Solutions for Midterm 2010 October + Suggested grading

1. Total points = 20 (2 per definition). Answers pretty obvious? *(let me know if not)*

2. Total points = 20
(a: 5 points b: 4 points c: 4 points d: 3 points e: 4 points)

a) For plotting rough diagram, Sun's position and points A, B, and C correctly give 5 points total

b) $L = 4\pi R^2 \sigma T^4$ so $L_\odot = 4\pi R_\odot^2 \sigma T_\odot^4$ $\therefore R_*/R_\odot = (L_*/L_\odot)^{1/2} \times (T_\odot / T_*)^2$
For Sun, use T_{eff} = 6000K $\therefore$ $R_* = (54)^{1/2} \times (6000/9250)^2 = 3.1\ R_\odot = R_A$

c) $R_B = 3.63\ R_\odot$ and $R_C = 71\ R_\odot$

d) Spectral types OBAFGKM represent a temperature scale with O hottest and M coolest. Although Stars A, B, C, have different radii and luminosities, they all have temperatures around 10^4 K, thus same spectral type, with similar spectral shapes and strong emission from Balmer lines of hydrogen.

3. Total points = 20
(a: 2 points b: 5 points c: 4 points d: 5 points e: 4 points)

a) If the limiting parallax θ = 0″.005 mas, this implies that 0″005 is the resolution available for the parallax measurement
$\therefore d_{max} = 1''/0''.005$ pc = 200 pc

b) Mean distance between stars is 2 pc. ∴Each star occupies a sphere of mean radius 1 pc, volume = $4/3\pi \times (1pc)^3 = V_{star}$
From part a), volume in which we can measure parallax accurately = $4/3\pi \times (200\ pc)^3 = V_{sphere}$ ∴Number of stars whose distance can be measured = $V_{sphere}/V_{star} = (200)^3$
= 8×10^6 stars with measureable parallax

c) $\theta_{min} = 0''.5$ and $\lambda \sim 6000$Å.
And for single aperture telescopes, $\theta_{min} = 1.22\lambda/D$
$\therefore D = 1.22\lambda/\theta_{min(radians)} = 1.22 \times 6000 \times 10^{-8}/ 6 \times 10^{-6} \sim 13.2$ cm

Not very impressive! That's seeing for you.

Diameters of existing large single aperture telescopes: Palomar 5 m, Keck 10m, Gemini 8m, Subaru, VLT etc

d) For interferometers, $\theta_{min} = \lambda/d$ where d is the largest separation of the telescopes i.e. interferometers can resolve objects separated by angular distance λ/d.
Here, λ = 10 μm =10×10^{-6} m and d= 80m

$\therefore \theta_{min} = 10 \times 10^{-6} / 80$ radians = *(need to complete calculation)*
The maximum distance R to the star at which linear separation of star and planet is 5 AU, is then given by $\tan \theta_{min} = 5AU/R$,
$\therefore R = 5 \times 1.5 \times 10^{13} / \tan \theta_{min}$ = *(need to complete calculation)*

e) Again $\theta_{min} = \lambda/d$ = 5500Å/ separation of elements of interferometer(baseline)
$\therefore$ d = baseline = 5500Å/θ_{min}
$P'' = 1/D(pc)$ where D is distance to object and $P \geq \theta_{min} = \lambda/d$
$\therefore b \geq \lambda / (P'') \times (206265 \text{ arcsec} / 1 \text{ radian})$
$\therefore$ baseline ~ $5500 \times 10^{-10} \times 8 \times 10^{3} \times 206265$ ~ 90 km
Thus separation must be at least 90 km.

4. Total points = 20
(a: 2 points b: 3 points c: 5 points d: 4 points e: 6 points)

a) $d = 1/P'' = 250$ pc
b) $m = 2^m.85$ and d = 250 pc $\therefore M = m - 5\log(d/10) = 4.14$

c) M_{bol} = M + Bolometric Correction (BC). And B-V = -0.31 $\therefore$ in tables, take average BC between B-V = -0.4 (BC = -4.1) and B-V = -0.2 (BC = -1.9)
$\therefore$ BC ~ -3.1 $\therefore$ M_{bol} = M + Bolometric Correction (BC) = -4.14 -3.1 = - 7.24

d) Compare Sun and star: $L_*/L_\odot = 100^{(M_\odot - M*bol)/5}$ $\therefore L_* = 4 \times 10^4 L_\odot$

e) A B0V star such as T Sco should have strong HeI absorption features. Balmer lines will be weakening compared to A stars but still quite strong. A) stars a "white light" stars characterized by the strongest Balmer absorption lines and fairly strong CaII lines. M dwarfs are the coolest red stars; their spectra are dominated by molecular absorption bands, especially TiO as well as metal absorption lines

5. Total points = 20
a: 2 points b: 9 points c: 9 points)

a) $B_\lambda(T) = 2hc^2/\lambda \times 1/ e^{hc/\lambda kT} - 1$ = 2.54 ergs /cm^2/sec/Å/steradian

b) If Sun is a BB at 5760 K and $m_\odot$ = -26.80 at 5500Å and star has m = 0.0, the ratio of fluxes, $F/F_\odot = 100^{(m\odot - m)/5}$. $\therefore F = 100^{-26.8/5} F_\odot = 100^{-26.8/5} \times \pi B\lambda_\odot$.
Thus F = 1.52×10^{-4} ergs/cm^2/sec/Å
Photons of wavelength 5500 Å have energy $h\nu = hc/\lambda = 3.6 \times 10^{-12}$ ergs/sec
$\therefore$ Number of photons = $F/\nu = 4.2 \times 10^7$

c) The Balmer jump is the abrupt drop in the continuous spectrum at wavelength 3647 Å. It results from the fact that any photon with wavelength below 3647 Å can absorb radiation from the n=2 level of hydrogen. The opacity increases abruptly. The U filter is centered very close to this wavelength so that U-B color is heavily affected for star with strong Balmer lines. This causes a significant deviation from the blackbody line in a color color diagram

[Also refer to the (U-B, B-V) diagram given in class]