HW 5
AY 123, Spring 2003
Due Tues May 6, beginning of class

Problem 1.

Energetic neutrinos from the Sun are produced by the decay of B^8. Show that the rate of the reaction producing B^8, $Be^7(p,\gamma)B^8$ is approximately proptional to T^{14}, when the temperature T is near 1.5×10^7 K. If one attempted to explain the discrepancy between the observed and predicted neutrino flux in Ray Davis' experiment by postulating an error in the central temperature of the Sun, what change in the central temperature would be required ?

Problem 2.

A model was presented in class for the production of the heavy elements via addition of neutrons to some seed nucleus. Assume Fe is the seed nucleus.

a) Assume such neturon addition produces all the nuclei in the Sun heavier than zinc in the Sun. What fraction of the Fe currently present in the Sun was required to be used as seed to produce all the heavy elements. Comment on the importance of the result you obtain.

b)Assume roughly that each of the heavy elements has two stable isotopes and all neutron addition is via the s-process only. Explain semi-quantitatively the pattern of abundance in the Sun of the elements between barium (atomic number 56) and terbium (atomic number 65) in this model. Your explanation should include a sketch of the cross section as a function of atomic number in this region.

Problem 3.

Provide a flow chart for a program that computes model stellar atmospheres under the classical simplifying assumptions discussed in class. Your flow chart should contain at least 10 and not more than 25 subroutines. Provide a brief (2 sentences maximum) description of the function of each of the subroutines in your program. List the input and output variables for each of your subroutines.

Problem 4.

Consider a first order ordinary differential equation $dy/dx = f(x)$, where $f(x)$ is a known function, which we wish to solve for $y(x)$. The equation cannot be integrated analytically and we need to find the solution by numerical integration. We are going to evaluate the error in predicting $y(x_i + h)$ from a known initial value $y(x_i)$.

a) Show that the error for a single step using the Euler scheme (replacing derivatives by differences) for integration of such a first order ordinary differential equation is proportional to h^2, where h is the size of the step and the interval goes from 0 to 1.0.

b) Show that using a mid-point technique (i.e. a second order Runge-Kutta integration) makes the error of a step proportional to h^3.

We are now going to apply these techniques to integrate the first order ordinary differential equation

$$dy/dx = x^2 + sin(x) \tag{1}$$

over the interval in x from 0 to 1. (x is given in radians.)

c) Solve this equation analytically and plot $y(x)$ over that interval.

d) Next use a second-order Runge-Kutta numerical integration. Use 40 steps over the interval in x. Plot your result for $y(x)$. What is the largest error in y in absolute value and as a fraction of y ?

Techniques for numerical integration

The numerical solution of the differential equation,

$$dy/dx = f(x, y),$$

where $f(x, y)$ (the deriviative of the solution) is a known function of x and y, is discussed in many books on numerical analyses, including Numerical Recipes, chapter 15. There is also a brief discussion in your text by Hansen and Kawaler in Section 7.2.2 (stellar modeling).

To summarize, the Euler method uses the simple approximation that to integrate over an interval x_1 to x_2, one divides it into N steps of equal size h in x. If you know the value of $y(x_n)$, then to get the value of y at the next step $y(x_{n+1})$ you use the derivitive f calculated at the known initial point (x_n, y_n) to propagate the solution to the next step, i.e. $y_{n+1} = y_n + h \times f(x, y)$, with the last term evaluated at (x_n, y_n).

The second order Runge-Kutta technique uses instead a half step to the middle of the interval, and evaluates the function there, instead of at the beginning of the interval. Thus $k1 = h \times f(x, y)$ at (x_n, y_n). The the midpoint $(x, y[mid])$ is at $(x_n + h/2, y_n + k1/2)$. Now we calculate the value of y_{n+1} as $y_n + h \times f(x, y)$ at $(x, y[mid])$.

See the description in Numerical Recipes (section 15.1) for a more detailed discussion.