Ay 101 - The Physics of Stars – fall 2015 - J. Cohen

Homework 2, due Friday Oct 16 at class (2 pm)

1. (5 points) If the mass fractions in a star are X (hydrogen) 70%, Y (helium) 29%, and Z (everything heavier than He) 1%, then what are the corresponding fractions by number ?

Assume here and for the next problem that the nuclei of the "heavy" elements have equal numbers of protons and neutrons, hence an atomic mass of $2Z$.

2. (10 points) Show that for a fully ionized gas,

$$1/\mu = 2X + 3Y/4 + Z/2,$$

where μ is the mean atomic weight per particle, X is the fraction by mass of H, Y is the fraction by mass of He, and Z is the fraction by mass of all other elements. Show also that the mean atomic weight per electron is $1/\mu_e = (1 + X)/2$.

3. (10 points) **Energetics of a pure H gas**

a) A gas of pure hydrogen has $n_H = 10^{17}$ cm^{-3} and $T = 5000$ K. The volume of the gas is 1 cubic meter. What fraction of the H atoms in the gas are neutral ?

b) Keeping the volume fixed, the gas is heated by a an external source of energy to reach a temperature T =20,000 K. What fraction of the H atoms are neutral now ?

c) How much energy had to be added to ionize the gas ? How much energy had to be added to heat the gas ?

4. (3 points) **Constant Density Star** (LeBlanc problem 2.2)
Compute $P(r)$ inside a sphere of radius $R_\star$ with constant density ρ.

5. (4 points) Give two examples (phenomena or objects) in astrophysics of where the virial theorem does **not** hold. What assumptions fail in those cases? Write a paragraph discussing each case at a scientific/technical level.

6. (7 points) **Initial Mass Function exercise**

Consider a cluster of stars that follow a Salpeter IMF $dN = a\ M^{-2.35}\ dM$, and range in mass from 0.1 $M_\odot$ to 150 $M_\odot$.

a. (2 points) If all stars with $8\,M_\odot \leq M < 25\,M_\odot$ eventually produce neutron stars and all stars with $M \geq 25\,M_\odot$ eventually produce black holes, what is the the ratio of the number of black holes to the number of neutron stars formed in the cluster?

b. (2 points) On the main sequence, a star's luminosity is related to its mass by

$$\frac{L}{L_\odot} \approx (\frac{M}{M_\odot})^3.$$

Given a total cluster mass of 10^5 $M_\odot$ and assuming that all of the stars are on the main sequence, what is the luminosity of the cluster?

c. (3 points) Find the mass M_L such that stars with $M < M_L$ emit half of the cluster's total luminosity. That is, stars with masses between 0.1 $M_\odot$ and M_L produce the same total luminosity as stars with masses between M_L and 150 $M_\odot$. What fraction of stars have masses **larger** than M_L ?