What conditions must be placed on the neutrino mixing matrix in order to have the same amount of ν_e in ν_3 as ν_μ in ν_τ?

This question is the same as saying

$$\begin{aligned} \langle \nu_e | \nu_3 \rangle &= \langle \nu_\tau | \nu_1 \rangle \\ U_{e3} &= U_{\tau 1}. \end{aligned}$$

Looking to our U matrix multiplied out,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}s_{13} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

So $U_{e3} = U_{\tau 1}$ would imply that the upper right and lower left components are equal.

$$s_{13}e^{-i\delta} = s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta}$$

The mathematical requirement that gives the matrix this paramaterization is the requirement that it is Unitary. Any additional symmetry requirements are not physical.

Check the formula for $\tan\theta_m$.

The equation is indeed

$$\tan 2\theta_m \simeq \frac{\sin 2\theta}{\cos 2\theta - L/L_0}$$

The confusion came in taking the limit of $L \gg L_0$. It is true that This is the high N_e case so the matter interaction length scales are short meaning oscillations do not have a chance to develop. This should show up as $\theta_m \to 0$, or no oscillations.

$$L \gg L_0 \Rightarrow L/L_0 \gg 1 \geq \cos 2\theta$$

$$an2\theta_m \simeq \frac{-L_0}{L} \sin 2\theta \to 0$$

$$tan2\theta_m \to 0 \Rightarrow \theta_m \to 0$$

as expected from our physical understanding of the length scales.

Why do you need θ_{13} in order to determine the hierarchy from the matter effects?

The plot I showed is based on an Earth-bound long-baseline neutrino experiment. Here, you start with a ν_μ beam. Matter effects become visible primarily with ν_e interactions, so in this case in $\nu_\mu \to \nu_e$. Using the same basic math we did before but with more complicated matrices, this probability works out to be:

$$P(\nu_\mu \to \nu_e) = \sin^2\theta_{23}\sin^2 2\theta_{13}\sin^2(1.27\Delta m^2_{31}L/E).$$

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When matter effects are introduced, the equation becomes,

$$
\begin{aligned}
P_{\mu e}^{m} &= \sin^2\theta_{23}^{m}\sin^2 2\theta_{13}^{m}\sin^2(1.27\Delta_{31}^{m}L/E) \\
\Delta_{31}^{m} &= \sqrt{\left(\Delta m_{31}^2\cos 2\theta_{13} - A\right)^2 + \left(\Delta m_{31}^2\sin 2\theta_{13}\right)^2} \\
\sin 2\theta_{13}^{m} &= \sin 2\theta_{13}\frac{\Delta m_{31}^2}{\Delta_{31}^{m}} \\
A &= 2\sqrt{2}G_f N_e E
\end{aligned}
$$

First, you can see that this survival probability is heavily dependent on $\theta_1 3$. Also, notice in the Δ_{31}^{m} definition that we get the Δm_{31}^2 mass difference minus a term of definite sign. This means the overall value of this term will be different depending on the sign of Δm_{31}^2, resolving the ambiguity.

What happens to lepton number for Majorana neutrinos?

"...$\mathcal{L}$ mixes ν and $\bar{\nu}$. Thus, a Majorana mass term does not conserve L. There is then *no conserved Lepton number* to distinguish a neutrino mass eigenstate ν_i from its antiparticle."

-B. Kayser, *PDG 13. Neutrino Mass, Mixing, and Flavor Change*

Why is $\mu_\nu^d \simeq 3 \times 10^{-19} \mu_B \left(\frac{m_\nu}{1 \text{ eV}}\right)$ so small?

I have not been able to find any information on why the neutrino magnetic moment is necessarily so small.