Ay 20: Basic Astronomy and the Galaxy
Fall Term 2010

Solution Set 5

Kunal Mooley
(based on solutions by Swarnima Manohar, TA 2009)

Reporting an answer to unnecesary number of decimal places should be avoided. CGS units are popular among professional astronomers. SI system is considered universal, and so I will encourage you to stick to SI if you are presently using this system most frequently.

PROBLEM 1 (C&O Problem 10.10)

The energy generation rates for the pp chain and CNO cycle are given by Eq 10.46 and Eq 10.59:

$$\epsilon_{pp} \simeq \epsilon'_{0,pp}\, \rho\, X^2\, f_{pp}\, \psi_{pp}\, C_{pp}\, T_6^4$$
$$\epsilon_{CNO} \simeq \epsilon'_{0,CNO}\, \rho\, X\, X_{CNO}\, T_6^{19.9}$$

where both are written as power laws centered around 1.5×10^7K.
Using the pp chain screening factor $\psi_{pp}\approx 1$ and pp chain branching ratio $f_{pp}\approx 1$ and given the values of T, ρ, X and X_{CNO} in the question we get:

$$\epsilon_{pp} \to 1.08 \times 10^{-12}\, \rho\, X^2\, f_{pp}\, \psi_{pp}\, C_{pp}\, T_6^4\ /.\ \{\rho \to 1.527 \times 10^5,\ X \to 0.3397,\ f_{pp} \to 1,\ \psi_{pp} \to 1,\ C_{pp} \to 1,\ T_6 \to 15.696\}$$

$$\epsilon_{pp} \to 0.00116$$

$$\epsilon_{CNO} \to 8.24 \times 10^{-31}\, \rho\, X\, X_{CNO}\, T_6^{19.9}\ /.\ \{\rho \to 1.527 \times 10^5,\ X \to 0.3397,\ X_{CNO} \to 0.0141,\ T_6 \to 15.696\}$$

$$\epsilon_{CNO} \to 0.000377$$

These are both in W/kg. Ratio:

$$\frac{\epsilon_{pp}}{\epsilon_{CNO}} = 3.06$$

ϵ_{CNO} depends much more strongly on temperature than ϵ_{pp}. In low mass (cooler) stars H fusion is dominated by the pp chain whereas in high mass (hotter) stars it is dominated by CNO cycle. The transition where $\frac{\epsilon_{pp}}{\epsilon_{CNO}} \approx 1$ occurs at a mass slightly higher than the solar mass.
Thus, high mass stars end up with the more heavy elements as their reaction products - this has profound effects on their structure and subsequent evolution.

PROBLEM 2 (C&O Problem 10.21)

From secition 10.3 of C&O we know that 0.7% of the mass of hydrogen is converted to energy in forming Helium nucleaus, and therefore,

$$t_{nuclear} = \frac{E_{nuclear}}{L} = \frac{x \cdot (0.007) \cdot Mc^2}{L}$$

where ' x ' is the fraction of the total hydrogen in the star available for fusion.

(1) Lower end of the MS : $M = 0.072\, M_\odot$, $L = 10^{-4.3}\, L_\odot$, $x = 1$

$$\therefore\ t_{nuclear} = \frac{(0.1)\cdot(0.007)\cdot M_\odot c^2}{L_\odot} \times \frac{(10)\cdot(0.072)}{10^{-4.3}} = \left(10^{10}\ \text{yr}\right) \times 14\,366 = 1.4 \times 10^{14}\ \text{yr}$$

(1) Upper end of the MS : $M = 85\, M_\odot$, $L = 10^{6.006}\, L_\odot$, $x = 0.1$

$$\therefore\ t_{nuclear} = \frac{(0.1)\cdot(0.007)\cdot M_\odot c^2}{L_\odot} \times \frac{85}{10^{6.006}} = \left(10^{10}\ \text{yr}\right) \times 8.4 \times 10^{-5} = 8.4 \times 10^{5}\ \text{yr} = 0.8\ \text{Myr}$$

PROBLEM 3 (C&O Problem 11.2)

(a) Assuming (wrongly) that all the energy released by Sun comes from the PPI chain, our reaction of interest is basically:

$4\ {}_1H^1 \longrightarrow {}_2He^4 + 2\,e^+ + 2\,\nu_e + 2\,\gamma$

0.7% of the mass of hydrogen is converted to energy in forming Helium nucleaus, and therefore,

Mass loss per unit time through fusion reactions is,

= (Mass loss in one fusion reaction)·(No. of reactions per unit time)

$$= (0.007 \times 4\, m_p) \left(\frac{L_\odot}{0.007 \times 4\, m_p\, c^2}\right)$$

$\simeq 4 \times 10^{9}$ kg / s

$\simeq 1.2 \times 10^{17}$ kg / yr

$\simeq 6 \times 10^{-14}\, M_\odot$ / yr

(b) Mass loss per unit time through solar wind (from C & O Example 11.2 .1) is,

= $3 \times 10^{-14}\, M_\odot$ / yr

Thus, (Mass loss through fusion) > (Mass loss through wind) for the Sun by a factor of 2

(c) As done in C & O Example 10.3 .2,

$t_{nuclear} \sim 10^{10}$ yr

This is the Main – Sequence lifetime of Sun,

and thus total mass loss in this period of time would be :

(i) Due to fusion : $6 \times 10^{-4}\, M_\odot$ which is << $M_\odot$ (ii) Due to wind : $3 \times 10^{-4}\, M_\odot$ which is also << $M_\odot$

PROBLEM 4 (C&O Problem 11.10)

(a) From C&O Example 11.2.2 we know that,

$P = 140\ \text{N m}^{-2}$, $\rho = 4.9 \times 10^{-6}\ \text{kg m}^{-3}$

Also, at the base of photosphere we should have,

$g = G\,M_r / r^2 \simeq G\,M_\odot / R_\odot^2 = 272\ \text{m}\cdot\text{s}^{-2}$

$\therefore H_P = \dfrac{P}{\rho g} \simeq 10^5\ \text{m} = 100\ \text{km}$ (using C & O Equation 10.70)

(b) Mixing length $= \mathcal{L}_M = 2.2\,H_P \simeq 220$ km
From secion 11.2 of C & O we know that the measured Doppler velocity of solar granulation is,
$\overline{v}_D = 0.4$ km / s

$\therefore$ Time required by convective bubble to travel one mixing length is,
$t_c = \mathcal{L}_M / \overline{v}_D = 550\ \text{s} \simeq 9$ minutes

To calculate the lifetime of a typical granule, we use
$H_P = 100$ km (at the base of Sun ' s photosphere)
C & O Example 10.4 .3 $\Longrightarrow \alpha \simeq 1 \Longrightarrow \mathcal{L}_M = H_P = 100$ km
C & O Example 10.4 .3 $\Longrightarrow \overline{v}_c \simeq 50$ m / s
Therefore, the lifetime of a typical granule,
which is the amount of time needed for a convective eddy to rise and fall the distance of one mixing length (see C & O section 11.2 subsecion on Granulation), is $\approx 2\,\mathcal{L}_M / \overline{v}_C = 4000\ \text{s} \sim 1$ hour

PROBLEM 5 (C&O Problem 12.1)

(a) Apparent visual magnitude of a star without extinction

$m_V \to M_V + 5\,\text{Log}[10, d] - 5$ /. {$M_V \to -1.1$, $d \to 700$}

$m_V \to 8.12549$

(b) Apparent visual magnitude of star if lying behind the nebula

$m_V \to M_V + 5\,\text{Log}[10, d] - 5 + A_V$ /. {$M_V \to -1.1$, $d \to 720$, $A_V \to 1.1$}

$m_V \to 9.28666$

(c) Distance the star appears to be at:

$d \to 10^{\frac{m_V - M_V + 5}{5}}$ pc /. {$M_V \to -1.1$, $m_V \to 9.28666$}

$d \to 1194.9$ pc

Big difference!
Percentage error:

$$\frac{1194.9 - 720}{720} \times 100 = 66\ \%$$