HW 4
AY 123, Spring 2003
Due Tues April 29, beginning of class

Problem 1. Convection in the Sun

We are going to try to find the convective zones in the Sun. You already have a model for its atmosphere from the last problem set. Use the interior model given in Allen's Astrophysical Quantities, 4th edition, table 14.2. We adopt the mixing length approximation for convection. Assume that for any surface convection, the mixing length l is the pressure scale height, while for any core convection, the mixing length is $R^*/10$.

a) Calculate $\nabla(rad)$, $\nabla(ad)$ and $\nabla(\text{actual})$ for each depth in the interior and atmosphere model for the Sun.

b) What criterion is used to determine if a a zone is convectively unstable? What radial zone(s) in the Sun are convectively unstable?

c) Now we consider one particular zone, the unstable zone with the smallest r. Calculate w, the mean speed of the convective elements, v_s, the sound speed there, and the Mach number.

d) Compute also the cooling time for a rising column, the eddy turnover time ($\tau \approx l/w$), and the ratio of the flux carried by convection to the total flux in this zone.

Problem 2. Diffusion of elements

Since the abundance of elements is important for both nuclear burning rates, hence stellar ages, and atmospheric abundances, we attempt to determine if there is enough time during stellar evolution for heavy elements to sink to the center of a star.

a) What is the macroscopic force on an ion with a charge Ze and a mass Am_P in a star consisting otherwise (and predominantly) of pure hydrogen. (Don't forget the electric field $eE = m_p g/2$ that has to be present so as to keep the protons from sinking and the electrons from sailing out of the star.)

b) Approximately how often does an ion collide with the electrons and protons in the gas? Which collisions determine the rate of downward settling? Calculate the collision rate for ions near the half-mass radius of the Sun (which we take to be a main sequence star).

c) Combining (a) and (b), calculate the rate of downward drift of an ion as a function of ρ, T and gravity in the gas. How does the time to drift a radius R scale with the internal temperature of the gas? (scale to $T = 10^7$ K) How does this timescale compare to the main sequence lifetime as a function of the mass of the star? (Use equation 2-3 of Hansen and Kawaler here.)

d) Now we examine the case of a star more massive than the Sun, and look at the photosphere. Assume that $P_{ph} = g/\kappa_T$, where κ_T is the Thomson scattering opacity. Then use the main sequence T_{eff} for a massive main sequence star (see Allen's Astrophysical Quantities, table 15.7) to calculate t_{drift}. Should we see heavy elements in the photosphere, or just pure hydrogen?

e) Now do the same as (d) but for a 0.6 Solar mass white dwarf with $T_{eff} = 20,000$ K.

Problem 3.

Assume a star begins initially with pure hydrogen. The cross section S_0 for the reaction p + d becomes He^3 + photon is 2.5 x 10^{-4} keV barn. That for p + p becomes D + positron is 3.8 x 10^{-22} keV barn. Calculate and plot the equilibrium ratio of D/H after deuterium has achieved a steady state abundance during H burning as a function of temperature for T between 1 and 20 million degrees. Assume $\rho = 140$ gm/cm^3 for this problem and the next one.

Problem 4.

The only cross section not yet given for the three reactions in the main pp chain is that of $He^3 + He^3$. Set the cross section S_0 for the reaction $He^3(He^3,2p)He^4$ to 5.0×10^3 keV-barns. For a star composed initially only of hydrogen, how long does it take for the main pp chain to reach equilibrium so none of the intermediate reactions is slower than the first (p + p) reaction as a function of T for T between 1 and 20 million degrees? Plot this time as a function of T, then plot the abundance of each relevant species (H, D, He^3 and He^4) as a function of time for T = 10^6 K.