HW 7
AY 123, Spring 2003
Due Tues April 27, beginning of class

Problem 1.

Consider a gas consisting only of one ion in which a bound-bound transition occurs at frequency ν_0, where hν_0 is the difference in the electron energy levels. The absorption/ion (absorption coefficient per ion) for this transition has the form $\alpha_\nu = \alpha_0 \phi(\nu)$, where $\phi(\nu)$ is normalized so that its integral from $\nu = 0$ to infinity is 1.

a)Assume $\phi(\nu)$ is given by the 1D Maxwellian velocity distribution function for a gas with temperature T and mean atomic weight/ion of A. Write down, with all the constants included, what $\phi(\nu)$ is. Let us denote this as $\phi 1(\nu)$.

b) Now assume $\phi(\nu)$ is a Lorentz profile with $\gamma = 1000\gamma_{nat}$, and γ_{nat} is the natural damping constant. What is $\phi(\nu)$ now ? Let us denote this as $\phi 2(\nu)$.

We now adopt the specific values $T = 5780$ K and $A = 56$ (corresponding to iron).

c) What is the corresponding Doppler width in frequency and in wavelength units ? Recall that $\Delta(\nu_D)$ is the Doppler width, which is $\frac{\nu_0}{c}\sqrt{2kT/(Am_H)}$.

d) Plot the ratio $\phi 1(\nu)/\phi 2(\nu)$ as a function of $\nu - \nu_0$ for $\nu - \nu_0 = 0$ to $20\Delta(\nu_D)$. At what distance from the line center do the damping wings overpower the negative exponential in the Doppler profile ?

e) Assume the previous values given above hold for $N_p = 10^{18}$ cm^{-3}, where N_p is the number of perturbers. What is the value of N_p when $\phi 1(\nu)/\phi 2(\nu) = 0.05$ at $\nu - \nu_0 = 15\Delta(\nu_D)$?

Problem 2.

We are going to calculate a curve of growth for a Fe I line in the Sun. The wavelength of the line center is 5000 Å and the line arises from the ground state of Fe I, so $\chi = 0.0$ eV. We will need to make a lot of simplifying assumptions to make the required code easier to write. You should only consider the ionization equilibrium between Fe I and Fe II. Since the Voigt profile is not an analytic function, you should represent the line absorption profile per Fe I atom in the ground state as $A_0 \times 0.5[\phi 1(\nu) + \phi 2(\nu)]$, where $A_0 = \pi e^2/(mc)$. Ignore the changes in $\phi 1(\nu) + \phi 2(\nu)]$ as a function of depth, i.e. assume, for example, that $\Delta(\nu_D)$

is fixed. Also assume that the surface flux $F_\nu = B_\nu(T : \tau = 0.667)$, where B is the Planck function and τ is the appropriate optical depth scale corresponding to the opacities included.

We are going to use the Solar model atmosphere calculated by Bob Kurucz which was also used in homework problem set 3, and which is given on the class web site. We are going to assume that we can add the line opacity to the Rosseland mean opacity to get the total opacity, without taking into account the frequency dependence of the continuous opacity.

Adopt a suitable frequency step size covering from -5$\Delta(\nu_D)$ to 0$\Delta(\nu_D)$ with respect to the line center frequency. (Because of symmetry only half of the line profile needs to be calculated.) At each frequency, calculate the line absoprtion coefficient, then calculate the new optical depth including the line opacity for each depth in the model. This will yield the continuum and line flux at each frequency point. Then integrate (sum) the resulting surface flux for each frequency in the line to calculate (approximately) the equivalent width of the line. Carry out this calculation for a range of values of gf which is sufficiently large that you see the full behavior of the curve of growth from linear to damping. Indicate any additional assumptions you have to make beyond those listed above.

a) Plot the resulting curve of growth, W_ν or W_λ as a function of $N(Fe)gf$.

b) Give the equivalent width range (W_λ please) for the linear part of the curve of growth and for the damping part of the curve of growth for your Fe I line.

c). Appended is a table of equivalent widths of Fe I lines in the Solar spectrum of a multiplet with $\chi = 0.05$ eV. The table gives the line wavelength in Å, the equivalent width in mÅ (1 mÅ = 10^{-3}Å) and the log(gf) value. For each line, g is the statistical weight of the lower level and f is the transition probability. Use the information in this table plus the theoretical curve of growth you calculated to obtain an approximate value for the Fe abundance $N(Fe)/N(H)$ in the Sun.

Equivalent Widths of Lines of FeI in the Sun

FeI

Multiplet 1, Excitation Potential of Lower Level = 0.05 eV

λ	Equivalent Width W_λ	log(gf)
5166.29	115	−3.68
5247.06	59	−4.50
5254.96	92	−4.23
5110.41	126	−3.34
5168.90	114	−3.49
5225.53	68	−4.26