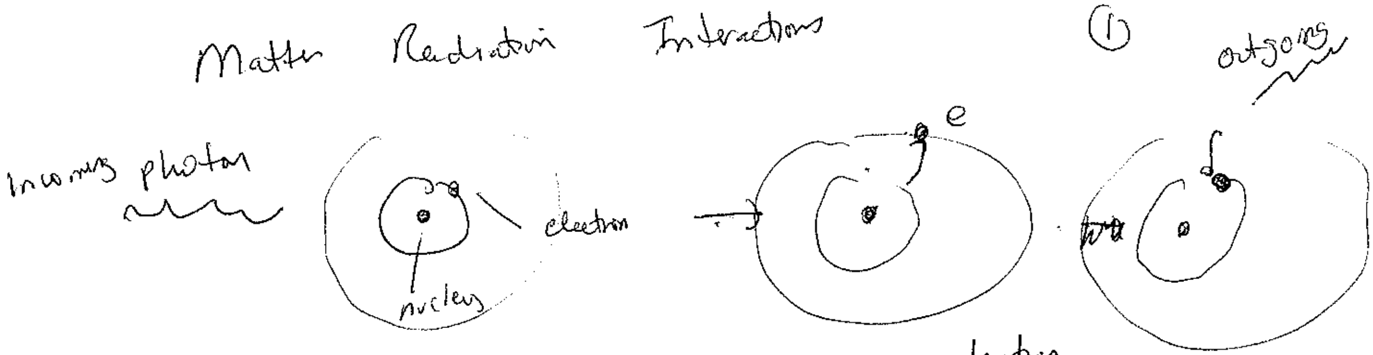
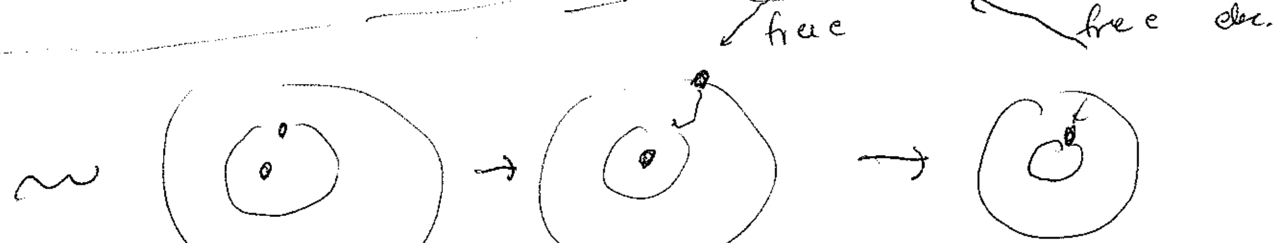
Matter Radiation Interactions (1)



excited electron

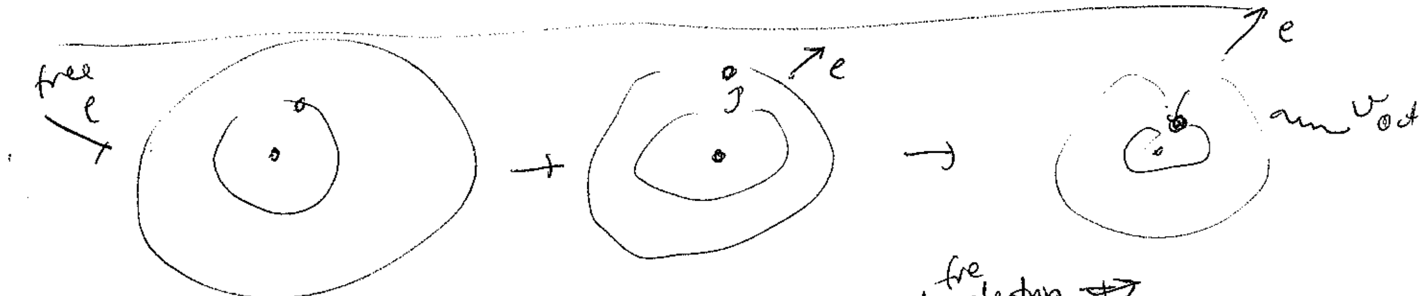
Scattering - out photon energy same as in photon energy



excite electron

de excite via collision with a free electron.

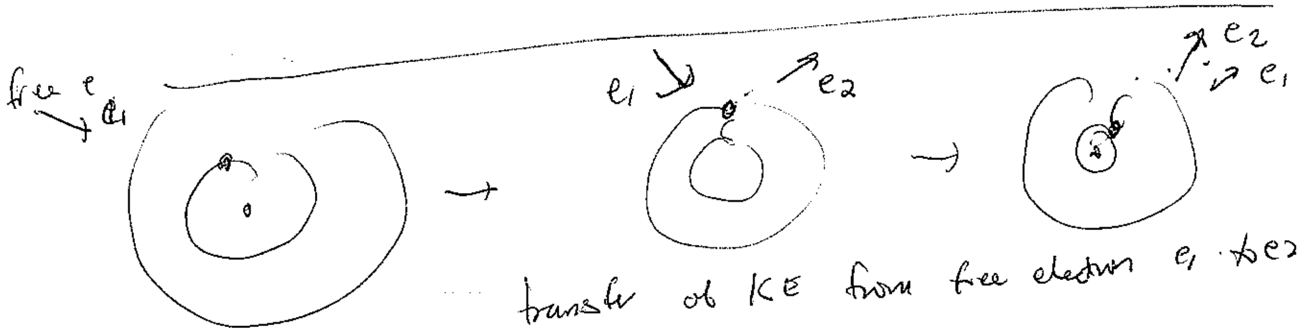
note $|\nu_f|$ of free electron $> |\nu_i|$



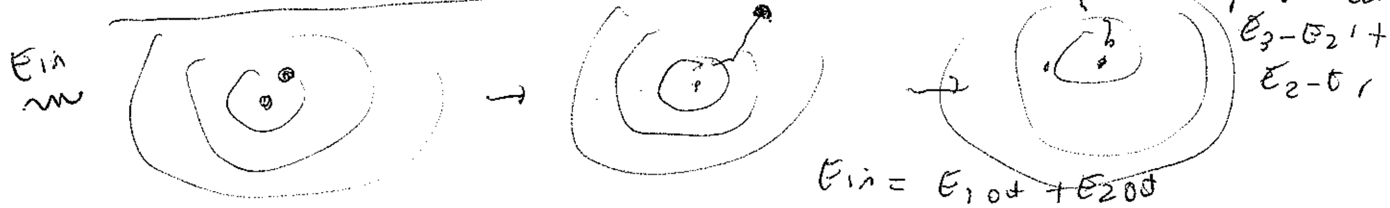
$V_e(i) > V_e(f)$

kinetic energy of free electron → bound electron; excitation energy of ~~at~~ bound electron

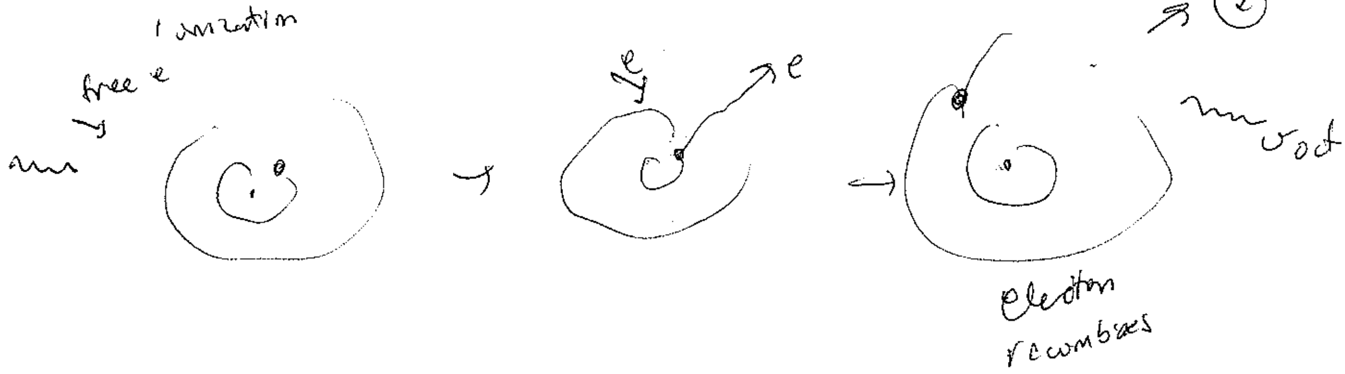
~~lose~~ falls to ground state → photon



transfer of KE from free electron e_1 to e_2



$E_{in} = E_{1\,out} + E_{2\,out}$



Note that mean KE (thermal equilibrium) per particle

$= \frac{3}{2} kT$

$KE = \frac{1}{2} m v^2 \qquad \frac{3}{2} kT = \frac{1}{2} m (\bar{v})^2$

$\bar{v} \approx 3kT/m$

So $v_e \gg v_{atoms}$, so

free electrons dominate excitations + photoionizations via collisions

#/unit time electron collides with atom $\gg$ atom collides with atom

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Absorption vs Scattering

$h\nu_{in} = h\nu_{out}$ scattering, just change direction

abs. $h\nu$ in $\rightarrow$ thermal energy ↑ of gas $\xrightarrow{\text{may yield}}$ $h\nu$ (out)

but $h\nu(out) \neq h\nu(in)$

B–B transitions
photon excites electron, fixed ν

B-F photon ionizes electron $h\nu \geq E_{ion}$

F-F photon excites a free electron on an unbound orbit around atom A to a higher energy orbit.

$$\frac{1}{2} m_e v_f^2 = h\nu + \frac{1}{2} m v_i^2$$

A free electron cannot abs both the energy & the momentum of a photon — need the atom there to absorb the extra momentum $E = h\nu$ $p = \frac{h\nu}{c}$

Can't simultaneously satisfy $\frac{1}{2} m_e v_i^2 + h\nu = \frac{1}{2} m_e v_f^2$ & $m_e v_i + \frac{h\nu}{c} = m_e v_f$

Scattering
by free electrons: Thomson scattering
by atoms or molecules Rayleigh scattering

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radiative Opacity – κ_ν (cm^2/g)

$\kappa_\nu \rho = \chi_\nu$ (units cm^{-1}) $= \frac{1}{\text{mean free path}}$

cm^2/gm gm/cm^3

$\kappa_{\nu,tot} = \kappa_\nu + \sigma_\nu$ — scattering

(~~abs.~~ processes
absorption

Thomson scattering easy, independent of ν
(for low ν typical in stellar astronomy)

$$\sigma_T = n_e \frac{(\pi r_{e0}^2)}{\rho}$$

Rayleigh Scattering

$$\sigma_R \propto \frac{1}{\lambda^4}$$

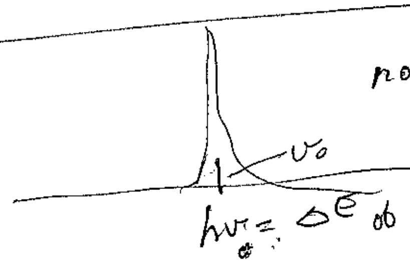
$\lambda_{blue} << \lambda_{red}$ $\qquad \sigma_R(\text{blue}) >> \sigma_R(\text{red})$

ISM absorption bigger in blue than red

Sun redder at sunset (blue all scattered)

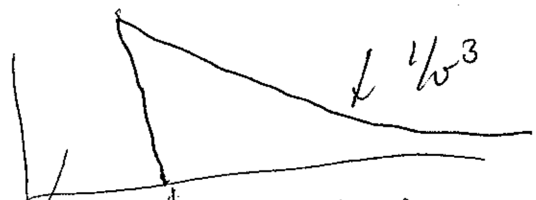
Sky is blue (scattered light, many scatterings, arrive from all over)

B-B



not quite a δ function due to finite velocity range of atoms + other effects

BF photoionization



H^- in cool stars: H (p+e) + e weakly bound

only 1 bound state E 0.754 eV $\Rightarrow \lambda = 16{,}500$ Å

important because peak of BB radiation in cool stars & few other opacity sources there (nothing involving normal H)

free electrons that provide 2nd e for H^- come from ionization of metals with IP << 13.6 eV of HI

Fundamental parameter of radiation field is I_ν, specific intensity. Units are ergs / sec / hz / cm^2 / steradian.

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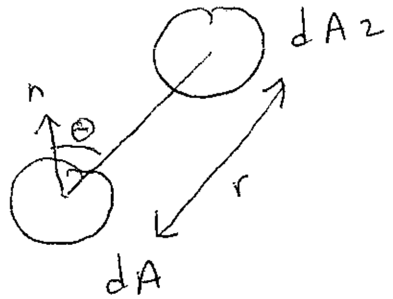


solid angle

$$d\Omega = dA_2 / r^2$$

Energy flow from dA to dA_2

$$[\text{ergs/sec/hz}] \quad dE_\nu = I_\nu(\Omega)\, d\Omega\, d\nu\, dt\, (\hat{n}\cdot\Omega)\, dA$$

$$\hat{n}\cdot\Omega = \cos\theta$$ = projected area normal to ray along separation

$$\int_{\text{over a sphere}} d\Omega = \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, d\theta = 4\pi$$ (spherical coordinates)

Beam travels a distance $dl = c\,dt$ $(v = c)$

Set $\underset{\text{volume}}{dV} = \cos\theta\, dA\, dl = \cos\theta\, dA\, c\,dt$

$$dE_\nu = \frac{I_\nu}{c}\, d\Omega\, d\nu\, dV.$$

Energy density $u_\nu d\nu = \frac{1}{\Delta V}\int_{\Delta V}\int_\Omega dE_\nu$

$$\boxed{u_\nu = \frac{1}{c}\int I_\nu\, d\Omega}$$

$$\frac{I_\nu\, d\Omega\, d\nu\, \cancel{dt}(\cancel{\cos\theta})\, \cancel{dA}}{\cancel{\cos\theta}\, \cancel{dA}\, c\, \cancel{dt}} = \frac{I_\nu\, d\Omega\, d\nu}{c}$$

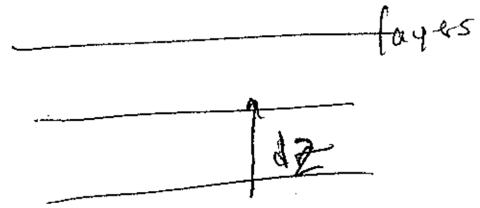
So $dE_\nu = \frac{I_\nu}{c}\, d\Omega\, d\nu\, dV$

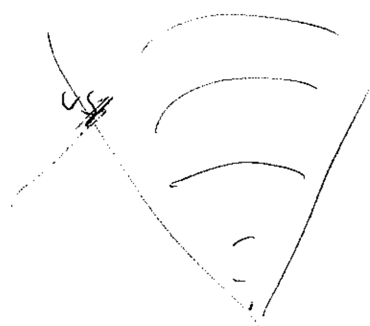
Can assume I_ν independent of time inside a star

ie Δt (nuclear etc) >> Δt (radiation field)

(except SN explosions etc.)

In stellar atmosphere, plane || approx





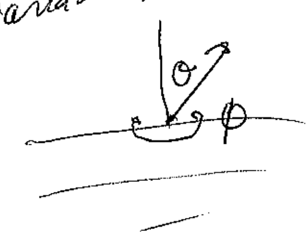
Simplifies equations a lot - to not have to take into account spherical geometry. Ignore curvature!

Just 1 variable, dr

Treat as semi-infinite plane || layers

OK for outer parts of ★

gets rid of all angle variables, independent of ϕ



$I_\nu(z, \theta)$

$u = \cos\theta$

Assume plane || layers

$\mathbb{P}_\nu$ = momentum flux per unit area, units dynes/cm^2/hz — pressure

E (photon) = $h\nu$ $p = h\nu/c$ — momentum

need component of p along beam $\perp$ to area

recall $\iint d\Omega = \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \, d\theta$

$$\mathbb{P}_\nu = \iint \frac{I_\nu \cos^2\theta}{c} \, d\Omega = \frac{2\pi}{c} \int_0^{\pi} I_\nu \cos^2\theta \sin\theta \, d\theta$$

If $I_\nu(\theta, \phi)$ independent of ϕ

If I_ν isotropic (independent of θ + ϕ, direction)

$$\mathbb{P}_\nu = \frac{2\pi}{c} I_\nu \left. \frac{\cos^3\theta}{3} \right|_0^{\pi} = \frac{4}{3} \frac{\pi}{c} I_\nu$$

Radiative flux: (F_ν ∝ Radiation pressure) ergs/sec/hz/cm^2

$$E_\nu = F_\nu \, dA \, dt \, d\nu = \iint dE_\nu \, d\Omega$$

$$dE_\nu^{d\Omega} = I_\nu \, d\Omega \, d\nu \, dt \, dA \cos\theta = F_\nu \, dA \, dt \, d\nu$$

$$F_\nu = \iint I \cos\theta \, d\Omega$$

If I is isotropic,

$$F_\nu = 2\pi I \int_0^{\pi} \cos\theta \sin\theta \, d\theta = 2\pi I \left. \left(\frac{\cos^2\theta}{2} \right) \right|_0^{\pi} = 0$$

forward flux = F^+ integral $0 \le \theta \le 90°$

Backward flux F^- " $90 \le \theta \le 180°$

(Net flux) $F = F^+ - F^-$ If isotropic radiation field, $F^+ = F^-$ and net flux = 0

Often use notation $\mu = \cos\theta$ in radiative transfer equations. (Not mean weight per particle!)

Moments of I_ν

$$J_\nu = \frac{1}{4\pi}\int I \, d\Omega \quad \text{(mean intensity)}$$

$$F_\nu = \int I \mu \, d\Omega$$

$$K_\nu = \frac{1}{4\pi}\int I \mu^2 \, d\Omega$$

related to P_{rad} & to energy density

$$P_\nu = \frac{4\pi}{c} K_\nu \qquad u_\nu = \frac{4\pi}{c} J_\nu$$

Isotropic – I independent of θ, ϕ

$$J_\nu = I_\nu$$

$$F_\nu = 0 \quad \text{(no net flux)}$$

$$F_- = F_+ = \pi I_\nu$$

$$K_\nu = \frac{1}{3} I_\nu$$

so $P_\nu = \frac{1}{3} u_\nu$

Axially symmetric, I independent of ϕ, only $I(\theta)$

$$J_\nu = \frac{1}{2}\int_{-1}^{1} I \, d\mu = 2\pi\int_0^\pi I \sin\theta \, d\theta$$

$$F_\nu = 2\pi\int_{-1}^{1} I(\mu)\,\mu \, d\mu$$

Neither assumption is generally true.

In a stellar atmosphere, where all energy is from nuclear reactions close to the center of the ☆ & energy is transported by radiation, & plane // as OK, the integrated radiative flux is constant throughout the atmosphere.

This applies to $\int F_\nu d\nu$, not to F_ν at a specific ν, which depends on T, i.e. on $B_\nu(T)$

Not true in spherical geometry, F increases with depth as L constant but r decreasing. Once reach core $L\downarrow$ as $r\downarrow$, so $F\downarrow$ as $r\downarrow$

Black Body Radiation

$$I_\nu = B_\nu(T) = \frac{2h\nu^3}{c^2}\left[e^{h\nu/kT} - 1\right]^{-1}$$

$$B(T) = \int_0^\infty B_\nu(T) = \frac{2h}{c^2}\left(\frac{kT}{h}\right)^4 \int_0^\infty \frac{x^3}{e^x - 1}\,dx$$

$$= \frac{\sigma T^4}{\pi}$$

$\sigma =$ Stefan-Boltzmann constant
$= 5.67 \times 10^{-5}$ ergs/cm^2/sec/deg^4

$$u = \frac{4\pi}{c} B(T) = \frac{4\sigma T^4}{c} = aT^4$$

$a =$ radiation constant
$= 4\sigma/c$
$= 7.56 \times 10^{-15}$ erg/cm^3/deg^4

$$P_{rad} = \frac{4}{3}\frac{\pi}{c} B(T) = \frac{aT^4}{3}$$

Photon # density (E/photon $= h\nu$)

$$= \int_0^\infty \frac{B_\nu(T)}{h\nu}\,d\nu = 20\,T^3/\text{cm}^3$$

Brightness temperature – units of I_ν are hard to remember

Set T_B: at $\nu = \nu_0$ (λ of interest)

$I_{\nu_0}(T_B) =$ actual I_{ν_0}

I_ν (actual) may not be a BB function at all!

T_B much easier to use intuitively than I_B

10^{12} vs 3 K easy to remember

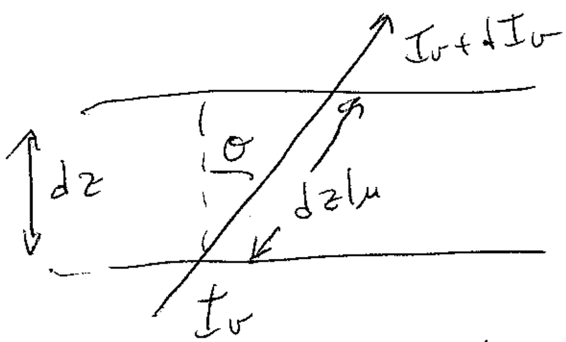
Radiative Transfer Equation For Plane Parallel Layers

$$\frac{\mu}{\rho}\frac{dI_\nu(z,\mu)}{dz} = \underbrace{-\kappa_\nu I_\nu}_{\text{absorption of radiation}} + \underbrace{j_\nu}_{\text{emission of radiation}}$$

Change in I with z

j_ν emission coefficient / unit mass, so $j_\nu \rho = \sigma T^+$
 ↳ ergs/sec/Hz/steradium/gm $\kappa_\nu \rho = \sigma I^-$

κ_ν abs coefficient / unit mass
 ↳ ergs/hz/gm $j_\nu \approx \underbrace{\kappa_\nu B_\nu}_{\text{abs}} + \underbrace{\sigma_\nu J_\nu}_{\text{scattering}}$



stellar surface

z increases ↑ ↓ τ increase

Define optical depth $d\tau_\nu = -\kappa_\nu dz$

• center

$$\boxed{\mu \frac{dI_\nu(\tau,\mu)}{d\tau_\nu} = I_\nu(\tau,\mu) - \frac{j_\nu}{\kappa_\nu}}$$

ratio: atomic physics

$$\frac{j_\nu}{\kappa_\nu} = S_\nu \quad \text{source function}$$

$$\mu \frac{\partial I_\nu(\tau,\mu)}{\partial \tau} = I_\nu(\tau,\mu) - S_\nu(\tau)$$

Formal Solution (does not ~~not a~~ solve for I_ν as need to know S_ν i.e. temperature structure) (11)

$$I_\nu(\tau, \mu > 0) = \int_\tau^\infty e^{-(t-\tau)/\mu} \frac{S(t)}{\mu}\, dt$$ outward rays

$$I_\nu(\tau, \mu < 0) = \int_\tau^0 e^{-(t-\tau)/\mu} \frac{S(t)}{\mu}\, dt$$ inward rays

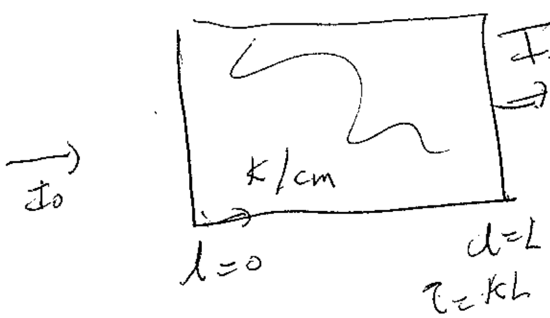
At surface of *, no inward rays unless surface is illuminated by another object (binary *, etc)

Get flux by integrating $\int I(\mu, \tau)\, \mu\, d\Omega$

$F(\tau)$

Homogeneous slab

Case a) absorption only

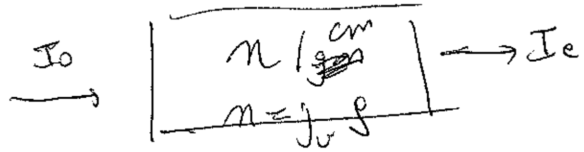


$$\boxed{\frac{dI}{dl} = -kI}$$

$$I = I_0 e^{-kl} = I_0 e^{-\tau}$$

if $I_0 = 0$, $I_e = 0$

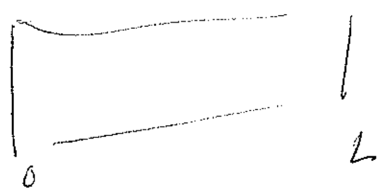
Case b) emission only



$$\frac{dI}{dl} = n$$

$$I = I_0 + nl$$

case e - both abs + emission



$$\frac{dI}{dl} = \eta - k I$$

$$\frac{dI}{dl} > 0 \quad \text{if} \quad \eta > \kappa I$$

$$S = \frac{\eta}{k} > I$$

If $S > I$, emission dominates
$S < I$ abs. dominates

In thermodynamic eq., I constant, and
$I_\nu = B_\nu(T)$, so $S_\nu = B_\nu(T)$ also, since $\frac{dI}{dl} = 0$

Note that $S_\nu^{(z)} = \frac{\eta_\nu(z)}{k_\nu(z)}$ is a ratio of atomic processes only, so perhaps ok to say $S_\nu = B_\nu$ ALWAYS, at least approximately

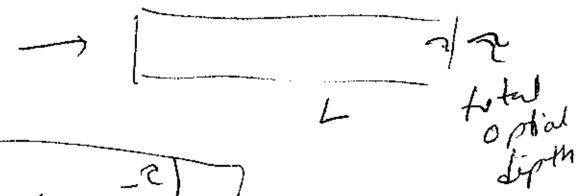
So $\frac{dI_\nu}{d\tau_\nu} > 0$ if $S > I$ ie if $B > I$

B is a local $f(T)$, I is determined by what is happening at distant locations

In usual stellar case, very hot interior determines I, surface is cooler, $S < I$, absorption is seen.

Slab Again

$$\frac{dI}{d\tau} = S - I$$



$$\boxed{I(\tau) = I_0 e^{-\tau} + S(1-e^{-\tau})}$$

formal solution, useless unless you know $S(\tau)$, i e $T(\tau)$

For 0 incident light, $I_0 = 0$ $\quad I(\tau) = S(1-e^{-\tau})$

$I_{max} = S = B_\nu(T)$ if $\tau >> 1$

For τ small and $I_0 = 0$

$$I(\tau) = S(1-e^{-\tau}) \approx S[1-(1-\tau)] \approx S\tau$$

Emission from an optically thin slab.

$$I_\nu(L) = S\kappa L \quad \text{for } \tau << 1$$

$$= S_\nu(T)\underbrace{\kappa L}_{\tau} = B_\nu(T)\,\tau_\nu$$

Thermodynamic Eq. / LTE

for thermodynamic eq., I constant, ind. of t + position. So $I_\nu = B_\nu(T)$

$$S_\nu = B_\nu(T)$$

this is required as $\frac{dI}{dl} = 0$

$$\frac{dI}{dl} = \underset{\substack{\uparrow \\ \text{atomic physics} \\ \text{(local)}}}{n} - \underset{\uparrow}{k} \underset{\text{determined by distant locations}}{I} = 0 \qquad \text{So } \frac{n}{k} = B_\nu(T) = I_\nu = S_\nu$$

In thermodynamic eq. Boltzmann + Saha eqtn hold

$$\frac{n_i}{n_j} = \frac{g_i}{g_j} e^{-(E_i - E_j)/kT} \qquad \text{Boltzmann eqtn}$$

+ for $\cancel{\frac{n_{II}}{n_{I}}}$ $\frac{n_{II} n_e}{n_I} = f(T)$ Saha eqt

$$\left(\frac{2\pi m_e kT}{h^2}\right)^{\frac{3}{2}} \left(2\frac{U_2}{U_1}\right) e^{-IP/kT}$$

(equivalent to electron collision dominate excitation & ionization, rather than photons dominating)

Local Thermodynamic Eq - this applies locally, although over very large scales it may not be true.

This works because particle/particle + photon/particle mean free paths are ~~small~~ short + collision rates are rapid compared to other stellar lengths or timescales (except nuclear reactions, SN, core collapse etc)

So 2 widely separated regions of star are effectively isolated from each other as far as thermodynamics is concerned.

Time for photon to diffuse out from stellar interior

Minimum opacity - electron scattering

Bohr radius for H ($n=1$ level) $\sim 5\times10^{-13}$ cm

cross section πr_e^2

Thompson scat. 6×10^{-25} cm^2/electron

$\langle\rho\rangle$ (Sun) $\simeq 1.4$ gm/cm^3

$\langle\mu\rangle$ (Sun) $= 1/2$ (fully ionized H)

$\langle n_e\rangle \approx 10^{24}$/cm^3

$\langle n_e\sigma_T\rangle \sim 0.5$

mean free path $= \frac{1}{\tau} = \frac{1}{n_e\sigma_T} = 2$ cm

[Note how small this is compared to size of star, so ~~thermodyn~~ LTE eq is ok

frequency shift in electron scattering

$\frac{\delta\nu}{\nu} \simeq \frac{v_e}{c}$ and $\langle\frac{\delta\nu}{\nu}\rangle = 0$ since v_e isotropic

$\langle(\frac{\delta\nu}{\nu})^2\rangle = \frac{\langle v_e^2\rangle}{c^2}$ random walk

scatterings for a photon to forget its frequency is N ~~$\delta\nu$~~ $\Delta\nu \approx \nu$ so $N(\frac{v_e}{c})^2 = 1$ $N = \frac{c^2}{\langle v_e^2\rangle}$

For $T = 10^7$ K, $v_e = 10^9$ cm/sec, $\frac{v_e}{c} \approx \frac{1}{30}$

So $N \sim 1000$

D for random walk of 1000 steps $= \left(\frac{1}{\tau}\right) N^{1/2} = l N^{1/2}$

$= 2\sqrt{1000} = 60$ cm

mean free path

Other opacity sources besides electron scattering $\uparrow$ $\kappa\rho$ from $0.5\,\text{cm}^{-1}$ to $10\,\text{cm}^{-1}$ in Sun (averages)

mean free path

$$\boxed{\langle l \rangle = \langle \tfrac{1}{\kappa\rho} \rangle \simeq 0.1\text{ cm} \quad \text{in Sun}}$$

$$\frac{dT}{dr} \approx \frac{T_c}{R_\odot} \approx 10^{-4} \text{ for Sun}$$

timescale for photon to diffuse out of Sun

1st find N required $\quad lN^{1/2} = R_\odot \quad N = (R_\odot/l)^2$

$$t = N\,(l/c) = \frac{(R_\odot)^2}{lc} \approx \frac{1}{3} \times 10^{13}\text{ sec}$$

$$\boxed{t \approx 10^5 \text{ years}}$$ photon diffuse out of Sun

Mean T of $\odot \sim 4 \times 10^6$ K

$$L_{rad} = 4\pi R_\odot^2 F_{rad} = 4\pi R_\odot^2 \left[\frac{4\pi}{3\kappa_R \rho} a c T^3 \frac{dT}{dr}\right]$$

$$\simeq 10^{34} \text{ ergs/sec} > L_{\text{actual}} \text{ of Sun}$$

So radiation can carry the entire solar flux without problems.

Stellar Interiors; Use the Diffusion Approximation

In stellar interior, use diffusion approx

I_ν almost isotropic (so S_ν almost isotropic)

almost constant with depth

$$S_\nu(t) = B_\nu(\tau) + (t-\tau)\frac{\partial B_\nu}{\partial \tau} + \ldots$$ (Taylor series expansion)

Keep only 1st order term of this Taylor series

$$\int_{\mu>0} I(\tau,\mu) = \int_\tau^\infty S_\nu(t)\, e^{-(t-\tau)/\mu}\frac{dt}{\mu}$$ (integrate this over dt)

$$\underset{(\text{Outward})}{I(\tau)} = B_\nu(\tau) + \mu\frac{\partial B_\nu}{\partial \tau}$$

J_ν (mean intensity) $\approx B_\nu(\tau)$

$$\frac{L(r)}{4\pi r^2} = \text{Flux} \approx \frac{1}{3}\frac{\partial B_\nu}{\partial \tau} = -\frac{1}{3\kappa\rho}\frac{\partial B}{\partial T}\frac{\partial T}{\partial z}$$

Valid only at high optical depth

$$\int_\tau^\infty I\mu\, d\mu = \int_0^1 \mu^2\frac{dB_\nu}{\partial \tau}\, d\mu$$

$$\int \mu^2 d\mu = \frac{1}{3}$$

$$\int B\, d\nu = \sigma T^4$$ So

$$\boxed{L(r) = -\frac{16\pi ac}{3\bar{\kappa}\rho} r^2 T^3 \frac{dT}{dr}}$$

note that $\frac{1}{\bar{K}} \propto \frac{\partial B_\nu}{\partial T}$ in the above formula for $L(r)$

Define Rosseland mean opacity K_R such that

$$\frac{1}{K_R} = \frac{\int_0^\infty \frac{1}{K_\nu} \frac{\partial B_\nu}{\partial T} d\nu}{\int_0^\infty \frac{\partial B_\nu}{\partial T} d\nu}$$

So setting $\bar{K} = K_R$ will work ok in above formula for $L(r)$

2 points of interest in diffusion equation:

1) $L(r) \propto \frac{dT}{dr}$ steeper T gradient → higher $L(r)$

2) $F_\nu(r) = \frac{L_\nu(r)}{4\pi r^2} \propto \frac{\partial B_\nu}{\partial T} \frac{1}{K_\nu \rho}$

For higher K_ν, F_ν is smaller

$\int_0^\infty F_\nu \, d\nu = L(r) =$ constant (assuming outside core where nuclear reactions occur)

So radiative ~~flow~~ flux transport through the star passes via frequencies that are less opaque (ie have lower K_ν)

Another common opacity: often used in approximate simple calculations — (Kramer's opacity)

generic opacity $K = K_0 \rho^n T^{-s}$ cm^2/gm

~~Thompson~~ Thomson scattering of electrons, $n = s = 0$ dominates in completely ionized regions

Kramer's opacity ($n = 1$, $s = 3.5$) is characteristic of radiative processes involving atoms.

Dependence on ρ:

let η be the emission coefficient

we consider b-f or f-f emission

$$\eta\rho \propto n_a n_e f(T) \quad \text{so } \boxed{\eta \propto \rho}$$

this (and below) are valid for b-f or f-f processes – these dominate in the ~~outer~~ outer parts of stars the opacity

Dependence on T: assume LTE so $\frac{\eta_\nu}{\kappa_\nu} = S_\nu = B_\nu(T)$

Maxwell Boltzman vel. distribution for electrons

$$n_e(v)\,dv = 4\pi n_e \left(\frac{m_e}{kT}\right)^{3/2} e^{-(m_e v^2/2kT)} v^2 dv$$

η = emission coeff

$$4\pi\eta\rho = n_I \int_{v_{min}}^{\infty} E\, n_e(v)\, v\, dv$$

ions; energy emitted/interaction/Hz

where assume η independent of angle of light beam (4π)

v_{min} : $\frac{1}{2} m v_{min}^2 = h\nu_{min}$ = minimum energy for transitional collision

$$4\pi\eta\rho \propto \frac{1}{(kT)^{1/2}} n_e n_i e^{-h\nu/kT} d\nu$$

Now integrate over ν

$$4\pi\eta\rho \propto n_i n_e \sqrt{T}$$

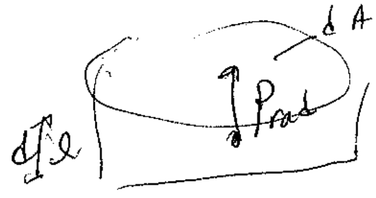
Recall LTE so $\frac{\eta_\nu}{\kappa_\nu} = S_\nu = B_\nu(T)$

So $\boxed{\kappa \propto \frac{T^{1/2}}{T^4}\rho \propto \rho T^{-3.5} \quad \text{Kramers' opacity}}$

Energy transferred to matter when light is absorbed

$$dE_\nu = \kappa_\nu \rho I_\nu \, d\Omega \, d\nu \, dt \, dA \, dl$$

momentum $p = \frac{E}{c}$



towards center of star / light flows toward surface

$$dP_\nu = \frac{\overbrace{\cos\theta}^{\mu} E_\nu}{c} = \frac{\mu}{c} \kappa_\nu \rho I_\nu \, d\Omega \, d\nu \, dt \, dA \, dl$$

Integrate over solid angle

$$dP_\nu = \frac{4\pi}{c} \kappa_\nu \rho \overbrace{H_\nu}^{\text{flux}} d\nu \, dt \, dA \, dl$$

$$dp = \frac{4\pi}{c} \rho \, dt \, dA \, dl \int_0^\infty \kappa_\nu H_\nu \, d\nu$$

force $= \frac{dp}{dt}$ Force | radiation $= \frac{4\pi}{c} \rho \, dA \, dl \int_0^\infty \kappa_\nu H_\nu \, d\nu$

$$= \rho \, dA \, dl \, g_{rad}$$

$$g_{rad} = \frac{4\pi}{c} \int_0^\infty \kappa_\nu H_\nu \, d\nu$$

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hydrostatic eq.

$$\frac{dP(r)}{dr} = -\rho(r) \left[\overbrace{g(r)}^{\text{to center / gravity}} - \underbrace{g_{rad}(r)}_{\text{up / radiation pressure}} \right]$$

If $g_{rad} > g_{grav}$, matter pushed out of star

$g_{rad} \propto T_*^4$ so can be large in hot stars

Example 3.6 of Le Blanc

Assume opacity = Thomson scat.

~~Find~~ Find upper limit to T_{eff} for a star before $P_{rad} > P_{grav}$. This is $\approx$ maximum T_{eff} for a stable star

$$g_{rad} = \frac{4\pi}{c}\left(\frac{n_e \sigma_T}{\rho}\right)\int_0^\infty H_\nu \, d\nu$$

(constant κ independent of ν

$$g_{rad} = \left(\frac{n_e \sigma_T}{\rho}\right)\left(\frac{\sigma T_{eff}^4}{c}\right)$$

Assume pure H star $\rho = n_p m_p + n_e m_e \approx n_p m_p$

Solve for T_{eff}

get $T_{eff}^{max} = 60{,}000\,K$