Ay 101 - The Physics of Stars – fall 2015 - J. Cohen

Homework 7, due Friday Nov 20 by 5 pm

1. (4 points) **Powering the Sun via Chemical Reactions** (LeBlanc problem 6.1)

Assuming that 10 eV of energy per atom found in the Sun is emitted during some chemical reaction (ionization for example) taking place there, calculate the time the Sun could shine at its present intensity if the only energy source was this chemical process. Assume that the Sun is composed of pure hydrogen. Is it then possible that the energy source of the Sun is chemical in nature ? Why or why not ?

2. (2 points) **Main Sequence Lifetimes**

Look up typical masses and luminosities for B, G, and M main sequence stars and estimate their main sequence lifetimes under the assumption that 10% of their hydrogen can be burned on the main sequence.

3. (6 points) **Nuclear Energy Generation** LeBlanc problems 6.4 to 6.6

Calculate the energy released in:

(2 points) Triple-α process,

(2 points) $^{1}\mathrm{H} + {}^{2}\mathrm{H} \rightarrow {}^{3}\mathrm{He} + \gamma\,,$

(2 points) $^{12}\mathrm{C} + {}^{4}\mathrm{He} \rightarrow {}^{16}\mathrm{O} + \gamma\,.$

4. (3 points) **Solar Helium Burning** LeBlanc problem 6-7

Estimate the time the Sun will spend on the horizontal branch supposing that helium burns via the triple-α reaction. Note that at this phase of evolution the luminosity of the Sun will be approximately equal to $100 L_{\odot}$. What is the ratio of the approximate lifetime of the Sun on the HB to the time it is on the main sequence found in problem 2 ?

5. (15 pts) **Nuclear Reaction Cross Sections**

We have shown that the cross section for a nuclear reaction is determined by the quantum mechanical tunneling, and is a function of v (or equivalently of energy E), both evaluated in the center of mass frame of the reacting particles A and B. Then it must be averaged over the Maxwellian velocity distribution.

$$< \sigma v > = 4\pi \, [\frac{m}{2\pi kT}]^{3/2} \int_0^\infty v \, \frac{S(E)}{E} \, exp[-\frac{mv^2}{2kT}] \, exp[-\frac{-2\pi Z_A Z_B e^2}{\hbar v}] \, v^2 dv$$

Let us write $\sigma \propto exp(-b/\sqrt{E})$, where

$$b = \frac{\sqrt{2}\pi Z_A Z_B e^2 \sqrt{m_{AB}}}{\hbar} = 0.99 \, Z_A Z_B \sqrt{m_{AB}} \, (MeV)^{1/2}.$$

5a. (10 pts) To make it possible to do the integral (at least approximately), we decide to replace this complex integral with the integral of a Gaussian. The Gaussian is centered at energy E_0, with a width Δ and an amplitude C, $g(E) = C \, exp[(E-E_0)^2/(\Delta/2)^2]$. Find the appropriate parameters of this Gaussian, C, E_0 and Δ, and do the integral. (Hint: to get C and E_0 you should match the central peak of the Gaussian in both position and height to the properties of the complex integral. Match some other appropriate property to find Δ.)

5b. (5 pts) Then define τ and derive the expression

$$< \sigma v > \propto \frac{1}{Z_A Z_B m} S_0 \, \tau^2 \, e^{-\tau}$$

6. (5 pts) **A Simpler But Approximate Expression of Nuclear Energy rates**

We often desire to have the nuclear reaction energy generation rate expressed as a power law, $\epsilon = \epsilon_0 \rho^n T^\eta$. Consider the reaction $p + p$. Write an expression for ϵ as a function of $< \sigma v >$, ρ, etc. Use that to determine the value of n, i.e. the appropriate power of ρ. Now use the last expression given in problem 5 above to find η in terms of τ (and constants).