ULTRAVIOLET OBSERVATIONS OF SUPERNOVAE

Nino Panagia

STScI, Baltimore, USA; panagia@stsci.edu INAF - Catania Observatory, Italy Supernova Ltd., Virgin Gorda, BVI

Abstract. The motivations to make ultraviolet (UV) studies of supernovae (SNe) are reviewed and discussed in the light of the results obtained so far by means of IUE and HST observations. It appears that UV studies of SNe can, and do lead to fundamental results not only for our understanding of the SN phenomenon, such as the kinematics and the metallicity of the ejecta, but also for exciting new findings in Cosmology, such as the tantalizing evidence for "dark energy" that seems to pervade the Universe and to dominate its energetics. The need for additional and more detailed UV observations is also considered and discussed.

Keywords: Supernovae: general, Ultraviolet: stars, Binaries: general, Cosmology: miscellaneous **PACS:** 97.60.Bw, 97.80.-d, 98.80.-k

1. INTRODUCTION

Supernovae (SNe) are the explosive death of massive stars as well as moderate mass stars in binary systems. They enrich the interstellar medium of galaxies of most heavy elements (only C and N can efficiently be produced and ejected into the ISM by red giants winds and by planetary nebulae, as well as pre-SN massive star winds): nuclear detonation supernovae, i.e., Type Ia SNe (SNIa), provide mostly Fe and iron-peak elements, while core collapse supernovae, i.e., Type II (SNII) and Type Ib/c (SNIb/c), mostly O and alpha-elements (see below for type definitions). Therefore, they are the primary factors to determine the chemical evolution of the Universe. Moreover, SN ejecta carry approximately 10^{51} erg in the form of kinetic energy, which constitute a large injection of energy into the ISM of a galaxy (for a Milky Way class galaxy $E_{kin}^{MW} \simeq 3 \times 10^{57}$ erg). This energy input is very important for the evolution of the entire galaxy, both dynamically and for star-formation through cloud compression/energetics.

In addition SNe are bright events that can be detected and studied up to very large distances. Therefore: (1) SN observations can be used trace the evolution of the Universe. (2) SNe can be used as measuring sticks to determine cosmologically interesting distances, either as "standard candles" (SNIa, which at maximum are about 10 billion times bright than the Sun, with a dispersion of the order of 10%) or employing a refined Baade-Wesselink method (SNII in which strong lines provide ideal conditions for the application of the method, with a distance accuracy of $\pm 20\%$). (3) Their intense radiation can be used to study the ISM/IGM properties through measurements of the absorption lines. Since most of the strong absorption lines are found in the UV, this is best done observing SNII at early phases, when the UV continuum is still quite strong. Additional

studies in the optical (mostly CaII and NaI lines) are possible using **all** bright SNe. However, only combining optical and UV observations can one obtain the whole picture and, therefore, SNII are the preferred targets for these studies. (4) Finally, the strong light pulse provided by a SN explosion (the typical HPW of a light curve in the optical is about a month for SNIa and about two-three months for SNII; in the UV the light curve evolution is much faster) can used to probe the intervening ISM in a SN parent galaxy by observing the brightness, and the time evolution of associated light echoes.

2. ULTRAVIOLET OBSERVATIONS

The launch of the International Ultraviolet Explorer (IUE) satellite in early 1978 marked the beginning of a new era for SN studies because of its capability of measuring the ultraviolet emission from objects as faint as $m_B=15$. Moreover, just around that time, other powerful astronomical instruments became available, such as the Einstein Observatory X-ray measurements, the VLA for radio observations, and a number of telescopes either dedicated to infrared observations (e.g. UKIRT and IRTF at Mauna Kea) or equipped with new and highly efficient IR instrumentation (e.g. AAT and ESO observatories). As a result, starting in the late 70's a wealth of new information become available that, thanks to the coordinated effort of astronomers operating at widely different wavelengths, has provided us with fresh insights as for the properties and the *Nature* of supernovae of all types. Eventually, the successful launch of the Hubble Space Telescope (HST) opened new possibilities for the study of supernovae, allowing us to study SNe with an accuracy unthinkable before and to reach the edge of the Universe.

Even after 18 years of IUE observations and 16 more of HST observations, the number of SN events that have been monitored with UV spectroscopy is quite small and hardly include more than two objects per SN type and hardly with good quality spectra for more than three epochs each. As a consequence, we still know very little about the properties and the evolution of the ultraviolet emission of SNe. On the other hand, it is just the UV spectrum of a SN, especially at early epochs, that contains a wealth of valuable and crucial information that cannot be obtained with any other means. Therefore, we truly want to monitor many more SNe with much more frequent observations.

We have learned at this Conference that SWIFT, in addition to doing UV photometry, is also able to obtain spectra of SNe with a sensitivity and a resolution comparable, or higher than IUE and that spectroscopic observations of SNe with SWIFT are in the planning (see, e.g., the contribution by F. Bufano). This is an exciting possibility that promises to provide very valuable results and to fill the gaps in our knowledge about UV properties of SNe.

Here, I present a short summary of the UV observations of supernovae. A more detailed review on this subject can be found in Panagia (2003).

3. TYPE IA SUPERNOVAE

Type Ia supernovae are characterized by a lack of hydrogen in their spectra at all epochs and by a number of typically broad, deep absorption bands, most notably the Si II 6150Å



FIGURE 1. Ultraviolet spectra of ten Type Ia supernovae observed with IUE around maximum light. The dashed line is SN 1992A spectrum as measured with HST-FOS.

(actually the blue-shifted absorption of the 6347-6371Å Si II doublet; see e.g. Filippenko 1997), which dominate their spectral distributions at early epochs. SNIa are found in all types of galaxies, from giant ellipticals to dwarf irregulars. However, the SNIa explosion rate, normalized relative to the galaxy H or K band luminosity and, therefore, relative to the galaxy mass, is much higher, up to a factor of 16 when comparing the extreme cases of irregulars and ellipticals (Della Valle & Livio 1994, Panagia 2000, Mannucci et al. 2005) in late type galaxies than in early type galaxies. This suggests that, contrary to common belief, a considerable fraction of SNIa belong to a relatively young (age much younger that 1~Gyr), moderately massive (~5M_☉< M(SNIa progenitor)< 8M_☉) stellar population (Mannucci, Della Valle & Panagia 2006), and that in present day ellipticals SNIa are mostly the result of capture of dwarf galaxies by massive ellipticals (Della Valle & Panagia, 2003, Della Valle et al. 2005)

3.1. Existing Samples of UV Spectra of SNIa

Although 12 type Ia SNe were observed with IUE, only two events, namely SN1990N and SN1992A, had extensive time coverage, whereas all others were observed only around maximum light either because of their intrinsic UV faintness or because of satellite pointing constraints. Even so, one can reach important conclusions of general validity, which are confirmed by the detailed data obtained for a few SNIa.

The UV spectra of type Ia SNe are found to decline rapidly with frequency, making it hard to detect any signal at short wavelengths. This aspect is illustrated in Fig. 1, which displays the UV long wavelength spectra of 10 type Ia SNe observed with IUE. It appears that the spectra do not have a smooth continuum but rather consist of a number of



FIGURE 2. The spectral evolution of SN1992A [adapted from Kirshner et al. 1993].

"bands" that are observed with somewhat different strengths. The fact that the spectrum is so similar for most of the SNe supports the idea of an overall homogeneity in the properties of type Ia SNe.

On the other hand, some clear deviations from "normal" can be recognized for some SNIa. In particular, one can notice that both SN1983G and SN1986G display excess flux around 2850 Å, and a deficient flux around 2950 Å. This suggests that the Mg II resonance line is much weaker, which may indicate a lower abundance of Mg in these fast-decline, under-luminous SNIa. On the other hand, SN1990N, SN1991T, and, possibly, SN1989M show excess flux around \sim 2750 Å and \sim 2950 Å and a clear deficit around \sim 3100 Å, which may be ascribed to enhanced Mg II and Fe II features in these slow-decline, over-luminous SNIa.

The best studied SNIa event so far is the "normal" type Ia supernova SN1992A in the S0 galaxy NGC1380 that was observed as a TOO by both IUE and HST (Kirshner et al. 1993). The HST-FOS spectra, from 5 to 45 days past maximum light, are the best UV spectra available for a SNIa (see Fig. 2) and reveal, with good signal to noise ratio, the spectral region blueward of ~ 2650 Å.

An LTE analysis of the SN1992A spectra shows that the features in the region shortward of ~2650Å are P Cygni absorptions due to blends of iron peak element multiplets and the Mg II resonance multiplet. Newly synthesized Mg, S, and Si probably extend to velocities at least as high as ~19,000 km s⁻¹. Newly synthesized Ni and Co may dominate the iron peak elements out to ~13,000 km s⁻¹ in the ejecta of SN1992A. On the other hand, an analysis of the O I λ 7773 line in SN1992A and other SNIa implies that the oxygen rich layer in typical SNIa extends over a velocity range of at least ~11,000-19,000 km s⁻¹, but none of the "canonical" models has an O-rich layer that completely covers this range. Even higher velocities were inferred by Jeffery et al. (1992) for the overluminous, slow-decline SNIa SN1990N and SN1991T through an LT analysis of their photospheric epoch optical and UV spectra. In particular, matter moving as fast as 40,000 and 20,000 km s⁻¹ were found for SN1990N and SN1991T, respectively. It thus appears that type Ia supernovae are consistently weak UV emitters, and even at maximum light their UV spectra fall well below a blackbody extrapolation of their optical spectra. Broad features due to P Cygni absorption of Mg II and Fe II are present in all SNIa spectra, with remarkable constancy of properties for normal SNIa and systematic deviations for slow-decline, over-luminous SNIa (enhanced Mg II and Fe II absorptions) and fast-decline, under-luminous SNIa (weaker Mg II lines).

4. CORE COLLAPSE SUPERNOVAE: TYPES II AND IB/C

Massive stars $(M^*>8M_{\odot})$ are believed to end their evolution collapsing over their inner Fe core and producing an explosion by a gigantic bounce that launches a shock wave that propagates through the star and eventually erupts through the progenitor photosphere, ejecting several solar masses of material at velocities of several thousand km s⁻¹. The current view is that single stars (as well as stars in wide binary systems in which the companion does not affect the evolution of the primary star) explode as type II supernovae, while supernovae of types Ib and Ic originate from massive stars in interacting binary systems. Although the explosion mechanism is essentially the same in both types, the spectral characteristics and light curve evolution are markedly different among the different types.

4.1. Type Ib/c Supernovae

Type Ib/c supernovae (SNIb/c) are similar to SNIa in not displaying any hydrogen lines in their spectra and are dominated by broad P Cygni-like metal absorptions, but they lack the characteristic SiII 6150Å trough of SNIa. The finer distinction into SNIb and SNIc was introduced by Wheeler and Harkness (1986) and is based on the strength of He I absorption lines, most importantly He I 5876Å, so that the spectra of SNIb display strong He I absorptions and those of SNIc do not. SNIb and SNIc are found only in late type galaxies, often (but not always) associated with spiral arms and/or H II regions. They are generally believed to be the result of the evolution of massive stars in close binary systems.

Although the properties of some peculiarly red and under-luminous SNI (SN1962L and SN1964L) were already noticed by Bertola and collaborators in the mid-1960s (Bertola 1964, Bertola et al. 1965), the first widely recognized member and prototype of the SNIb class was SN1983N in NGC5236=M83.

Because of its bright magnitude (B~11.6 mag at maximum light), SN1983N is one of the best-studied SNe with IUE (see Panagia 1985). The UV spectrum of SN1983N closely resembles that of type Ia SNe at comparable epochs and, as such, only a minor fraction of the SN energy is radiated in the UV. In particular, only ~13% of the total luminosity was emitted by SN1983N shortward of 3400Å at the time of the UV maximum. Moreover, there is no indication of any stronger emission in the UV at very early epochs; this implies that the initial radius of the SN, i.e. the radius the stellar progenitor had when the shock front reached the photosphere, was probably < 10^{12} cm, ruling out



FIGURE 3. The spectrum of SN 1983N near maximum optical light, dereddened with E(B-V)=0.16. Both UV and optical spectra have been boxcar smoothed with a 100Å bandwith. The triangle is the IUE *Fine Error Sensor (FES)* photometric point, and the dots represent the J, H, and K data. The dash-dotted curve is a blackbody spectrum at T=8300K [adapted from Panagia 1985].

a RSG progenitor. From the bolometric light curve Panagia (1985) estimated that ~ 0.15 M_{\odot} of ⁵⁶Ni was synthesized in the explosion.

The best observed SNIc is SN1994I that was discovered on 2 April 1994 in the grand design spiral galaxy M51 and was promptly observed both with IUE (as early as 3 April) and with HST- FOS (19 April). The UV spectra were remarkably similar to those obtained for SN1983N and, although they were taken only at two epochs well past maximum light (10 days and 35 days), they were of high quality. From synthetic spectra matching the observed spectra from 4 days before to 26 days after the time of maximum brightness, the inferred velocity at the photosphere decreased from 17,500 to 7,000 km s⁻¹ (Millard et al. 1999). Simple estimates of the kinetic energy carried by the ejected mass gave values that were near the canonical supernova energy of 10^{51} erg. Such velocities and kinetic energies for SN1994I are "normal" for SNe and are much lower than those found for the peculiar type Ic SN1997ef and SN1998bw (see, e.g. Branch 2000) which appear to have been hyper-energetic.

Thus, as type Ia, type Ib/c supernovae are weak UV emitters with their UV spectra much fainter than a blackbody extrapolation of both optical and NIR spectra, and their typical luminosity is about a factor of 4 lower than that of SNIa. The mass of ⁵⁶Ni synthesized in a typical SNIb/c is, therefore, $\sim 0.15 \text{ M}_{\odot}$.

4.2. Type II Supernovae

Type II supernovae display prominent hydrogen lines in their spectra (Balmer series in the optical) and their spectral energy distributions are mostly a continuum with relatively few broad P Cygni-like lines superimposed, rather than being dominated by discrete features as is the case of all type I supernovae. SNII are believed to be the result of a core collapse of massive stars exploding at the end of their RSG phase. SN1987A was



FIGURE 4. UV spectral evolution of SN1998S (SINS project, unpublished). Shown are spectra obtained near maximum light (March 16,1998), about two weeks past maximum (March 30, 1998), and about two months after maximum (May 13, 1998).

both a confirmation and an exception to this model. It was clearly the product of the collapse of a massive star, but it exploded when it was a BSG, not an RSG. Since its properties are amply discussed in many detailed papers presented at this Conference, we do not include SN1987A in this summary of the UV properties of "normal" SNII.

Among the other five SNII that were observed with IUE, only two, SN1979C and SN1980K, were bright enough to allow a detailed study of their properties in the UV (Panagia et al. 1980). They were both of the so-called "linear" type (SNIIL), which is characterized by an almost straight-line decay of the B and V-band light curves, rather than of the more common "plateau" type (SNIIP) which display a flattening in their light curves starting a few weeks after maximum light.

The SNII studied best in the UV so far is possibly SN1998S in NGC3877, a type II with relatively narrow emission lines (SNIIn). SN1998S was discovered several days before maximum. Its first UV spectrum, obtained on 16 March 1998, near maximum light, was very blue and displayed lines with extended blue wings, which indicate expansion velocities up to 18,000 km s⁻¹ (Panagia 2003). The UV spectral evolution of SN1998S (Fig. 5) showed the spectrum to gradually steepen in the UV, from near maximum light on 16 March 1998 to about two weeks past maximum on 30 March, and the blue absorptions to weaken or disappear completely. About two months after maximum (13 May 1998) the continuum was much weaker, although its UV slope had not changed appreciably, and it had developed broad emission lines, the most noticeable being the Mg II doublet at about 2800Å. This type of evolution is quite similar to that of SN1979C (Panagia 2003) and suggests that the two sub-types are related to each other, especially in their circumstellar interaction properties.

A detailed analysis of early observations of SN1998S (Lentz et al. 2001) indicated that early spectra originated primarily in the circumstellar region itself, and later spectra are

due primarily to the supernova ejecta. Intermediate spectra are affected by both regions. A mass-loss rate of order of $\sim 10^{-4}$ [v/(100km s⁻¹)] M_☉/yr was inferred from these calculations but with a fairly large uncertainty.

Despite the fact that type II plateau (SNIIP) supernovae account for a large fraction of all SNII, so far SN1999em in NGC1637 is the only SNIIP that has been studied in some detail in the ultraviolet. Although caught at an early stage, SN1999em was already past maximum light (see, e.g. Hamuy et al. 2001). An early analysis of the optical and UV spectra (Baron et al. 2000) indicates that, spectroscopically, this supernova appears to be a normal type II. Also, the analysis suggests the presence of enhanced N as found in other SNII.

Another sub-type of the SNII family is the so-called type IIb SNe, dubbed so because at early phases their spectra display strong Balmer lines, typical of type II SNe, but at more advanced phases the Balmer lines weaken significantly or disappear altogether (see, e.g. Filippenko et al. 1997) and their spectra become more similar to those of type Ib SNe. A prototypical member of this class is SN1993J that was discovered in early April 1993 in the nearby galaxy M81. An HST-FOS UV spectrum of SN1993J was obtained on 15 April 1993, about 18 days after explosion, and rather close to maximum light. The study of this spectrum (Jeffery et al. 1994) shows that the approximately 1650-2900Å region is smoother than observed for SN1987A and SN1992A and lacks strong P Cygni lines absorptions caused by iron peak element lines. It is of interest to note that the UV spectrum of SN1993J is appreciably fainter than observed in most SNII, thus revealing its "hybrid" Nature and some resemblance to a SNIb. Synthetic spectra calculated using a parameterized LT procedure and a simple model atmosphere do not fit the UV observations. Radio observations suggest that SN1993J is embedded in a thick circumstellar medium envelope (Van Dyk et al. 1994, Weiler et al. 2007). Interaction of supernova ejecta with circumstellar matter may be the origin of the smooth UV spectrum so that UV observations of supernovae could provide insight into the circumstellar environment of the supernova progenitors.

Thus, despite their different characteristics in the detailed optical and UV spectra, all type II supernovae of the various sub-types appear to provide clear evidence for the presence of a dense CSM and, in many cases, enhanced nitrogen abundance. Their UV spectra at early phases are very blue, possibly with strong UV excess relative to a blackbody extrapolation of their optical spectra.

5. SUPERNOVAE AND COSMOLOGY

SNIa have gained additional prominence because of their cosmological utility, in that one can use their observed light curve shape and color to standardize their luminosities. Thus, SNIa are virtually ideal standard candles (e.g. Macchetto and Panagia 1999) to measure distances of truly distant galaxies, currently up to redshift around 1 and, considerably more in the foreseeable future. In particular, Hubble Space Telescope observations of Cepheids in parent galaxies of SNIa (an international project lead by Allan Sandage) have produced very accurate determinations of their distances and the absolute magnitudes of normal SNIa at maximum light that, in turn, have lead to the most modern measure of the Hubble constant (i.e. the expansion rate of the local Universe),



FIGURE 5. Top: Intermediate resolution (R=1500) spectra of three SNe near maximum light, SNIa 1992A (Kirshner et al. 1993), SNIIn/L 1998S (Lentz et al. 2001) and SNIIP 1999em (Baron et al. 2000) normalized in the V band. Bottom: Low-resolution rendition of the observed spectra convolved to a R=4 resolution showing the color differences in the UV.

 $H_0 = 62.3 \pm 1.3$ (*random*) ± 5.0 (*systematic*) km s⁻¹Mpc⁻¹ (Sandage et al. 2006, and references therein).

Observations of high redshift (i.e. z>0.1) SNIa have provided evidence for a recent (past several billion years) acceleration of the expansion of the Universe, pushed by some mysterious "dark energy". This is an exciting result that, if confirmed, may shake the foundations of physics. The results of two competing teams (Perlmutter et al. 1998, 1999, Riess et al. 1998, Knop et al. 2003, Tonry et al. 2003, Riess et al. 2004) appear to agree in indicating a non-empty inflationary Universe, which can be characterized by $\Omega_M \simeq 0.3$ and $\Omega_{\Lambda} \simeq 0.7$. Correspondingly, the age of the Universe can be bracketed within the interval 12.3-15.3 Gyrs to a 99.7% confidence level (Perlmutter et al. 1999).

However, the uncertainties, especially the systematic ones, are still uncomfortably large and, therefore, the discovery and the accurate measurement of more high-z SNIa are absolutely needed. This is a challenging proposition, both for technical reasons, in that searching for SNe at high redshifts one has to make observations in the near IR (because of redshift) of increasingly faint objects (because of distance) and for more subtle scientific reasons, i.e. one has to verify that the discovered SNe are indeed SNIa and that these share the same properties as their local Universe relatives.

One can discern Type I from Type II SNe on the basis of the overall properties of their UV spectral distributions (Panagia 2003), because Type II SNe are strong UV emitters, whereas all Type I SNe, irrespective of whether they are Ia or Ib/c, have spectra steeply declining at high frequencies (see Figure 5). This technique of recognizing SNIa from their steep UV spectral slope was devised by Panagia (2003), and has been successfully employed by Riess et al. (2004a,b) to select their best candidates for HST follow-up of

high-z SNIa. However, we have to keep in mind that by using this technique one is barely separation the SNe with low UV emission (SNe Ia, Ib, Ic and, possibly, IIb) from the ones with high UV emission (most type II SNe). While it is a convenient approach to select interesting candidates, it cannot be a substitute for detailed spectroscopy, possibly at an R>100 resolution, to reliably characterize the SN type.

ACKNOWLEDGMENTS

I would like to acknowledge partial support from STScI, through DDRF grant #82367, and from the Conference organization that allowed me to attend this conference.

REFERENCES

- . E. Baron et al., 2000, *ApJ*, 545, 444
- . F. Bertola, 1964, Ann.Ap, 27, 319
- . F. Bertola, A. Mammano, M. Perinotto, Asiago Contr., 174, 51
- . D. Branch, 2000, in "The Largest Explosions since the Big Bang: Supernovae and Gamma Ray Bursts", eds. M. Livio, N. Panagia, K. Sahu (Cambridge University Press, Cambridge) p. 96
- . M. Della Valle , M. Livio, 1994, *ApJ*, 423, L31
- M. Della Valle, N. Panagia, 2003, *ApJ*, 587, L71
- . M. Della Valle et al., 2005, *ApJ*, 629, 750
- . A. Filippenko, 1997, ARAAp, 35, 309
- . M. Hamuy et al., 2001, ApJ, 558, 615
- . R.P. Kirshner et al., 1993, ApJ, 415, 589
- . R.D. Knop, et al., 2003, *ApJ*, 598, 102
- . E.J. Lentz et al., 2001, *ApJ*, 547, 406
- D.J. Jeffery, B. Leibundgut, R.P. Kirshner, S. Benetti, D. Branch, G. Sonneborn: ApJ, 397, 304
- . D.J. Jeffery et al., 1994, *ApJ*, 421, L27
- F.D. Macchetto, N. Panagia, in "Post-Hipparcos Cosmic Candles", eds. A. Heck, F. Caputo (Kluwer, Holland 1999) p. 225
- . F. Mannucci et al., 2005, A&A, 433, 807
- . F. Mannucci, M. Della Valle & N. Panagia, 2006, MNRAS, 370, 773
- . J. Millard, et al., 1999, *ApJ*, 527, 746
- . N. Panagia, 1985, in "Supernovae As Distance Indicators", LNP 224, (Springer, Berlin) p. 226
- . N. Panagia, 2000, in "Experimental Physics of Gravitational Waves", eds. G. Calamai, M. Mazzoni, R. Stanga, F. Vetrano (World Scientific, Singapore 2000) p. 107
- . N. Panagia, 2003, in "Supernovae and Gamma-Ray Bursters", ed. K. W. Weiler (Springer-Verlag: Berlin), p. 113-144.
- . N. Panagia, 2005, in "Frontier Objects in Astrophysics and Particle Physics", eds. F. Giovannelli & G. Mannocchi, It. Phys. Soc., in press [astro-ph/0502247]
- . N. Panagia et al., 1980, *MNRAS*, 192, 861
- . S. Perlmutter et al., 1998, *Nature*, 391, 51
- S. Perlmutter et al., 1999, *ApJ*, 517, 565
- . A.G. Riess et al., 1998, AJ, 116, 1009
- . A.G. Riess et al., 2004a, ApJ, 600, L163
- . A.G. Riess et al., 2004b, *ApJ*, 607, 665
- . A. Sandage, G.A. Tammann, A. Saha, B. Reindl, F.D. Macchetto, N. Panagia, 2006, ApJ, 653, 843
- . J.L. Tonry et al., 2003, ApJ, 594, 1
- . S. Van Dyk et al., 1994, *ApJ*, 432, L115
- . K.W. Weiler et al., 2007, ApJ, in press
- . J.C. Wheeler, R.P. Harkness, 1986 in "Galaxy Distances and Deviations from Universal Expansion", eds. B.F. Madore, R.B. Tully (Reidel, Dordrecht) p. 45