Chapter 9

Testing Universal Mass Functions

The concept of a universal function describing the distribution of stellar masses originated with the finding by Salpeter (1955) that the luminosity function in the solar neighborhood, adjusted for stellar evolution, translated into an "original" mass function which followed a simple power law, $\xi(M) \propto M^{-1.35}$. This issue was revisited in detail by Miller & Scalo (1979) and by Scalo (1986), who each proposed more complicated "initial" mass functions. [1] As stars in the solar neighborhood are 10^9 - 10^{10} years old and were formed over spatial scales of many kiloparsecs, the goal of these works was to construct the present day mass function and then correct this function for the star formation history of the Galaxy (a three-part correction comprised of the stellar birth rate, the stellar death rate, and the Galactic age) in order to derive the "initial" mass function. In effect, these authors assumed that the time- and space-averaged mass function they observed today derived from a universal mass function at some point in the past, for which they could solve.

The path we would like to take is the inverse of that above, namely, to assume our observed mass functions *are initial*, that is, coherent in time and/or in space, so that we can estimate one of the following quantities: 1) *the number* of independent time- and space-coherent star-forming events necessary to reproduce a universal mass function; 2) *the time scale* over which one must integrate a single space-coherent event in order to reproduce a universal mass function; 3) *the spatial*

[1] The Miller & Scalo representation is a three-power-law function which is flat at the low-mass end, while the Scalo representation is a continuous function which turns over at the low-mass end.

scale over which one must integrate a single time-coherent event in order to reproduce a universal mass function. With data on stellar masses in young clusters, we avoid the need for correction factors from present-day to initial mass functions and can ask, directly, the question of interest: Is the spectrum of masses produced by star forming regions uniform, on all spatial scales and over all time? Alternatively, is the star formation process bi-modal in mass, with certain regions biased towards producing higher-mass stars and others biased towards producing lower-mass stars?

Comparisons between universal mass functions and data in young star-forming regions have been carried out previously. In most cases these investigations have used the relative monochromatic infrared brightnesses of stars (*e. g.* K-band "luminosity" functions computed simply as histograms of *apparent* magnitudes) or their far-infrared and submillimeter "colors" (*e. g.* "bolometric" luminosity functions computed by inverting the wavelengths of peak emission to get characteristic temperatures) to perform these comparisons. However, until recently, none of these types of analyses in star-forming regions accounted for the effects of 1) significant and nonuniform interstellar reddening, 2) pre-main sequence luminosity evolution, and 3) excess emission arising in the circumstellar environment, The severe implications of these combined effects have begun to be realized and, one-by-one, they have gained consideration (*cf.* Strom, Strom & Merrill, 1993; Zinnecker, McCaughren, & Wilking, 1993).

Our approach is different from those presented in the literature to date in two regards. The first difference is that we attempt to estimate actual stellar masses (instead of using apparent brightness distributions), correcting or accounting for all of the following: 1) interstellar reddening, 2) pre-main sequence evolution, and 3) circumstellar excesses. We have described our methods of obtaining stellar masses in previous Chapters. For optically visible stars we use photometric and

spectroscopic information to determine intrinsic stellar fluxes and temperatures, with which we interpolate masses from theoretical tracks on HR diagrams (as described in Chapter 8). For optically invisible stars we use photometric information along with the cluster age derived from the optically visible population, to deredden stars and determine their masses (as described in Chapter 7). The second difference is that we compare our derived distribution of stellar masses to a *simulated* distribution of stellar masses drawn from a standard universal mass function (instead of trying to compare the observed and theoretical functions at a detailed level), as described next.

9.1 A Statistical Approach

Because of the effects of small number statistics in our sparse clusters (N = 5-40), any conclusions regarding the detailed shapes of observed stellar mass spectra are tentative. However, we reiterate that we have selected these regions based on the presence of stars with a mininum mass, $M > 3$ $M_{\odot}$. If other relatively massive stars are also present, these clusters could reflect either 1) random fluctuations from a standard universal mass function, or 2) a real mass bias in the star formation scenario for particular regions. To attempt to distinguish between these two possibilities, we compare our observed mass distributions to simulated clusters in which stellar masses are randomly drawn from a standard mass function. In this way we can assess the theoretical likelihood, under the assumption that mass spectra are universal, of observing any given mass distribution.

The method is to use an analytical form of the corrected solar neighborhood mass function to determine the integrated probability of finding a star within a particular mass range. This procedure is discussed below. Knowing these probabilities, we then employ monte carlo sampling to predict the distribution of

masses in a cluster with the same number of members as our sample cluster, over a large number of trials. As we have large uncertainties in our assignment of specific masses to individual stars (estimated at 30-100% when uncertainties in the observations and in the theory used to calibrate the observations are combined; see Chapters 7 and 8), we consider stellar masses integrated over bin sizes which are larger than our mass errors: from $0.1 - 1$ $M_{\odot}$ and from $1 - 10$ $M_{\odot}$. This aproach minimizes the effects of inherent uncertainties in the observed mass distribution. We present the results of our simulations in the form of a predicted probability distribution, defined as the ratio: [number of stars with mass $1 < M/M_{\odot} < 10$] to [number of stars with mass $0.1 < M/M_{\odot} < 1$]. Finally, we compute the fraction of cases in which the observed mass distribution is reproduced by the models. We thereby determine the likelihood that any particular star formation site has yielded a stellar mass distribution drawn strictly from a "universal" mass function.

The probabilities of finding stars within each of our mass bins can be calculated from the tabulations of Miller & Scalo (1979), and are recorded in Table 9.1. As shown by Scalo (1986), of the currently considered shapes for the initial mass function (derived under a range of assumptions regarding the stellar birth function), the Miller-Scalo initial mass function is that which produces the largest fraction of intermediate- and high-mass stars. In other words, numerically, the Miller-Scalo initial mass function predicts 1 of every 300 or so stars to have mass $M > 10$ $M_{\odot}$, while the prediction obtained from the opposite extreme of the birth function is only 1 of every 3000 or so stars to have mass $M > 10$ $M_{\odot}$. The implication for our simulation is that we create predicted mass distributions in which the highest possible number of intermediate- and high-mass stars are expected. This limit is important in that it strengthens any case in which we find a stellar mass distribution unusually skewed towards intermediate- and high-mass stars, since our computational bias is in the opposite direction.

Table 9.1. The Miller-Scalo Initial Mass Function

	$M > 10$ M$_\odot$	$1 < M/M_\odot < 10$	$0.1 < M/M_\odot < 1$
Fraction by Number	0.324%	13.801%	85.875%
Fraction by Mass	8.511%	49.033%	42.456%

We show in Figure 9.1 the probability distribution of the ratio of the number of intermediate-mass stars, N($1 < M/M_{\odot} < 10$), to the number of low-mass stars, N($0.1 < M/M_{\odot} < 1$), in 1 million random samplings of 33, 53, and 140 stars from the Miller-Scalo initial mass function. If our sample were to contain an infinite number of stars, this distribution would be sharply peaked at a value of 0.16, as can be seen from Table 9.1. However, our finite sample sizes introduce some breadth to the probability distribution; the distribution width increases as the number of stars in the sample decreases. How do the data compare to these predicted distributions of mass ratios?

9.2 Results of Comparisons to the Miller-Scalo IMF

The problem we pose with respect to the Ae/Be clusters is the following. If all stellar masses are drawn from a universal mass function, how many star-forming "events" which produce the number of stars in each our samples, produce a star with mass $M \gtrsim M_{most\ massive\ star}$? We can answer this question with our simulations, and for each of the "events"/trials in which we do find a single high mass star, we then ask how well the remainder of the stellar mass distribution matches the universal mass function. Thus, our approach to testing universal mass functions accounts for the fact that we have selected regions for study via the presence of a high-mass star.

We proceed by comparing theoretical to observed ratios of intermediate - to - low mass stars for the few Ae/Be stellar groups for which we have been able to construct optical HR diagrams. As noted previously, we use the HR diagram to derive masses for optically visible populations, as well as to estimate the ages which are applied in our infrared dereddening procedure for optically invisible populations. We present our results on mass distributions in the BD +40° 4124, Lk Hα 215, and NGC 7129 regions in Table 9.2, and discuss each of these regions

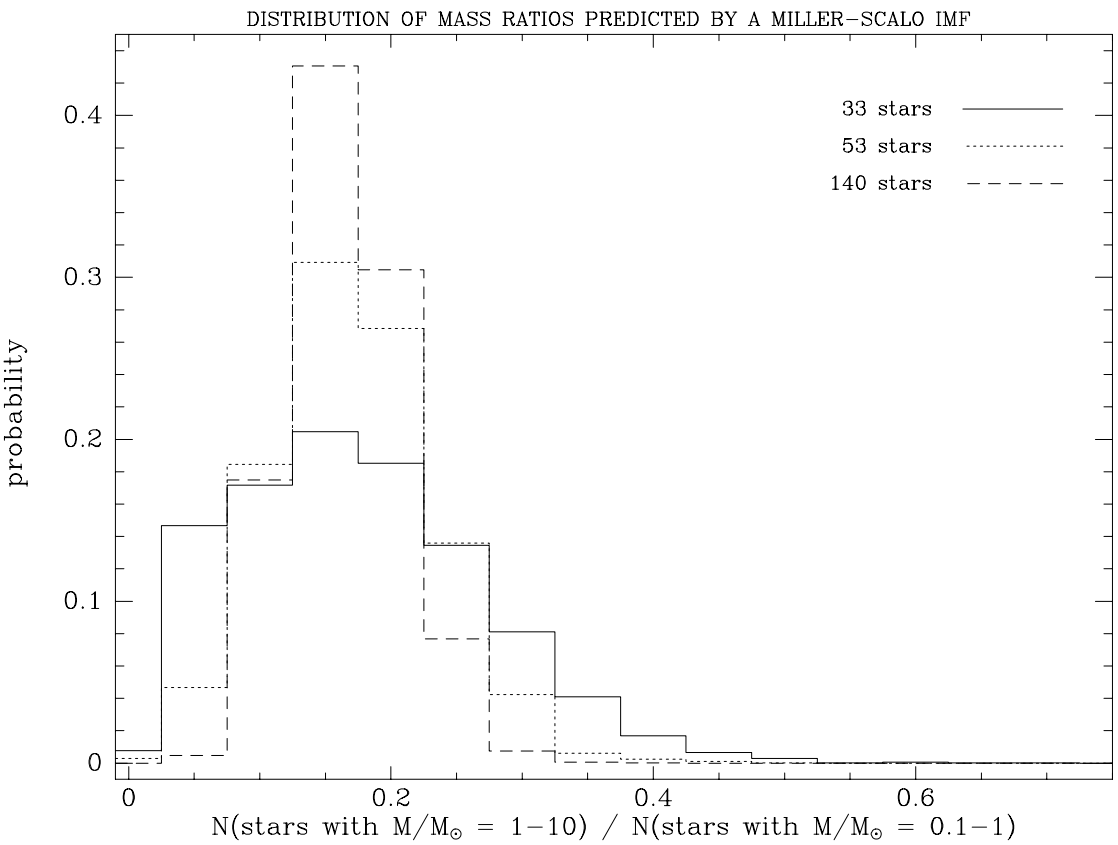

Figure 9.1. *Simulated predictions of the Miller-Scalo initial mass function for the ratio of intermediate- to low-mass stars.* A probability distribution of the ratio of intermediate- to low-mass stars ([number of stars with mass $1 < M/M_{\odot} < 10$]: [number of stars with mass $0.1 < M/M_{\odot} < 1$]) is computed from the Miller & Scalo (1979) initial mass function for sample sizes of 33, 53, and 140 stars. In the case of an infinite number of stars, the value of this ratio is 0.16 with a probability of unity. As the number of stars decreases the probability distribution broadens. We compare the probability distribution of mass ratios to the observationally derived in the Ae/Be clusters, in order to probe the consistency of all sites of star formation with universal mass functions.

individually below. For clusters without fully assembled HR diagrams, we are able to discuss the mass distributions on a qualitative level only.

For the BD +40° 4124 region, in the inner 0.167 pc we find 1 star with $10 < M/M_{\odot}$, 7 stars with $1 < M/M_{\odot} < 10$, and 14 stars with $0.1 < M/M_{\odot} < 1$. The ratio of intermediate- to low-mass stars is thus 0.50. From Figure 9.1, the likelihood of producing such a mass distribution from a Miller-Scalo mass function for a cluster of this size (N=22) is less than 0.003! As discussed by Hillenbrand, Meyer, Strom, & Skrutskie (1995), based on our photometric completeness limits in this region, we are confident that we include in our observed mass distribution all objects more massive than the hydrogen burning limit as long as they are extincted by $A_V < 10$ mag. Thus, our result is not explainable, for example, by the preferential detection of higher mass stars in our imaging survey. This region appears to be biased towards producing intermediate- and high-mass stars, those with masses $M > 1$ $M_{\odot}$. We do offer the caveat, however, that if star formation has not yet ceased in the BD +40° 4124 region, our results may simply reflect a sequence of star formation from high to low stellar masses.

For the Lk Hα 215 region, in addition to the B star, we find only sub-solar mass stars within the inner 0.167 pc. The ratio of intermediate- to low - mass stars is 0.07, perfectly consistent with the Miller-Scalo function for a cluster of this size (N = 15).

For the NGC 7129 region, despite the appearance from optical images that this cluster contains a large number of bright, intermediate-mass stars, there does not appear to be a bias in this region towards producing intermediate-mass stars. A near-full complement of low-mass stars also exists. Although the ratio of intermediate- to low-mass stars is slightly above the nominal value, at 0.28, it is still within the peak of the expected distribution in Figure 9.1 and therefore the apparent bias is not statistically sound.

Table 9.2. Observed and Predicted Ratios of Intermediate- to Low-Mass Stars

Ae/Be Star	N(> 10 $M_\odot$)	N($1 < M/M_\odot < 10$)	N($0.1 < M/M_\odot < 1$)	Expectation
BD +40° 4124	1	7	14	0.003
Lk Hα 215	0	1	14	0.175
NGC 7129	0	11	40	0.250

Table 9.2.

In summary, compared to the predictions of a Miller-Scalo mass function, we have identified one Ae/Be region with a significantly deviant mass distribution, one with a marginally deviant mass distribution, and one with an apparently normal mass distribution. What about the remainder of our sample? As can be seen from Table 5.1, there are 10 other Ae/Be clusters in which at least some classification spectra have been obtained. However, we are not prepared at the current time to discuss characteristic ages or masses in these regions to the same level of rigor as we have for the BD +40° 4124, Lk Hα 215, and NGC 7129 regions. Completion of our spectroscopic survey should enable us to do so. We can, however, make the following qualitative characterization of the sample: a majority of the optically visible companions to the Ae/Be stars appear to have spectral types later than K5 (see Table 5.1). This implies that they reside on convective pre-main sequence evolutionary tracks and are likely to have masses less than a solar mass. We note a difference in this regard, between the majority of the Ae/Be clusters and the BD +40° 4124 region, for which the distribution of optical spectral types is more heavily weighted towards earlier types. Turning to our infrared images, examination of the relative source brightnesses (see Figure 5.2) shows an obvious difference, as well, between the apparent magnitude distribution in the BD +40° 4124 region and those in other Ae/Be clusters. From the analysis of stellar masses completed to date, we conclude that the perceived difference in the distribution of spectral types and apparent magnitudes does reflect a real difference in the distribution of stellar masses. Thus our results in the BD +40° 4124 region, although robust, appear anomalous amongst Ae/Be sites. None of the other regions show a similar effect. In general, the stellar populations of the molecular cores defined by Ae/Be stars are comprised of single early-type stars surrounded by several late-type (> K5), presumably sub-solar mass, emission-line companions, with no other intermediate-mass in the vicinity.

Properly speaking, we have one and only one region in which the likelihood of finding the observed mass distribution is extremely small (0.3%), if all stellar masses are indeed drawn from the Miller-Scalo function. However, this statement does not translate directly into an assertion that there is only a 0.3% chance that the masses in this particular cluster were drawn from a standard mass function. In order to prove this latter statement, we would have to survey several hundred regions with the same selection effects as are inherent in our investigation of the BD +40° 4124 region, and find a statistically large number of them ($> 1/333$) to have the same apparent mass bias as does the BD +40° 4124 region.

9.3 Conclusions on Mass Spectra in Ae/Be Clusters

We have attempted to investigate the consistency of the distributions of stellar masses in the Ae/Be clusters with the Miller-Scalo mass function. In doing so, we have developed a statistical technique which does not require us to know the detailed shape of the observed stellar mass distribution (only the ratio of intermediate- to low-mass stars) and thus which accounts for the unavoidably large uncertainties in derived stellar masses. Our method accounts both for the selection effect that we have chosen star formation sites for study in which intermediate- and high-mass stars have already formed, and for the small number statistics in these regions. We conclude, based on our analysis, that while it is possible to find isolated star-forming sites in which the initial mass function is apparently biased towards high-mass stars (*e. g.* the BD +40° 4124 region), that in most cases the mass spectra are consistent with those predicted by universal functions, at least at the level to which our statistical approach allows us to address the problem.

Upon the accurate determination of stellar masses in *all* of the Ae/Be cluster sites in our sample, we hope to improve our statistical arguments above, and also to refine the level at which we address questions related to the initial mass

function. There are two outstanding questions: First, regarding the universality of the initial mass function, by comparing the results on mass distributions in our sample of Ae/Be regions with each other, we can ask whether any significant differences exist in the mass spectra emerging from different, individual, isolated star-forming regions. We believe, based on the conclusions above, that the answer to this question is affirmative. Second, regarding the particular shape of the initial mass function, if universal, given that some differences appear to exist between the mass spectra output by similarly selected, isolated star-forming "events," by averaging all regions in our sample together to obtain an integrated mass distribution, we can ask how many of these independept time- and space- coherent star-forming "events" are necesary in order to reproduce a universal mass function?