Order-of-Magnitude Astrophysics

The real point of figuring out these questions is to obtain, and then to convey to the rest of the group, an understanding for the central physics issues and a feel for the numbers. The solutions are supposed to be estimates, not detailed calculations. You can look to the literature or ask colleagues if there is astronomical or physical information you need (e.g., how big is the Milky Way), but you should try to work out the numbers yourself. Some of the questions may not be well-defined or perhaps not have clear solutions; if so, you should come up with a better defined question relevant to the problem and answer that.

Weinberg/Gould Order-of-Magnitude questions

1. Is proper rotation of nearby galaxies observable?
2. An unfriendly alien uses advanced technology to cool all of the gas in the Sun to 0.01 degrees Kelvin (and keep it there). How long does the sun take to collapse? Does it collapse to a black hole? If not, at what radius does it stop?
3. What fraction of the light from an old stellar population comes from main sequence stars?
4. What galaxies dominate the luminosity density of the universe? What M/L trend would be required in order for low luminosity galaxies to dominate the mass density?
5. What is the internal temperature of an old disk white dwarf?
6. A ball of gas collapses under gravity to form sun. What temperature is it heated to?
7. What is the slope of the white dwarf sequence in a V vs. V-I color-magnitude diagram? Is this steeper or shallower than the main sequence?
8. A star like the sun is made out of phydrogen and phelium, which are exactly like hydrogen and helium except that they cannot undergo nuclear fusion. What is the star's luminosity? What is the timescale on which its luminosity and radius change?
9. What is the kinetic, optical, and neutrino energy of a Type Ia supernova?
10. What accuracy is required to detect extra-solar planets by astrometric wobble? By Doppler shift?
11. How big a detector is required to detect neutrinos from active galactic nuclei?

12. How many e-folds of inflation are required to solve the horizon problem?

13. How does the value of h ($=H_0/100$ km/s/Mpc) enter into estimates of galaxy masses, galaxy mass-to-light ratios, and dynamical estimates of Omega?

14. Could the energy in the cosmic microwave background have been produced by stars? (Use metallicity as a constraint.)

15. What is the optical depth of a globular cluster to self-lensing?

16. Calculate the mass of a galaxy assuming that it forms from gas cooling in a Hubble time in an isothermal dark halo of circular velocity v_c.

17. Derive the α^3 dependence of Bremmstrahlung emission.

18. How do you find white dwarfs in a proper motion survey?

19. Show that if extreme scattering events are caused by self-gravitating neutral clouds, then these clouds are likely to be a significant fraction of the dark matter. [Based on Walker & Wardle, astro-ph/9802111]

20. How accurately can the globular cluster distance scale be determined using geometrical methods, with 30 orbits of HST time and appropriate supporting ground-based observations?

21. What is the ratio of the number of neutrinos emitted by the sun in its lifetime to the number of neutrinos emitted by a Type-II supernova?

22. Suppose that structure grows by gravitational clustering in an Omega=1 cosmology with an initial fluctuation power spectrum $P(k) \sim k^n$. If the halo mass function is $\phi_1(M)$ at time t_1, what is the halo mass function $\phi_2(M)$ at time t_2?

23. With diffraction limited speckle imaging at K-band from Keck, how many years of observation would be required to map the orbit of stars about the Galactic Center black hole and thereby determine the distance to the Galactic Center to within 10%?

24. Your starship accidentally gets transported into the middle of a globular cluster, and you have no control over the motion of the ship; it will continue in a straight line until you escape the cluster, unless you run into a star first. In order to assess your odds of escaping, you transport a lump of coal with an attached thermometer into empty space. The temperature of the lump of coal drops rapidly at first, then more slowly, asymptoting at a temperature of 25K. What are your odds of escaping the cluster without hitting a star?

25. The central surface brightness of a globular cluster is 17 mag/arcsec^2. What is the covering fraction of stars?

26. A globular cluster has 10^5 stars and a 1-d velocity dispersion of 5 km/s. What is the covering fraction of stars?

27. In the intergalactic medium of a group with line-of-sight velocity dispersion of 200 km/s, what are the lightest atoms that will not be fully ionized?

28. The Space Interferometry Mission (SIM) will require a grid of about 4000 stars over the sky, brighter than 12th magnitude, that are astrometrically stable to an accuracy of 4 micro-arcseconds over the five-year mission (i.e., their motions can be described by annual parallax and straight-line proper motions to within 4 micro-arcseconds). What type of stars should be used for the SIM grid?

29. What is the minimum black hole mass for a quasar of luminosity L? How long does it take to form such a black hole by accretion, if the radiative efficiency is ϵ? What quasar lifetime can be inferred by comparing the black hole population in present day galaxies to the peak space density of high-z quasars? [Based on Richstone et al., astro-ph/9810378]

30. How big is a meteor that produces a shooting star of apparent magnitude -6? [Suggested by H.-W. Rix.]

31. You observe an elliptical galaxy in 2" seeing. At a radius where the mean surface brightness is $\mu = 19$ mag/arcsec^2, the rms fluctuations about the mean surface brightness are $\Delta\mu = 0.01$ mag/arcsec^2. What is the distance of the galaxy?

32. Two identical twins, with identical, perfectly accurate, initially synchronized watches, travel from Columbus to Los Angeles. One of them takes an airplane, the other one walks. When they meet up in Los Angeles, what is the difference in time between their watches?

    Suppose that instead of flying, the first twin also walks (and rows) to Los Angeles, but goes around the world in the long direction. What is the difference between the watches in this case?

33. In hydrodynamic simulations of structure in the high-redshift ($z \sim 2$–4) universe, the diffuse, photoionized, intergalactic gas with density less than about 10 times the cosmic mean exhibits a tight correlation between density and temperature. Long after the gas has been reionized, this relation approaches a power-law, $T \propto rho^{\alpha}$. What is the value of α?

34. The main interferometer of the Space Interferometry Mission will consist of two mirrors, each 0.3m in diameter, separated by 10m. (SIM contains other mirrors as well, but they are used for other purposes, like alignment.) What will the point-spread function of SIM look like? How accurately can SIM measure the position of a 12th-magnitude star in a 60-second exposure?

35. An evolving brown dwarf (with mass below the $0.08 M_\odot$ limit for igniting steady hydrogen fusion) follows a track of approximately constant surface temperature T in an HR diagram. Taking this surface temperature as given, derive a formula for the luminosity of a brown dwarf as a function of mass and time. When necessary, you can assume that deuterium will be fused to helium-3 at a temperature of $\sim 10^6$ K; how long does the deuterium burning phase last? (Recall that brown dwarfs are fully convective.)

36. It has been proposed that the dark matter is made of superheavy charged particles, with mass $\sim 10^6$ GeV: CHIMPS. Suppose that CHIMPS captured by the envelope of a massive red giant accumulate in the post-supernova neutron star. Describe the subsequent evolution of the CHIMP population.

    (See Gould 1990, Phys Lett B, 238, 337)

37. Consider a star near the Chandrasekhar mass limit, in which particles of mass m_1 are supported by the degeneracy pressure of particles (similar in number) of mass m_2. (e.g., for a carbon white dwarf, m_1 is the mass of a carbon nucleus and m_2 is the mass of an electron; for a neutron star m_1 and m_2 are both the mass of a neutron.) At an order-of-magnitude (not factor-of-two) level, show that the number of particles in the star is $N_p \sim (m_p l/m_1)^3$, where m_{pl} is the Planck mass. At the same level of approximation, estimate the radius of the star. Why do neutron stars and white dwarfs have similar masses but radically different radii? Show that if $m_1 = m_2$ (as in a neutron star), a Chandrasekhar-mass object is always near the border of collapse to a black hole.

38. What is the best observational upper limit that Isaac Newton could have put on the amount of uniformly distributed dark matter within the earth's orbit, if he had considered this an issue worth his attention?

39. A supernova of energy E explodes in a medium of density rho. What is the radius R of the supernova blast wave as a function of time t? What are the conditions required for your solution to be valid? Now suppose that energy E is released at time t_i in an expanding universe, with Omega=1. What is R(t) once $t \gg t_i$? How much blast energy would be required to "blow a bubble" of radius 10 Mpc, comparable to the size of garden-variety cosmic voids? What is the efficiency E/Mc^2 required to create a cosmic void of radius R? Do the energetic requirements for creating large scale structure by explosions seem reasonable or unreasonable?

40. An elliptical galaxy with a stellar velocity dispersion of 200 km/s and a supermassive central black hole (1e9 Msun) merges with a smaller elliptical galaxy with a smaller supermassive black hole (1e8 Msun). The smaller supermassive black hole is dragged in towards the center of the larger galaxy by dynamical friction. Do the two black holes get close enough

to radiate away their remaining orbital energy in gravity waves, or does the process of bringing them together by dynamical friction get hung up at some larger radius? If it does get hung up, at what radius does this happen, and why?

41. Calculate, from first principles, the radius and luminosity of a 2 solar mass, main sequence star. (No scaling to the sun, no empirical mass-luminosity relations, etc.) You can assume that the dominant opacity source is Thomson scattering by free electrons. Approximate the temperature dependence of CNO nuclear energy generation as zero below T = 2 x 107 K and extremely steep above this temperature.

42. A "soft" binary in a star cluster is a gravitationally bound binary whose orbital speed is smaller than the cluster's velocity dispersion. According to "Heggie's Law," hard binaries in a star cluster get harder, and soft binaries get softer and eventually get disrupted; this behavior is a consequence of the negative specific heat of a self-gravitating system. (A system with positive specific heat, by constrast, evolves towards a state in which all degrees of freedom have energy kT/2.)

    How many soft binaries are there in a globular cluster (on average)?

43. Over the history of a typical galaxy, what is the ratio En/Eg of the amount of nuclear energy released to the amount of gravitational potential energy released?

44. How much U-235 is required to make an atomic bomb?

    How much energy does it release, in kilotons?

    Why was the yield of the Hiroshima bomb (20 kilotons) so low?

45. Observations of X-ray emission and the Sunyaev-Zel'dovich effect (distortion of the cosmic microwave background spectrum by electron scattering) from a cluster of galaxies can be used to determine the distance to the cluster, and hence to measure the Hubble constant. How? What are the implicit assumptions of this approach, and how would the breakdown of these assumptions influence the result?

46. The Luyten proper motion survey covered 2/3 of the sky to a magnitude limit V=18 and detected all objects with proper motion 0.2 "/yr ¡ mu ¡ 2.5 "/yr. The Hubble Deep Field reaches magnitude V=28 over an area 3 x (80")2. If the Galactic halo contains a population of old white dwarfs, which of these surveys is more sensitive for detecting them, and by what factor?

47. In 1994, VLBI studies of the source GRS 1915+105 showed a blob of radio emission that moved 0.5" in 4 weeks. The distance to GRS 1915+105 is 12 kpc. What is the minimum possible space velocity of this moving blob? If the blob has mass m, how much energy was required to accelerate it to this velocity?

(See Mirabel & Rodriguez 1998, Nature, 392)

48. The FAME (Full-Sky Astrometric Mapping Explorer) satellite mission is expected to last 2.5-5 years and to have an astrometric precision of roughly 17, 60, and 150 micro-arcseconds respectively for sources with apparent magnitudes V = 9, 12, and 15. How many extra-solar planets will FAME find?

49. A cloud of primordial composition (hydrogen/helium) gas collapses to form a disk galaxy like the Milky Way. In order to do so, the gas must dissipate energy by radiative cooling. How much cooling radiation is released during this galaxy formation process, and at what wavelengths is it released?

50. A planet has a ring composed of chunks of ice a few meters in size. The ring has radius r and orbital period P. It is very flat, with vertical thickness h much smaller than radius r, and it is very thin in the radial direction, with radial range delta-r=fr much smaller than r. The optical depth of the ring is tau. If the central planet is the only significant source of gravitational forces, how long can the ring remain this thin?

    Evaluate the result for the parameters r=1010cm, P=104s, f=0.01, tau=10, h=104cm.

51. Roughly a dozen Kuiper Belt Objects (KBOs) have been found in searches for distant supernova, which are sensitive to about 22nd magnitude. From this information, and the fact that the first KBO was discovered by Tombaugh in 1930, estimate the logarithmic slope of the KBO mass function and the total mass of the Kuiper Belt.

52. What is the angular size of the first acoustic peak in the cosmic microwave background anisotropy power spectrum? Assume an $\Omega_m = 1$ universe with very small Ω_b.

    How big does Ω_b have to be to have a significant effect on the result? How does the result change if $\Omega_m < 1$ and the universe is flat? How does the result change if $\Omega_m < 1$ and the universe is open?

53. What observational constraints can be placed on the amount of solid material (not dust, but chunks of rock or ice) ejected by a typical stellar system over its lifetime? Focus first on the size scale R 10 km, then consider other sizes.

54. A 15 solar mass, main sequence, Population II star with metallicity Z=10-4 has a luminosity 104.4Lsun and an effective surface temperature of 104.6K. What are the luminosity and surface temperature of a 15 solar mass, main sequence, Population III star? What are the implications of this result for ionization of the intergalactic medium?

    Assume that the Population III star has a core metallicity of Z=10-10 (it can enrich itself to this metallicity through triple-alpha reactions even if it

starts with no metals) and that both the Pop II and Pop III stars generate most of their energy by the CNO cycle, whose efficiency is approximately proportional to T12 in the relevant temperature range.

(See Tumlinson & Shull 2000, ApJ 528, L65.)

55. A black hole of mass M accretes gas at a rate Mdot. The gas forms a geometrically thin, optically thick, steady-state accretion disk. What is the total luminosity of the disk? What is the radial temperature profile? What is the frequency distribution of the emitted radiation?

56. Do the tidal interactions between the moon and the earth make the moon's orbit more eccentric over time or more circular? What about Triton (which has a retrograde orbit around Neptune)? For Io, resonance with Europa tends to make the orbit more elliptical, but the orbit is nonetheless very close to circular. Why are Io-Jupiter tidal interactions very different from earth-moon interactions, and what is the effect on Io?

57. Using the virial theorem, describe the behavior of a spherical gas cloud as the pressure Pext of the confining external medium is steadily increased. In a molecular cloud with T=10K, nH2=100 cm-3, and velocity dispersion sigma=1 km/s, what is the characteristic mass of gravitationally unstable clumps?

58. What observational factors determine the accuracy of mass measurements from visual binaries and from eclipsing binaries? Nearly all mass determinations for high-mass stars are from eclipsing binaries, and nearly all mass determinations for low-mass stars are from visual binaries. Why?

59. If you devoted a 1m telescope to searching for planets around G stars via transit detection, how many Jupiters could you find? How many Earths? What would be the main limitations for such a program?

60. What would be the radio surface brightness of a galaxy composed of a population of "normal" stars, with no interstellar medium? Which stars would dominate the radio emission?

61. Does a 15 solar mass star release more energy during its stellar lifetime or when it explodes as a supernova? Which form of energy release is likely to have more impact on the surrounding interstellar medium.

62. During the lifetime of the Sun, what is the probability of having an encounter with another star close enough to perturb the earth from its orbit? What if the Sun were in a globular cluster?

63. Analyses of X-ray observations of hot gas in galaxy clusters show that the gas is reasonably close to hydrostatic equilibrium based on its thermal pressure. Given this fact, derive an upper limit on the strength of intergalactic magnetic fields in the cores of galaxy clusters.

64. What fraction of the infrared photons emitted by a z=5.8 quasar make it to the earth?

65. If the earth were kicked out of the solar system by Jupiter and orbited on its own around the Galaxy, what would its surface temperature be?

66. An explosion of energy E goes off in the middle of a star with density profile $\rho(r) = \rho_0(r/r_0)x$ for $r > r_0$. For what values of x does the resulting blast wave shock accelerate rather than slow down, at late times (when $R \gg r_0$)? Assume that radiative losses are negligible.

67. By studying residuals about the Tully-Fisher relation, Courteau & Rix (1999, ApJ, 513, 561) find that partial(V)/partial(Rexp)—L – the partial derivative of the disk rotation speed V (at 2.2 scale lengths) with respect to the exponential scale length Rexp for galaxies of fixed luminosity – is -0.2 for unbarred, late-type spiral galaxies. If dark halos made a negligible contribution to V, what would this derivative be? What can one say about the relative contributions of disks and halos to the rotation velocity from the measured value?

68. The quantity $\lambda = J|E|^{1/2}\mathrm{G}^{-1}M^{-5/2}$ is a dimensionless measure of the angular momentum of a self-gravitating system. Suppose that a dark matter halo of radius R has $\lambda = 0.05$ (gravitational perturbation theory and numerical simulations imply this is a typical value acquired through tidal torques). If the baryons in the halo dissipate their energy while conserving their angular momentum, what will be the size of the resulting disk?

69. Suppose that the absolute magnitude of a white dwarf as a function of its age t is MV=F(t). If the initial mass function of a star cluster is a power law dN/dM = KMx, what is the white dwarf luminosity function dN/dMV of the cluster when the cluster age is t0?

70. The white dwarf cooling curves of Richer et al. (2000, ApJ, 529, 318) are very roughly described by MV = 17.5 + 0.3(tgyr-12), for tgyr (the age in Gyr) greater than 3. Sketch the white dwarf luminosity function of a globular cluster of age t0=12 Gyr, assuming x=-2.35 (a Salpeter IMF). What is the absolute magnitude of a white dwarf whose main sequence progenitor was twice the current turnoff mass?

71. What might one hope to learn about the slope of the IMF from the white dwarf luminosity functions of globular clusters?

72. What is the density profile rho(r) of a red giant envelope, in the range rsh ¡ r ¡¡ R, where rsh is the radius of the hydrogen burning shell and R is the radius of the photosphere?

73. A galaxy initially has a constant density core with all stars on nearly circular orbits. A black hole of mass M_{H} grows in the middle of the

galaxy on a timescale that is long compared to the central orbital time. What is the final density profile near the center of the galaxy?

74. If galaxies have central black holes of mass MH=1.5e8(sigma/300 km/s)4, what is the angular scale where this approximate calculation applies for a galaxy at 5 Mpc?

75. The star HD 209458 has a transiting planet with a period of 3.5 days. The extremely precise light curve allows determination of the start of the eclipse to a precision of a few minutes.

76. Suppose there is another planet in the system, at larger orbital radius, with an orbital plane inclined to that of the transiting planet by an angle i. What would be the period of precession of the transiting planet's orbit induced by the second planet? Would this effect be detectable?

77. A proto-star of radius R* and surface temperature T* is surrounded by an annulus of dust of total mass Md at orbital radius ad. What is the ratio of flux from the dust to flux from the star at wavelengths Lambda ¿ Lambda* = hc/(kT*)?

78. What is the radius of the Oort comet cloud?

79. What is the bolometric mass-to-light ratio of a stellar population of age t ¿ 1 Gyr, as a function of t?

80. What is the intensity of the cosmic background of Lyman-alpha radiation "scattered" fluorescently by the Lyman-alpha forest?

81. What is the mass of a black hole that evaporates via Hawking radiation over the age of the universe?

82. If the cross-section of the slow step of the proton-proton chain increased by a factor of 100, how would the luminosity, radius, and lifetime of the sun be different?

83. If the Kepler mission can detect transits of earth-like planets, can it detect (non-transiting) hot jupiters?

84. How long does it take a photon to escape from a 2-solar mass star? What are the implications of this result for the stellar mass-luminosity relation?

85. How does the luminosity of the giant branch change with time? How does it compare to the luminosity of the main sequence?

86. How old was the universe when deuterium formed?

87. The globular cluster M13 is barely visible as an extended blob to the naked eye in a dark site. If you traveled in a rocket at 10,000 km/s aimed at the edge of the cluster, what are the chances that you would hit a star on your way through? How does the answer change if your velocity is 100 km/s? 10 km/s?

88. Assume that the average metallicity of baryonic matter in the universe today is 1/10 solar. Predict the mean bolometric surface brightness of the extragalactic sky, in $L_\odot$/Mpc^2/sr and in mag/arcsec^2.

89. How many stars are ejected from a globular cluster by interactions with binaries, over its lifetime?

90. Assume that the UV background is produced by quasars that have a space density 10 per square degree per unit redshift. What is the range of the proximity effect in the Lyman-alpha forest?

91. You observe an X-ray cluster and find that its spectrum implies a gas temperature kT=8 keV. The surface brightness profile is flat to a radius r0, then proportional to 1/r3 out to radius R, with central intensity I0 = 3e-4 erg/s/sr/cm2, R=2.5 Mpc, and r0=0.1R. The Bremmstrahlung emissivity law is approximately Lambda=2.5e-23 sqrt(T/108K) n2 erg/cm3/s, where n is the hydrogen number density.

    What is the cluster velocity dispersion? What is the cluster's total mass? What is the ratio of baryon mass to total mass? What is Omegam? What is the central cooling time?

92. Suppose that the value of G were larger by a factor of two. What would the luminosity and radius of the sun be?

    You can use the observed scalings of main sequence stars, R M0.7 and L M3.5.

93. The WMAP team reports: $\Omega_b = 0.045$, $\Omega_m = 0.28$, h=0.7, and an optical depth of 0.17 to Thomson scattering. At what redshift was the universe reionized?

94. The estimated bolometric surface brightness contributed by all AGN in the observable universe is I 4 nW/m^2/sr = 4e-6 erg/s/cm^2/sr. Predict the present day density (in Msun/Mpc^3) of supermassive black holes.

95. What fraction of main sequence stars in a proper motion survey with limits mu=0.2"/yr, V=19, are halo stars? What fraction of the white dwarfs are halo stars?

96. A 1 solar mass star reaches the main sequence turnoff with a 0.1 solar mass helium core. During the subsequent red giant evolution, what fraction of the energy is produced by hydrogen fusion, what fraction by helium fusion, and what fraction by gravitational contraction?

97. If a 15 solar mass star ejects 3 solar masses of metals and explodes as a Type II supernova, what is the ratio of its gravitational energy release to its nuclear energy release over its lifetime (including the supernova)?

98. What is the lowest mass (non-transiting) planet that could be detected by 3 years of dedicated 1-meter telescope observations of a V=8 mag eclipsing binary with a 1-day period?

99. How many Type Ia supernovae at z=0.5 are needed to distinguish dark energy with an equation of state parameter w=-0.9 from a true cosmological constant (w=-1)?

100. Assume a flat universe with $\Omega_m = 1/3$ and that Type Ia supernovae have an rms dispersion of 0.1 magnitude in peak luminosity. We would like a 3-sigma detection of the difference.

101. The minimum mass solar nebula has a surface density profile of solid material Sigma 30 g/cm^2 (a/1 AU)$^{-3/2}$. How long does it take a "planetary embryo" of mass $M \sim 10^{25}$ g to double in mass at a = 1, 5, and 20 AU?

     Assume solid material has rho = 1 g/cm^3 and approximate pi=1.

102. Assume that the first star in some large region of the universe forms in a halo of 10^6 Msun at z 20 containing 2e5 Msun of baryons, and that the star has a mass of 100 Msun. The halo initially has a profile $\rho \sim r^{-2}$ within the virial radius of 170 pc (giving it a mean baryon number density of 0.3 cm^{-3}). Will this star alone be able to ionize the gas in its halo and emit its ionizing photons to the intergalactic medium, assuming no other stars form in the vicinity during the star's lifetime?

103. Give an upper limit for the mean mass of comets that have been ejected by G stars in the Milky Way.

     What is the ratio of this limit to the mass of objects in the Kuiper Belt?

104. How long until the Earth "stops rotating" (more precisely, becomes tidally locked with the Moon)?

     What can be done to stop it?

105. Imagine we could resolve the angular size of a Gamma-ray burst occurring at redshift z, where the angular diameter distance is $D_a(z)$. If the fireball that is responsible for emitting the gamma-rays or the optical afterglow which we may be resolving is moving out from the central engine with a bulk velocity v, what is the apparent angular motion of expansion of the emitting source we would observe?

106. You discover an object in the ecliptic, at opposition, moving with angular speed of 1.4 arc-seconds per hour, with apparent visual magnitude of 18.8. What is its distance from the Sun, and what is a lower limit to its size? Hint: it is beyond Pluto.

107. Suppose the Milky Way dark matter consists of black holes with mass M less than Msun. At what values of M (if any) would one of these black holes have a noticeable effect if it were to hit the Sun?

108. Consider a gravitational potential of the form Psi = - GM/(r-rg). What is the innermost stable circular orbit in such a potential?

109. If a parcel of gas of mass Delta-M spirals in to 3rg as part of a thin viscous accretion disk, then spirals from 3rg into the event horizon with no further interactions, how much radiation does it emit?

110. A proto-star is surrounded by an annulus of dust of total mass Md at radius ad. If the stellar ratius and surface temperature are R* and T*: What is the temperature of the dust? What is the ratio of flux from the dust to flux from the star at wavelengths lambda ¿ lambda* = hc / kT*?

111. Can you detect the acceleration of a pulsar by the gravitational field of a globular cluster?

112. At what mass does a moon become spherical?

113. How many core-collapse supernovae can the LBT find per square degree?

114. How far away can Keck detect Jupiter-like extra-solar planets?

115. If the star formation history of the Universe had been different, would it be possible for the brightness of the sky at night to equal the brightness of our observed daytime sky?

116. 1. You observe the central regions of a nearby galaxy with an X-ray telescope, whose effective area is $A_{\rm eff}$. The galaxy distance is d, and your exposure time is t = 1 ksec. The X-ray photons are counted in pixels, which are large compared to the telescope PSF. You collect an average of 100 counts per pixel, and there are no systematic trends across your field of view. However, the standard deviation from pixel to pixel is 20 counts. If the X-ray photons are produced by a population of sources each with luminosity L, what is L in photons/sec?

117. 2. With a t=10 ksec Chandra exposure, you detect $N_\gamma = 100$ distributed over an area in the Galactic plane, but because of Chandra's high angular resolution you can definitively say that no two of these photons came from the same source. Assuming that the photons are produced by sources of luminosity L that are all at essentially the same distance d, what limit can you put on the value of L, at 95% confidence?

118. What is the gravitational microlensing optical depth of a self-gravitating stellar system? What is the typical timescale of events? What is the event rate (per source star)? What is the planetary microlensing event rate (per source star)? What is the total planet event rate? How many planets could be detected given reasonable resources?

119. 1. A mass of gas is converted into stars with a Salpeter IMF, dN/dM M-x with x=2.4, truncated at a minimum mass Mmin=0.1Msun. After

10 Gyr, what fraction of this mass has been "recycled" to the interstellar medium?

2. What is the mass of metals ejected by this population, relative to the mass that is locked up in main sequence stars and stellar remnants? This ratio is called the yield, y. Estimate (very roughly) the yield from Type II Sne, then guess at the relative importance of other processes.

3. Suppose that this population of stars forms from initially primoridal gas (zero metallicity) and that recycled gas is formed into new stars. What is the gas phase metallicity Z at the time that the gas fraction (gas mass / total mass) has fallen to Mg/M0=0.1? Assume that no gas enters or leaves the system (the "closed box" model) and make the "instantaneous recycling" approximation that mass and metals are ejected as soon as stars form, with no time delay.

120. Cluster Weak Lensing

1. The formula for the Einstein angular radius for a mass M is $\theta_e = sqrt[(4Gm/c2)(DLS/DLDS)]$. What is the Einstein angular radius of a 1015 Msun cluster at z=0.1, lensing background galaxies at z=0.2? How does this compare to the angular size of the cluster, assuming it has a 2 Mpc physical radius?

2. A source at an angular distance $x\theta_e$ from a point lens is deflected to an apparent angular distance $x_1\theta_e$ where $x_1 = f(x) = [sqrt(x2 + 4) + x]/2$, where θ_e is the Einstein ring radius. Consider a circular galaxy whose center is $x\theta_e$ from the cluster center and whose angular radius is $r\theta_e$. What is the axis ratio of the lensed galaxy?

3. How many background galaxies per square arcminute are required to measure the cluster mass with a signal-to-noise ratio of 5:1?

121. Detecting Gravity Waves from Black Hole Binaries

A binary supermassive black hole in another galaxy has an orbital period of one year. Out to what distance D could the gravitational radiation from this binary be detected with a timing array of 100 millisecond pulsars? (Consider just a 1-sigma detection threshold, and assume that average pulse arrival times can be measured to a precision of 1 microsecond.)

122. The Transverse Proximity Effect

You observe a quasar Q2 with redshift z 3 at a transverse distance r from the line of sight to a background quasar Q1. You obtain a high resolution spectrum of Q1 and measure the Lya forest. What is the expected (specifically, the median) value of the Lya optical depth at the redshift of Q2, relative to the median value tau-med-u in randomly selected selected pixels (not near foreground quasars) at this redshift? Case 1: Assume that quasars live in random environments. Case 2: Estimate the effect of quasar environment assuming that the correlation length of quasars is the same as the present day correlation length of dark matter.

123. SIM Sensitivity to Long-Period Planets

1. What is the lowest mass planet that SIM can detect at 5-sigma significance from 25 1-d measurements in each direction, each of 1 micro-arcsec precision, assuming a face-on circular orbit and a period P less than Delta-t, where Delta-t is the (10-yr) mission lifetime?

2. What if P is much greater than Delta-t?

124. The Tully-Fisher Relation and Its Residuals

Assume that disk galaxies form when a fraction $m_d = M_d/M_h$ of the halo mass settles into an exponential disk of scale length $R_d = \lambda R_h/2$, where the mean density inside the halo radius R_h is $200\rho_{crit}$ and the halo density profile is that of a singular isothermal sphere. Take lambda = 0.03-0.12 with a median value of 0.06. What is the baryonic Tully-Fisher relation, $V_{2.2}$ vs. M_d, where $V_{2.2}$ is the disk rotation speed at 2.2 scale lengths, and what is the correlation between TF residual $\Delta V_{2.2}$ and mass-radius residual $\Delta \log R_d$? Approximation 1: Ignore disk gravity entirely. Approximation 2: Include disk self-gravity, but ignore the effect of the disk on the halo. Approximation 3: Include a correction for the influence of baryons on the halo profile, assuming that dark matter particles are on circular orbits and that when the disk forms their orbits contract but preserve their angular momentum.

Other Order-of-Magnitude questions

Initials in [] indicate who devised each question.

1. Small-scale cutoff in $P(k)$: At what distance scales (wavenumbers k) is the primordial power spectrum $P(k)$ cut off by the end of inflation? At what distance scales is $P(k)$ suppressed by kinetic decoupling of WIMPs? What about axions? [MK]

2. SMBH masses: What is the maximum possible mass for a supermassive black hole at a redshift $z \sim 6$? [MK]

3. High-energy neutrino astrophysics: What is the lowest energy at which an astrophysical neutrino can be detected in IceCube? What is the largest energy? [MK]

4. Observational BBN: What is the smallest D/H that could have been detected in a Keck QSO spectrum? [MK]

5. Galaxy formation: What is the natural size for a galaxy disk in a $10^{11} M_\odot$ galactic halo? [MK]

6. UHECRs: The trajectory of an ultra-high-energy cosmic ray (UHECR) is deflected by the Galactic magnetic field. Above what UHECR energy does the UHECR point to within one degree of its source? What is the UHECR energy (the GZK bound) above which the UHECR can scatter from a CMB photon to produce a pion? What is the mean-free path of higher- energy UHECRs? How many AGN are there within this distance? And how many galaxy clusters? What is the deflection angle, in the Galactic magnetic field, of UHECRs above the GZK limit? and how does that compare with the mean spacing of potential sources? If super-GZK UHECRs came from AGN, how many UHECRs would have to be detected to discover a 3sigma correlation with AGN? or with clusters? [MK]

7. Stellar populations: How many stellar-mass black holes are there in the Galactic disk? [MK]

8. Cosmological B fields: What is the largest Faraday rotation of the CMB due to IGM magnetic fields? Suppose we postulate some magnetic field in the early Universe (at the time of recombination)? What is the largest Faraday rotation from these fields? Is this detectable with Planck? [MK]

9. Maximum halo DM mass: Suppose a very massive black hole is on an orbit in the Milky Way halo that takes it through the Milky Way disk. Estimate the dynamical-friction timescale for such an object to inspiral to the Galactic center. Assume the thing starts at a distance of 10 kpc. What is the largest mass that might still be circling in such an orbit today? [MK]

10. Al26: Aluminum-26 nuclei are produced in stars and then injected into the Milky Way IGM. These undergo a nuclear decay that emits a 1.808 MeV gamma ray. Such gamma rays have been observed throughout the Milky Way with gamma-ray telescopes. Estimate the total flux of such gamma rays from the Milky Way. How should these gamma rays be distributed? [MK]

11. Diffuse backgrounds: Estimate Omega (the fraction of critical density) of diffuse radiation backgrounds in radio, IR, optical, and gamma rays. [MK]

12. Suppose there is some mechanism that produces cosmological magnetic fields before recombination. What is the smallest post- recombination ionization fraction required to preserve these magnetic fields? What is the smallest scale below which magnetic fields get smoothed by diffusion? How do these answers depend on the magnitude of the magnetic field? [MK]

13. Estimate the amplitude of the stochastic gravitational-wave background at LISA frequencies from merging SMBHs, white-dwarf binaries, and NS-NS binaries. [MK]

14. What are the gravitational-wave aplitudes and frequencies that can be probed with pulsar timing? [MK]

15. Galactic astronomy: Derive the fundamental plane relating the half-light radius R, the surface brightness I, and the stellar velocity dispersion σ of elliptical galaxies. [AP]

16. Supernovae may eject all of the gas from a galaxy, if the galaxy is small enough. Estimate the critical galaxy mass below which the gas is fully ejected. [MK]

17. Estimate the cooling time of gas in hydrostatic equilibrium in a galactic halo of mass M. Above what mass M is the cooling time longer than the age of the Universe? [MK]

18. If there are large extra dimensions, gravitons may be radiated into the extra dimensions, leading to an extra source of cooling in a supernova. Find the bound to the size of these extra dimensions, assuming two extra dimensions, that arises from SN1987a. [Ref. arXiv:hep-ph/9903422]. [MK]

19. Almost all the energy of a core-collapse supernova is processed to MeV neutrinos. They accumulate and form diffuse background in the present universe, as they are unabsorbed by anything. Estimate the energy density of these neutrinos, and compare it with energy densities of other cosmic backgrounds such as CMB, CIB, CGB. [SA]

    Suppose that these neutrinos are monoenergetic, say 5 MeV, estimate the detection rate at the Super-Kamiokande detector (for now neglect all the backgrounds such as solar, atmospheric, reactor neutrinos).

20. GLAST launched recently. What is the flux sensitivity of LAT onboard GLAST to a steady point source, as a result of one-year all sky survey? Note that it is limited by the diffuse gamma-ray background (not by the detector noise). Use the following GLAST-LAT specification: field of view 2.4 sr, effective area 4×10^3 cm^2, angular resolution 3 deg (for 100 MeV photons).

    Argue how the sensitivity scales if the target is a transient source (as in the case of GRB, SGR, etc.). [SA]