charged dof counting: 2 of 3 Goldstone bosons are charged (to $W^{+/-}$), so 2 charged dof from Goldstone bosons, 2 charged dof matching $H^{+/-}$ → 4 charged dof
    - neutral dof counting: 1 of 3 Goldstone bosons is neutral (to Z^0), 3 scalars → 4 neutral dof
  - Two Higgs vevs, $\tan\beta = v_1/v_2$

Soft SUSY Breaking

- Unbroken SUSY:
  - No freedom in the Lagrangian -- superpartner fields must appear in Lagrangian in same forms as partner fields, with same coefficients (up to arithmetic factors)
  - Masses of superpartners are same as masses of partners
- So, must introduce SUSY-breaking terms
  - Soft: breaks SUSY without ruining quadratic divergence cancellation
  - Allowed terms that don't violate *R-parity* (Girardello and Grisaru):
    - sfermion mass/mixing matrices M_Q, M_L, M_U, M_D, M_E
    - gaugino mass terms M_1, M_2, M_3
    - Higgs mass terms m_1, m_2
    - higgsino mass term $B\mu$
    - trilinear couplings A_E, A_D, A_U
    - Still: 115 free parameters!

$$\begin{aligned}
V_{soft} \;=\; & \epsilon_{ij}(\widetilde{e}^*_R A_E h_E \widetilde{l}^i_L H^j_1 + \widetilde{d}^*_R A_D h_D \widetilde{q}^i_L H^j_1 \\
& -\widetilde{u}^*_R A_U h_U \widetilde{q}^i_L H^j_2 - B\mu H^i_1 H^j_2 + h.c.) \\
& +\widetilde{q}^{i*}_L M^2_Q \widetilde{q}^i_L + \widetilde{l}^{i*}_L M^2_L \widetilde{l}^i_L + \widetilde{u}^*_R M^2_U \widetilde{u}_R + \widetilde{d}^*_R M^2_D \widetilde{d}_R + \widetilde{e}^*_R M^2_E \widetilde{e}_R \\
& +H^{i*}_1 m^2_1 H^i_1 + H^{i*}_2 m^2_2 H^i_2 \\
& +\tfrac{1}{2}(M_1 \widetilde{B}\widetilde{B} + M_2 \widetilde{W}^a \widetilde{W}^a + M_3 \widetilde{g}^b \widetilde{g}^b),
\end{aligned}$$

  - R-parity: "Superpartner number": all SUSY-conserving and SUSY-breaking terms conserve superpartner #

Radiative Electroweak Symmetry Breaking

- How does SUSY result in electroweak symmetry breaking?
  - (that is, how does Higgs potential get set up so Higgs acquired vev?)
  - Higgs potential for EWSB arises from summation of higher-order diagrams, no additional parameters for Higgs potential. Provides a relation between Higgs parameters:

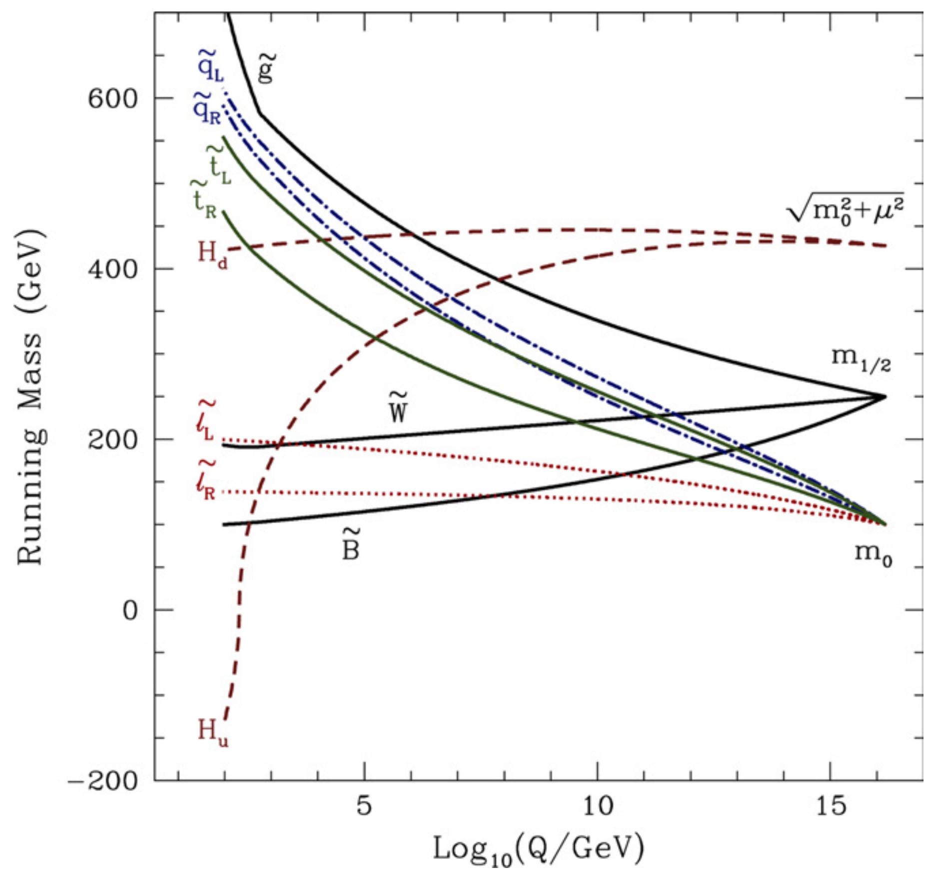
$$|\mu|^2 = \frac{m_{H_\mathrm{d}}^2 - m_{H_\mathrm{u}}^2 \tan^2\beta}{\tan^2\beta - 1} - \frac{1}{2} m_Z^2 \approx -m_{H_\mathrm{u}}^2 - \frac{1}{2} m_Z^2.$$

last line follows for all but the lowest values of $\tan\beta$

m_{Hu} is what we have called m_1 in the discussion of soft SUSY-breaking terms. We get a relation between m_1^2 and $|\mu|$ when we assume REWSB; only sign(μ) remains free.

Running of Masses in SUSY

- Masses evolve as one runs down from M_{GUT} to M_{EW}.
- Gauge couplings increase these parameters in going to M_{EW}.
- Yukawa couplings decrease them at the same time.
- Yukawa couplings don't care about gauge couplings, so the particles with biggest gauge couplings will be heaviest at M_{EW}: *squarks are heavier than sleptons, gluinos are heavier than binos and winos*
- But for Higgs, need to in fact get m_1^2 to be negative to get EWSB (need mexican hat potential). Large top Yukawa coupling can do this.



ino Mixing

- Neutral gauginos and higgsinos have same quantum numbers and mix to form 4 neutralinos
  - bino, wino, up and down higgsinos
- Charged gauginos and higgsinos also mix to give 4 charginos (2 of each sign)
  - +wino and +higgsino
  - -wino and -higgsino

Frameworks

- mSUGRA
  - minimal supergravity: convert global SUSY to local SUSY, creating gauge transformation and gen'l relativity. Broken at some scale below the Planck scale. But GUT scale is so much lower than Planck scale that no reason to expect differences between gauge groups and generations.
  - Universal fermion Yukawa-coupling unification (SM terms)
  - Universal gaugino mass $m_{1/2}$
  - Universal scalar mass m_0 (for sfermions and for Higgs m_1, m_2) and trilinear coupling A_0, mass and trilinear coupling matrices all equal to the identity matrix.
  - $m_{3/2} = m_0$: gravitino mass = scalar mass
  - $B = A_0 + m_0$ connects higgsino mass to scalar mass and trilinear coupling
  - Higgs sector dof: only three of m_1, m_2, tan β, B, μ are independent. Sometimes use a different parameter m_A derived from B, μ, and $\int$.
  - Assumption of a GUT gauge group that breaks to SU(3) x SU(2) x U(1)
  - Radiative EWSB provides additional Higgs sector relation, so 5 Higgs params (m_1, m_2, tan β, B, μ) are reduced to 2 (e.g., one mass and tan β)

Frameworks

- LEP and LHC mSUGRA: usual framework for LEP/LHC calcs
  - Don't require unification of SM fermion Yukawa couplings because quark masses don't really allow it
  - Don't check color/charge neutrality of the EW vacuum
- Generic SUGRA, also known as NUHM (non-universal Higgs mass)
  - Assume Higgs mass unification $m_1 = m_2$, but no connection between Higgs and sfermion masses. LEP Higgs mass constraints don't constrain the sfermion masses. (Presumably, also release $m_{3/2} = m_0$ constraint)
  - Don't assume radiative EWSB → three, not two, free Higgs parameters
  - Requirement that EW vacuum be reasonable (colorless, chargeless, stable) at all energies up to GUT scale places constraint on non-universality parameter sign(m_i^2) | m_i^2 / m_0^2 |
- Constrained MSSM (aka LEEST for Ellis)
  - Many variants on this. Ellis LEEST is mSUGRA without scalar mass universality assumption.

Frameworks

- Anomaly-mediated SUSY breaking:
  - Idea is that SUSY is broken not by some gravity-mediated mechanism (which would be universal and thus give the parameter universality of mSUGRA), but rather by a different mechanism that is insensitive to v. high energy behavior.
  - Predicts specific gaugino mass ratios: M_1: M_2: M_3 = 1:2.8:7.1
  - Also predicts v. heavy gluinos and squarks, so squark-mediated interactions are suppressed.
- Gauge-mediated SUSY breaking
  - SUSY breaking mediated by gauge interactions rather than gravity, so different gaugino masses naturally separate.
  - Also gives condition that the gravitino mass $m_{3/2}$ is much smaller than the SUSY-breaking scale
  - So the LSP becomes the gravitino, but will overclose unless T_{reheat} is kept low. Gravitinos could also be produced by decay of NLSP.

Collider Constraints

- LEP chargino searches, M > 100 GeV
  - can't see pair-produced neutralinos at LEP: no detectable energy
  - assumes gaugino mass unification to get constraint on neutralino mass
- sneutrino limits
  - search for charged sleptons
  - use $SU(2)_{EW}$ symmetry of broken SUSY to relate to sneutrino
  - M > 85 GeV
- Gluino and squark searches
  - colored particle searches at Tevatron
  - M > 200 GeV
- Flavor-changing neutral currents
  - There is no FCNC in Standard Model (e.g., u Z c terms linking charm and up quark via Z)
  - Non-diagonal sfermion mass matrices could yield FCNC
  - mSUGRA assumes they are diagonal to suppress FCNC

Accelerator Constraints

- BR($b \rightarrow s\gamma$)
  - Can proceed via light charged Higgs or charginos

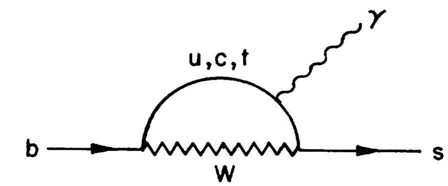


FIG. 1. Penguin diagram for $b \rightarrow s\gamma$. The photon may be radiated from any of the four lines.

- $B_s \rightarrow \mu^+\mu^-$
  - Excess above SM scales as $(\tan\beta)^6$, so very constraining
- $(g-2)_\mu$
- Precision electroweak