Ay 101 - The Physics of Stars – fall 2015 - J. Cohen

Homework 6, due Friday Nov 13 by 5 pm

1. (4 points) **Temperature Dependence of Earth's Atmosphere**

The Earth's atmosphere can be modelled as an isentropic fluid, for which

$$P^{1-\Gamma}T^{\Gamma} = const.$$

where Γ is the adiabatic index. In the following, assume that air is pure nitrogen (not quite right, but close) and the ideal gas equation of state ($\Gamma = 5/3$). You can use the density of air at sea level as a reference density.

(3 points) Combine above equation with hydrostatic equilibrium (at constant gravitational acceleration g) to find an expression for the temperature lapse rate, dT/dz.

(1 point) If the temperature on the Caltech campus is 75 F, what is the temperature at the top of Mount Wilson?

2. (20 pts) **Convection in the Sun.**

We are going to try to find the convective zones in the Sun.

We adopt the standard solar model of John Bahcall and collaborators from 2005, given in the file bahcall_solar_model.txt (which I extracted from the his web site, www.sns.ias.edu/(tilde)jnb/SNdata/solarmodels.html.)

You may find you need the Rosseland mean opacity as a function of depth. For $T > 10^6$ K, adopt κ_R from the paper "Opacities for the Solar Radiative Interior" by C. A. Iglesias and F. J. Rogers (Astrophysical Journal, 1991, volume 371, page 408). (This is the file called 1991ApJ...371..408I.pdf). Use the values in Table 5, column 2 (the first column gives T in units of 10^6 K). For cooler T use the values in the Kurucz model of the Solar atmosphere, file kurucz_solar_model.txt.

We adopt the mixing length approximation for convection. Assume that for any surface convection, the mixing length l is the pressure scale height, while for any core convection, the mixing length is $R^*/10$.

The bins in this model are very small. You should begin by rebining this model so that your adopted model has approximately 50 bins. Please submit a text file with your rebinned model, and with a plot of $L(r)$ and of $T(r)$ as a function of $M(r)$.

a) (10 pts) Calculate $\nabla(rad)$, $\nabla(ad)$ and $\nabla(\text{actual})$ for each depth in the model for the Sun.

b) (5 pts) What criterion is used to determine if a a zone is convectively unstable ? What radial zone(s) in the Sun are convectively unstable ?

c) (5 points) The Bahcall model gives at each depth the fraction by mass of H, ^{4}He, and ^{3}He. Ignore ^{3}He as it is a very small fraction throughout the star.

Plot the fraction of H versus radius. Then calculate for each bin the fraction of H that has been burned into He by assuming that the normal fraction of H and of ^{4}He is that of the outermost bin and make a plot of that with radius. Finally calculate the fraction of the total mass of the Sun that has been burned from H into ^{4}He.

3. (16 points) **Relative Strength of Absorption Lines in the Solar Spectrum**

The files needed to do this problem are being sent out by email.

We are going to predict the relative depth of absorption lines in the Solar spectrum and compare them with the ratios in the observed Solar spectrum. Appended is a small part of the Solar spectrum, covering wavelengths from approximately 5846 to 5881 Å. The element and ionization stage giving rise to most of the lines within this wavelength range are indicated on the plot. This plot is from the digital library of the National Solar Observatory (www.nso.edu).

Assume that the lines are formed in gas with $T = 5600$ K, $P = 7.2 \times 10^4$ dyn/cm^2, $n_e = 1.4 \times 10^{13}/cm^3$, $n_p = 3.3 \times 10^{12}/cm^3$, and $n_H = 8.3 \times 10^{16}/cm^3$ (i.e. neutral hydrogen).

a) (3 pts) Choose your lines:

Choose 10 of the weak lines (those where the flux at the center of the line is at least 70% of the continuum). Try to choose lines that come from as many different elements as possible. First measure the depth of absorption at the line center (which should be from a few to 30% of the continuum) of each of your selected lines. Normalize the measured central line depth for each line to that of the strongest (i.e. deepest) line you selected.

b) (5 points) Collect the necessary atomic data

First determine the approximate line center wavelength from the plot. Then find the exact central wavelength, the excitation potential, of the lower level, and the transition probability gf by using the NIST Atomic Spectra Database (www.nist.gov/pml/data/asd.cfm).

Enter into the query form the approximate line center you estimate from the plot, recall that you know the element giving rise to the line, and try to find from the database the specific transition which gives rise to the observed feature. Once you have the specific transition and its exact wavelength you can extract all the required atomic parameters for this line from the database output. Note that you need to change the tab "Transition Strength" from the default (A_{ki} (a lifetime) to "log(gf)".

Assemble any additional data (example – the relative abundance of the element that gives rise to the transition) as necessary. Make (and hand in) a table listing all the parameters and their values you have assembled for your 10 selected lines.

c) (5 points) The Line Absorption Coefficient

We are now going to find the ratio of the line absorption coefficient for each of your lines to that of the strongest line you have chosen. Write down the line absorption coefficient α_ν and calculate it's value at the central wavelength for each of your 10 lines. Assume all lines have the same profile function ϕ_ν and ignore all multiplicative constants. Derive the ratio of α for each line at the line center to that of the strongest line in your sample.

Hand in the formulae you are using and the value for your 10 lines, as well as the ratio of these to the value for the strongest line in your sample.

d) (3 points) How Good Are Your Predictions ?

Your final result should be shown as a plot with wavelength as the X axis and the line ratios as the Y axis (from 0.0 to 1.0).

Each of your 10 lines should be shown as a solid vertical line representing your by eye measurement of the line strength ratio to the strongest line, shown at the wavelength of the line center. Slightly offset in wavelength so it can be distinguished from the solid line, a dashed line should represent the predicted line strength ratio from the data you collected from NIST and other sources.

How well do your predicted line strength ratios agree with those actually observed in the Solar spectrum ? Comments ?