Ay 20: Basic Astronomy and the Galaxy
Fall Term 2010

Solution Set 1

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(based on solutions by Swarnima Manohar, TA 2009)

Notice that I have stated all numerical answers keeping in mind significant figures. Reporting an answer to unnecesary number of decimal places should be avoided. To know more about significant figures, see http://calculator.sig- figs.com/tutorial/why.

PROBLEM 1 (C&O Problem 6.8):

Rayleigh Criterion is $\theta_{res}=1.22\,\frac{\lambda}{D}$

(a) For a D of 20 cm and λ of 550 nm (mind the units!!):

$$\theta_{res} \to 1.22\,\frac{\lambda}{D}\,\text{radians} \;/.\; \{\lambda \to 550\times10^{-9}\,\text{m},\; D \to 0.20\,\text{m}\}$$

$$\therefore \theta_{res} \to 3.355\times10^{-6}\,\text{radians}$$

Convert to arcseconds,

$$\theta_{res} \to 3.355\times10^{-6}\,\text{radian}\left(\frac{180}{\pi}\,\frac{\text{degrees}}{\text{radians}}\right)\left(3600\,\frac{\text{arcsec}}{\text{degrees}}\right)$$

$$\theta_{res} \to 3.355\times10^{-6}\,\text{radian}\left(\frac{206\,265\,\text{arcsec}}{1\,\text{radian}}\right)$$

$$\therefore \theta_{res} \to 0.69\,\text{arcsec}$$

(b) Using the relationship between arclength and radius of a circle:

$$R_{crater} \to d\,\theta_{res} \;/.\; \{d \to 384.4\times10^{3}\,\text{km},\; \theta_{res} \to 3.355\times10^{-6}\}$$

$$\therefore R_{crater} \to 1.3\,\text{km}$$

Here 'd' is the distance to moon. Thus, you should be able to see a crater of size 1.3 km.

(c) No, this resolution limit would be very hard to achieve because of atmospheric turbulence. The limit is typically about 1" - 2" for optical wavelengths at moderately good sites.

PROBLEM 2 (C&O Problem 6.9):

(a) Focal ratio is f/2.2, meaning f/D = 2.2. NTT has a diameter of 3.58 m. Thus, the focal length is:

$$\mathcal{N} = \frac{f}{D} = 2.2$$

$$f \to \mathcal{N}\,\mathcal{D} \;/.\; \{\mathcal{N} \to 2.2,\; D \to 3.58\,\text{m}\}$$

$$\therefore f \to 7.9\,\text{m}$$

(b) Plate scale:

$$\frac{d\theta}{dy} = \frac{1}{f} = \frac{1\ \text{radian}}{7.9\ \text{m}} = 0.13\ \text{radian} \cdot \text{m}^{-1}$$

(c) From the last part we know that the plate scale is 0.13 rad$\cdot m^{-1}$. The separation of the binary is $\Delta\theta$ = 2.9". Thus, the linear separation is given by:

$$\frac{\Delta\theta}{\Delta y} = \frac{1\ \text{radian}}{f} \Rightarrow \Delta y = f\left(\frac{\Delta\theta}{1\ \text{radian}}\right)$$

$$\therefore \Delta y = (7.9\ \text{m})\left(\frac{2.9\ \text{arcsec}}{1\ \text{radian}}\right)\left(\frac{1\ \text{radian}}{206\,265\ \text{arcsec}}\right) = 0.11\ \text{mm}$$

PROBLEM 3: CCAT

Diameter, D = 25m.
Rayleigh Criterion for lower energy limit (λ_1=2200 μm):

$$\theta_{low} \to 1.22\,\frac{\lambda}{D}\ \text{radians}\ /.\ \{\lambda \to 2200 \times 10^{-6}\ \text{m},\ D \to 25\ \text{m}\}$$

$$\theta_{low} \to 0.00010736\ \text{radians}$$

$$\theta_{low} \to 0.00010736\ \text{radians}\left(\frac{206\,265\ \text{arcsec}}{1\ \text{radian}}\right)$$

$$\therefore \theta_{low} \to 22\ \text{arcsec}$$

Rayleigh Criterion for higher energy limit (λ_2=200 μm):

$$\theta_{high} \to 1.22\,\frac{\lambda}{D}\ \text{radians}\ /.\ \{\lambda \to 200 \times 10^{-6}\ \text{m},\ D \to 25\ \text{m}\}$$

$$\theta_{high} \to 9.76 \times 10^{-6}\ \text{radians}$$

$$\theta_{high} \to 9.76 \times 10^{-6}\ \text{radians}\left(\frac{206\,265\ \text{arcsec}}{1\ \text{radian}}\right)$$

$$\therefore \theta_{high} \to 2.0\ \text{arcsec}$$

So you see that CCAT achieves a lower resolution for longer wavelengths or lower energy radiation.
The binary is separated by 10 AU. The distance to which it can be resolved for the lower energy limit:

$$d \to \frac{a}{\theta_{low}}\ /.\ \{a \to 10\ \text{AU},\ \theta_{low} \to 0.00010736\ \text{rad}\}$$

$$\therefore d \to 9.3 \times 10^{4}\ \text{AU}$$

– OR –

$$d \to \frac{a}{\theta_{low}}\ /.\ \{a \to 10 \times 1.496 \times 10^{13}\ \text{cm},\ \theta_{low} \to 0.00010736\ \text{rad}\}$$

$$\therefore d \to 1.4 \times 10^{18}\ \text{cm} \simeq 0.5\ \text{pc}$$

The distance to which it can be resolved for the higher energy limit:

$$d \to \frac{a}{\theta_{high}}\ /.\ \{a \to 10\ \text{AU},\ \theta_{high} \to 9.76 \times 10^{-6}\}$$

$\therefore d \to 1.0 \times 10^{6}$ AU

– OR –

$$d \to \frac{a}{\theta_{high}} \;/.\; \{a \to 10 \times 1.496 \times 10^{13}\,\text{cm},\; \theta_{high} \to 9.76 \times 10^{-6}\}$$

$\therefore d \to 1.5 \times 10^{19}$ cm $\simeq$ 5 pc

PROBLEM 4: Telescopes Galore:

Telescope	Diameter	Wavelength	Frequency	Resolution, θ (arcsec)
CARMA	2 km	2.6 mm	115 GHz	0.33
HST	2.4 m	380 nm	789 THz	0.040
Spitzer	0.85 m	24 μm	12.5 THz	7.1
TMT	30 m	1.0 μm	300 THz	0.0084

PROBLEM 5 (C&O Problem 1.4):

Time of Year	Right Ascension	Declination
Vernal Equinox	0 h	0 °
Summer Solstice	6 h	+23.5 °
Autumnal Equinox	12 h	0 °
Winter Solstice	18 h	−23.5 °

PROBLEM 6 (C&O Problem 1.6):

(a) Stars that never set are:

Northern Hemisphere: L>0 $\quad$ 90°-L < Dec < 90°

Southern Hemisphere: L<0 $\quad$ -90° -L > Dec > -90°

Stars that never rise:

Northern Hemisphere: L>0 $\quad$ -90° +L > Dec > -90°

Southern Hemisphere: L<0 $\quad$ 90°+L < Dec < 90°

(b) The sun will never set at summer solstice at a latitude of:

$$90 - L = 23.5$$
$$\Rightarrow L \geq 66.5\,°$$

(c) Theoretically at least, the sun at either equinox will not set when observed from the North or the South Poles.

PROBLEM 7 (C&O Problem 1.8): α Centauri System

(a) Angular Separation: Looking at the coordinates, we see that the stars are close enough that we can use a planar approximation to solve the problem. The differences in right ascension and declination respectively are $\Delta\alpha$ = 0h9m53s and $\Delta\delta$ = 1°50'38". Before applying the textbook formula, note that the right ascension needs to be converted to degrees. The formula thus becomes

$$\Delta\theta = \sqrt{(\Delta\alpha \times 15 \times \text{Cos}[\delta])^2 + (\Delta\delta)^2}$$

So we get $\Delta\theta = 2.2°$. Don't neglect the cos δ factor here. Recall that RA differences do not directly measure angles on the sky (that is, as measured along the arc of a great cricle). Think of two points on just opposite sides of the celestial north pole. They have $\Delta\alpha$ = 12h = 180°, but can be at an arbitrarily small angular separation.

(b) Physical Distance: From the parallaxes, we can calculate the distances:

$$d = \frac{1}{\theta_{\text{parallax}}}$$

where d is in parsecs and θ in arcseconds. The distance to Centauri A is thus d_1=1.35 parsecs (pc), and that to Proxima is d_2=1.32 pc. The physical distance is then given by,

$$r = \sqrt{d_1^2 + d_2^2 - 2\, d_1\, d_2 \cos \Delta\theta}$$

Thus the physical separation in the two stars is 0.059 pc. (Which is also 12000 AU, 1.8×10^{17} cm, 0.19 light years...let's keep our answers to pc or AU)

(c) Angular Diameter: The sun subtends 32' at 1 AU. The distance to Centauri A is 1.35 pc. Since there are 206265 AU per parsec, this is 2.78×10^5 AU. (The conversion factor simply comes from geometry: it is the number of arcseconds in a radian.) The angular size of an object with the same physical size as the sun is then $32' \times \frac{1}{2.78 \times 10^5} = 1.15\times10^{-4}$ arcmin = 0.0069 arcsec. (The resolution of Hubble is only 0.1 arcsec, for comparison).

PROBLEM 8 (C&O Problem 3.3)

(a) Parallax given is p = 0.379". We know that $d = \frac{1}{p''}$pc (Eqn 3.1)

(i) In parsecs:

$$d \to \frac{1}{p}\text{pc} \;/.\; \{p \to 0.379\}$$

$$d \to 2.64\ \text{pc}$$

(ii) 1 pc = 3.26 ly

$$d \to \frac{1}{p}\text{pc} \;/.\; \{p \to 0.379,\ \text{pc} \to 3.26\ \text{ly}\}$$

$$d \to 8.60\ \text{ly}$$

(iii) 1 pc = 206265 AU (Remember also that 1 rad = 206265 arcsec):

$$d \to \frac{1}{p}\text{pc} \;/.\; \{p \to 0.379,\ \text{pc} \to 206\,265\ \text{A.U.}\}$$

$$d \to 5.44 \times 10^5\ \text{A.U.}$$

(iv) 1 pc = 3.086×10^{16} m

$$d \to \frac{1}{p}\text{pc} \;/.\; \{p \to 0.379,\ \text{pc} \to 3.086 \times 10^{16}\ \text{m}\}$$

$$d \to 8.14 \times 10^{16}\ \text{m}$$

(b) Using eqn 3.6:

$$\text{Distance Modulus} = m - M = 5\,\text{Log}_{10}\left[\frac{d}{10\,\text{pc}}\right]$$

$$\text{dmod} \rightarrow 5\,\text{Log}\left[10, \frac{1}{0.379}\,\frac{1}{10}\right]$$

$$\text{dmod} \rightarrow -2.89$$

Thus, the distance modulus is -2.89