

Physics for Proctologists



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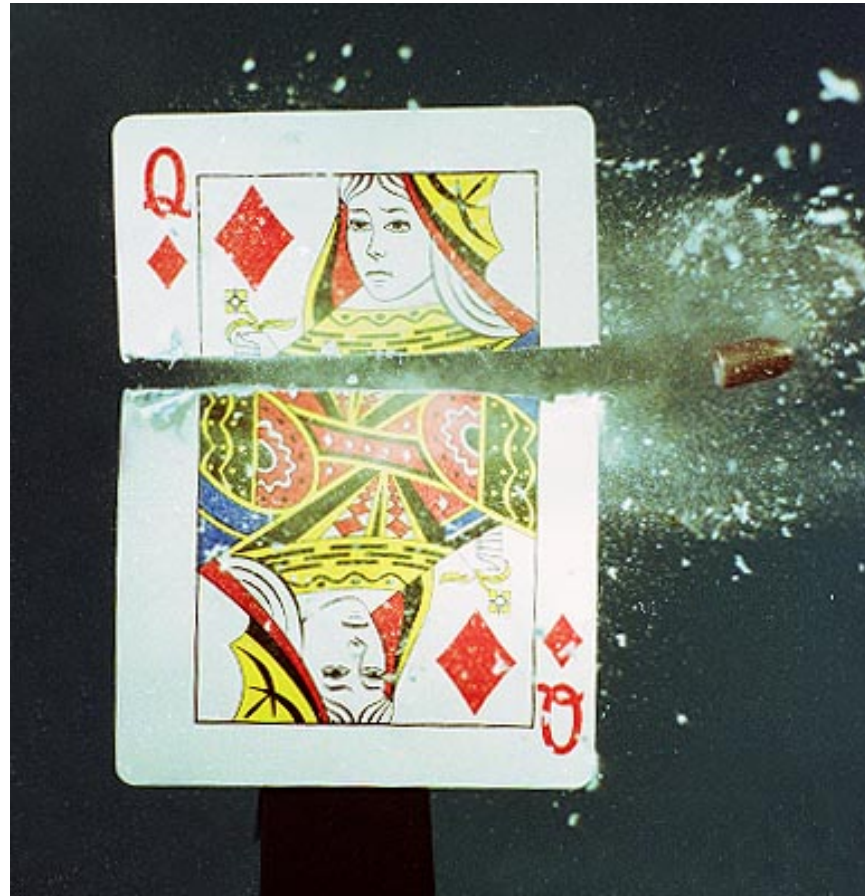
Contents

1. Kinematics	pg 5
2. Newton's Laws	pg 13
3. Center of Mass & centripetal acceleration	pg 24
4. Energy & Momentum	pg 33
5. Rotational Physics (extra Office Hours)	pg 40
6. Fluids	pg 48
7. Electricity	pg 57
8. Magnetic Fields & Circuits	pg 67
9. Right Hand Rule Worksheet	pg 79
10. Oscillations, waves, & sound	pg 82
11. Optics	pg 97
12. Interference of Waves (extra Office Hours)	pg 111
Index	pg 121

Things you *absolutely don't* need to know:

- **Relativity ☹**
- **Quantum Mechanics**

Physics 1: Kinematics



1. Fill in the Trig values in the table (radical & decimal form)

Theta	Sin	Cos
0	$(0)^{1/2}/2=0$	$(4)^{1/2}/2=1$
30	$(1)^{1/2}/2=0.5$	$(3)^{1/2}/2=0.86$
45	$(2)^{1/2}/2=0.7$	$(2)^{1/2}/2=0.7$
60	$(3)^{1/2}/2=0.86$	$(1)^{1/2}/2=0.5$
90	$(4)^{1/2}/2=1$	$(0)^{1/2}/2=0$

Use the units: we need m/s and hence must force kg to cancel. So we must divide these two numbers and they must be raised to the same power (namely 1)

2. At what speed v does sound travels through a medium with Bulk Modulus

β [kg s²/m] and density ρ [kg/m³]?

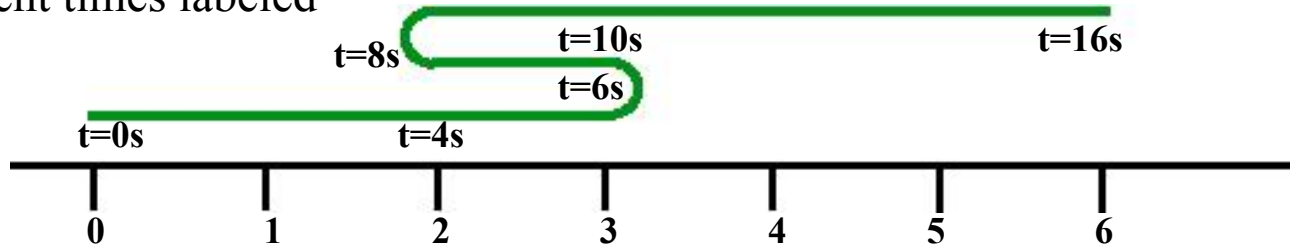
A) $\sqrt{\beta\rho}$

B) $\sqrt{\beta / \rho}$

C) $\sqrt{\beta\rho^2}$

D) $\sqrt{\beta / \rho^2}$

3. The 1-dimensional trajectory of a drunk man is depicted below with his position at different times labeled



- What is the total distance traveled? **8m**
- What is the man's total displacement? **+6m**
- What is the displacement between 6s and 8s? **-1m**
- What is the displacement between 6s and 8s? **0m**
- What is the velocity between 0s and 16s? **0.375m/s**
- What is the velocity between 6s and 8s? **-0.5m/s**

$$v = \frac{\text{displacement}}{\text{time}}$$

$$a = \frac{\Delta v}{\text{time}}$$

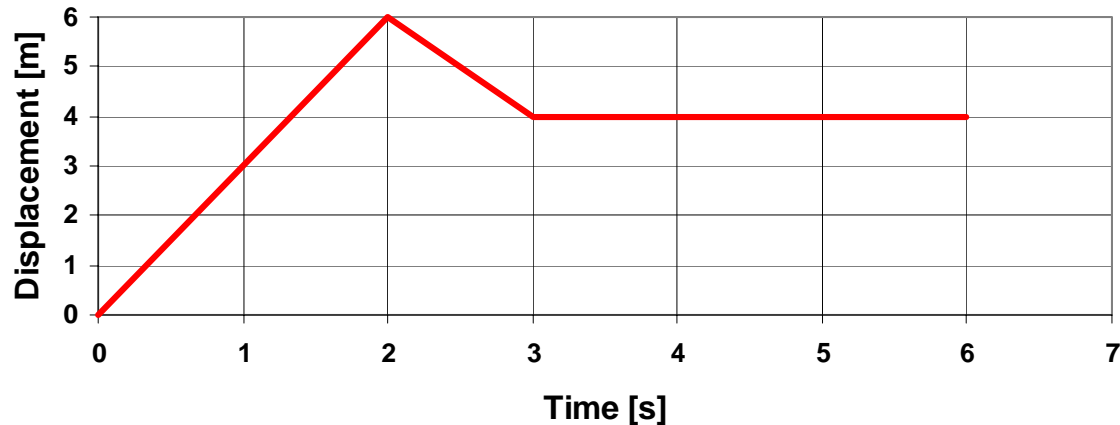
4. a. As I enter a freeway on-ramp at 10m/s and accelerate uniformly to a speed of 30m/s in 10s. What's my acceleration?

$$a=(v_f-v_o)/t=(30-10)/10=20\text{m/s}^2$$

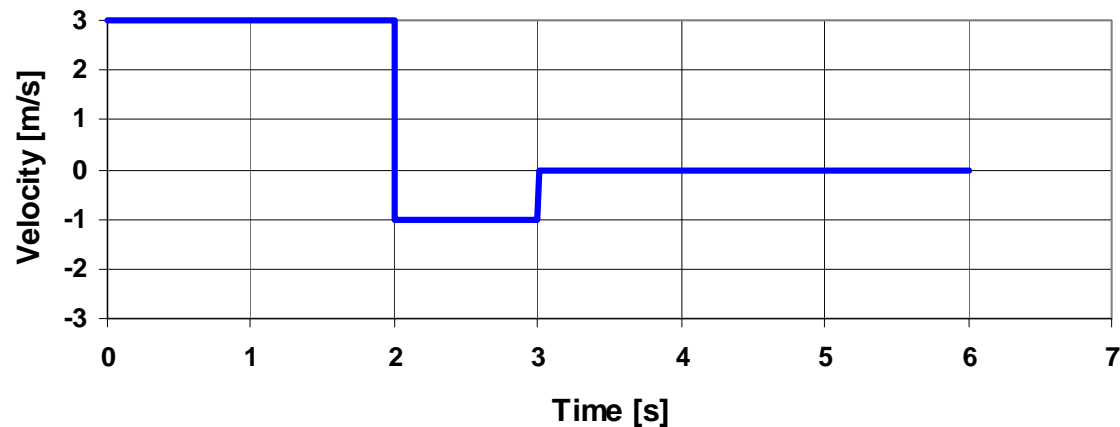
- b. Once on the freeway, I hit an overturned truck & stop with an acceleration of 60m/s². How long does it take me to stop?

$$t=(v_f-v_o)/a=(0-30)/-60=0.5\text{s}$$

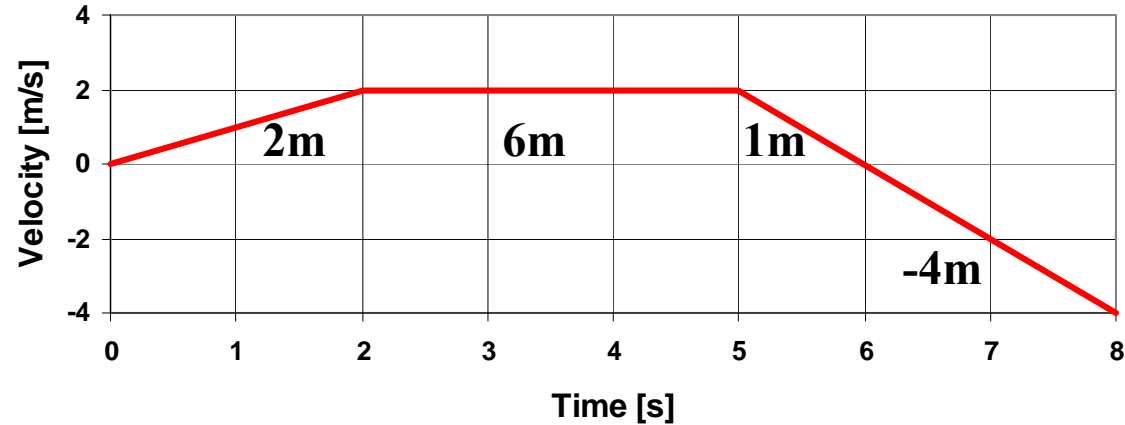
5. The 1-dimensional trajectory of another car's displacement vs. time is:



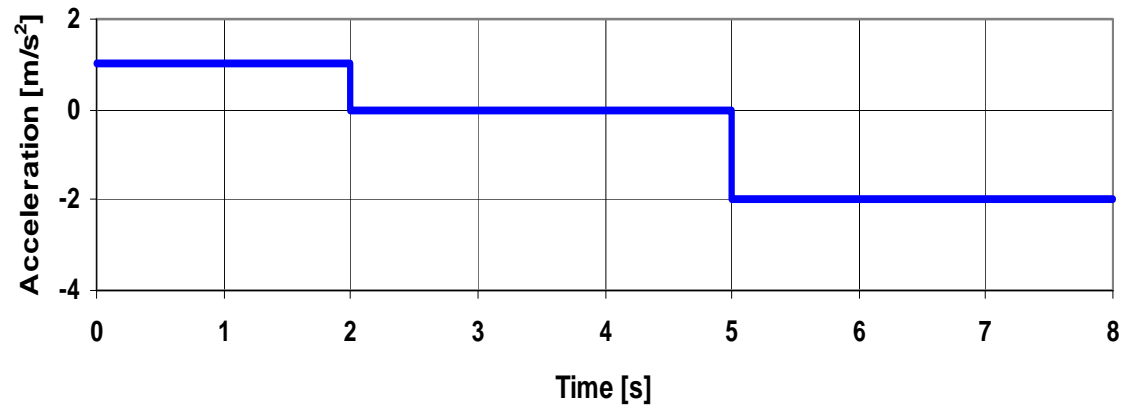
Plot the velocity vs time:



6. The 1-dimensional trajectory of yet another car's velocity vs. time is:



Plot the acceleration vs. time of this car:

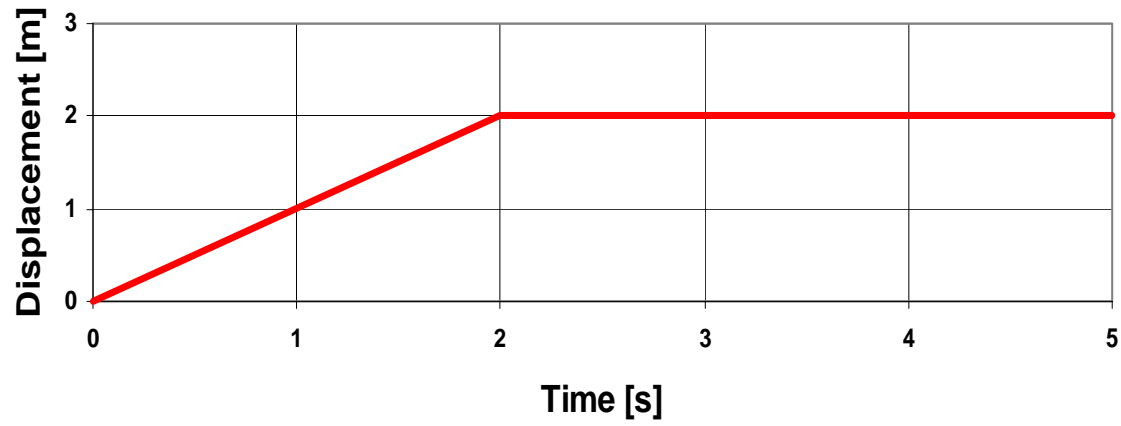


What is the displacement between 2s and 5s? $d = \text{area} = v * t = 2 * 3 = 6\text{m}$

What is the displacement between 0s and 2s? $d = v_f * t / 2 = 2 * 2 / 2 = 2\text{m}$

What is the displacement between 0s and 8s? $d = \text{total area} = 5\text{m}$

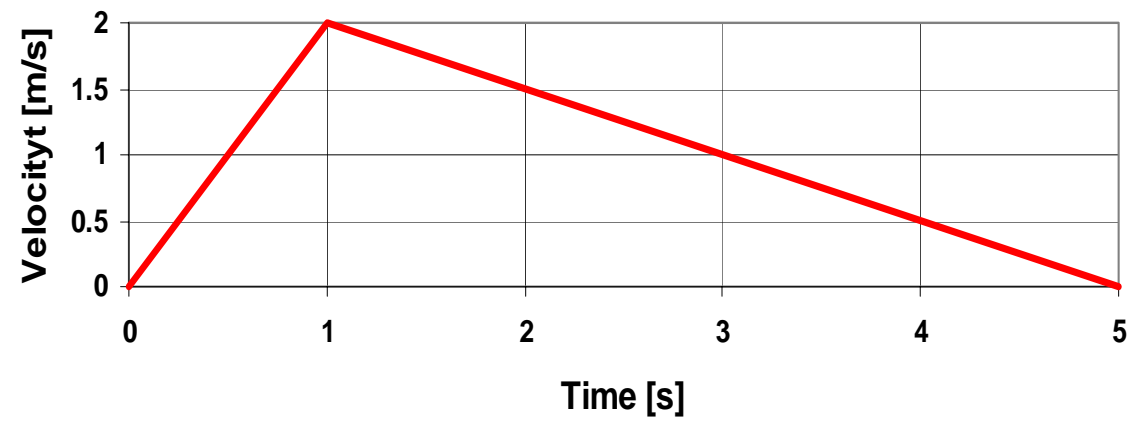
What is significant about the time t=6s? **It turns around**



I know his one's easy; just be sure not to rush so much that you mis-read it as *velocity vs time*

7. How far does the above object move between 0 s and 5 s?

- A) 2m B) 7m C) 8m D) 9m



To get avg velocity for 1st second, average initial & final (0 m/s & 5 m/s). To get it for last 4 seconds, average 5 m/s & 0 m/s. So the average velocity for both times is the same.

8. Which of the following are true concerning the above graph?

- I. At t=5s, the object returns to it's original position. It only turns around @ t=5, where v=0
- II. The avg speed betwn. t=0s & t=1s exceeds that betwn. t=1s & t=5s.
- III. The object changes its direction of travel at t=1s. It only turns around @ t=5, where v=0

- A) I & II only B) I & III only C) II & III only D) None are true

9. The following equations are useful in Problems with *constant* acceleration:

Number	Equation	Missing Variable
1	$d = ((v_o + v_f)/2) * t$	a
2	$v_f = v_o + a * t$	d
3	$d = v_o * t + 0.5a * t^2$	v_f
4	$d = v_f * t - 0.5a * t^2$	v_o
5	$v_f^2 = v_o^2 + 2a * d$	t

10. A particle with initial velocity of 4m/s moves along the x-axis under constant acceleration. Three seconds later, it's velocity is 14m/s. How far did it move?

- A) 21m B) 24m C) **27m** D) 30m

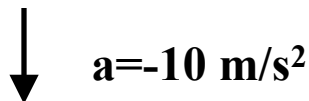
We don't care about a, so use #1:
 $d = ((4+14)/2) * 3 = 27 \text{ m}$

11. An object starting from rest is accelerated uniformly until it's velocity is v and it's displacement is D. If the object had been allowed to reach a final speed of 4v, what would it's displacement have been?

- A) 2D B) 4D C) 8D D) **16D**

We don't care about t, so use #5 to compare the two cases: $v^2 = v_o^2 + 2a d$, where $v_o = 0$.
 So: $d \sim v^2$. If final speed quadruples, then displacement increases by $(4)^2 = 16$

12. What is the direction and magnitude of acceleration when an object experiences free fall with no air resistance near the surface of the earth?



13. An object is dropped to the ground and hits the earth three seconds later. From what height was it dropped? We don't care about v_f , so use #3 with $V_0=0$ & $a=-10$: $d=0*3+0.5*(-10)*(3)^2=45$ m

- A) 15m **B) 45m** C) 90m D) 180m

14a. An object is thrown horizontally at 15m/s from a 100m tall tower. How far does it travel horizontally in the first 2s?

- A) 20m **B) 30m** C) 40m D) 50m

Horizontal motion: We don't care about v_f , so use #3 again with $V_0=15$ & $a=0$: $d=15*2+0.5*(0)*(2)^2=30$ m

14b. How far does the above object vertically in those same first 2s ?

|d|=20m

Vertical motion: We don't care about final v , so use #3 yet again with $V_0=0$ & $a=-10$
 $d=0*2+0.5*(-10)*(2)^2=20$ m

15a. If an object were throw upwards with an initial speed of 8m/s and it strikes the ground 3s later, from what height was it thrown?

- A) 21m** B) 24m C) 45m D) 69m

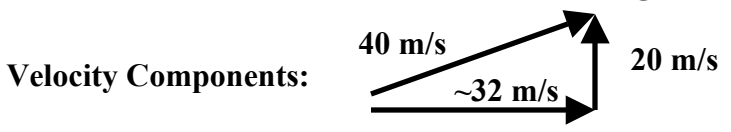
Don't care about V final; Use #3: $d=8*3+0.5*(-10)*(3)^2=-21$ m

15b. How high does that object reach? **3.2m** Don't care about t ; Use #5 with $v_f=0$: $v_f^2= (8)^2+ 2*(-10)*d$

15c. How long does it take to reach that zenith? **0.8s** Don't care about d ; Use #2 $v_f=0$: $0 = 8 + -10 * t$

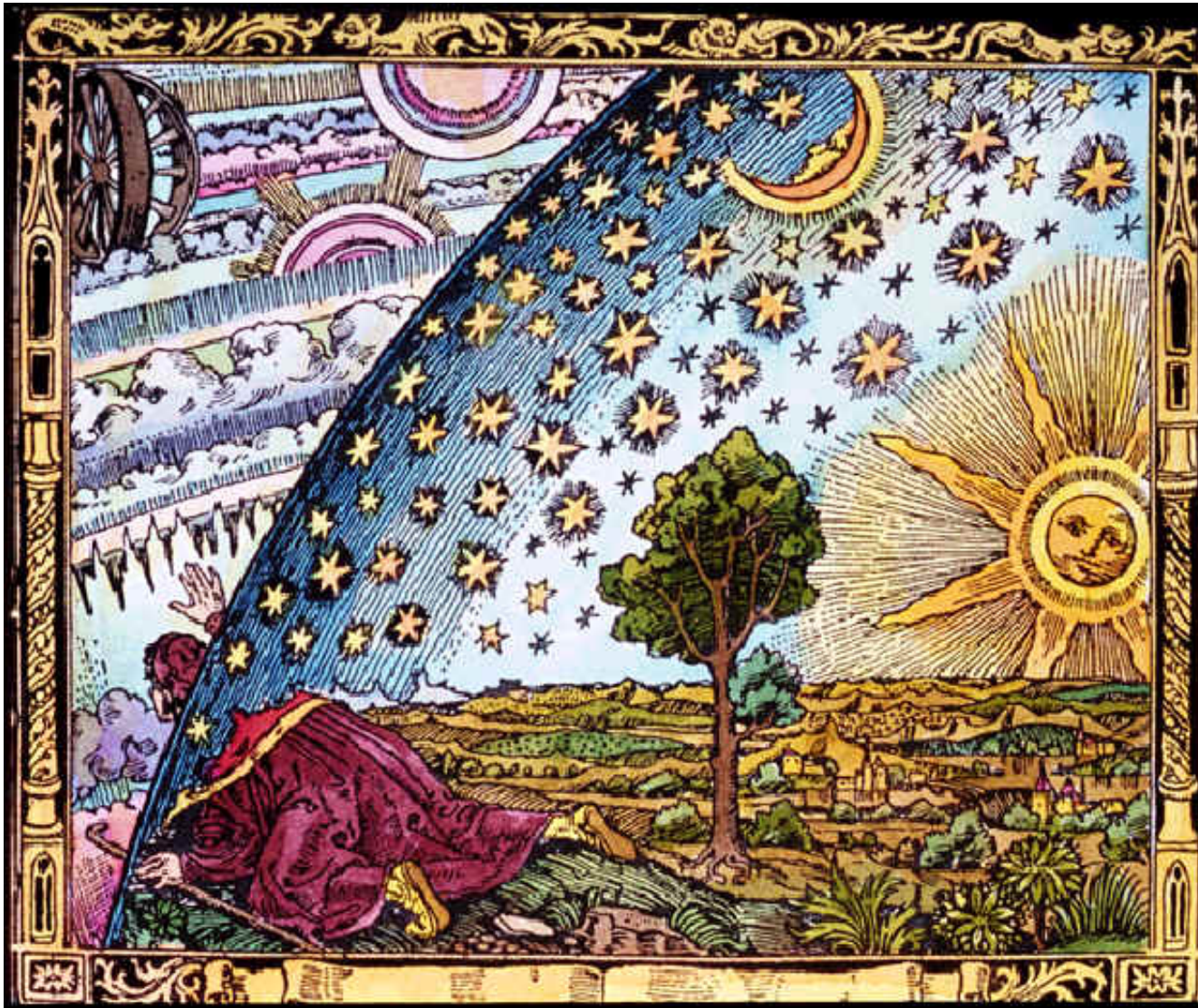
16a. If I fire a cannonball on a level field with a muzzle velocity of 40m/s at 30° above the horizon, what is the range? **~128m** Do part b 1st, then multiply TOF by V_h (This is really eqn #3): $d\sim 32*4 + 0.5*0*4^2=128$ m

16b. What is the time of flight? **t=4s**

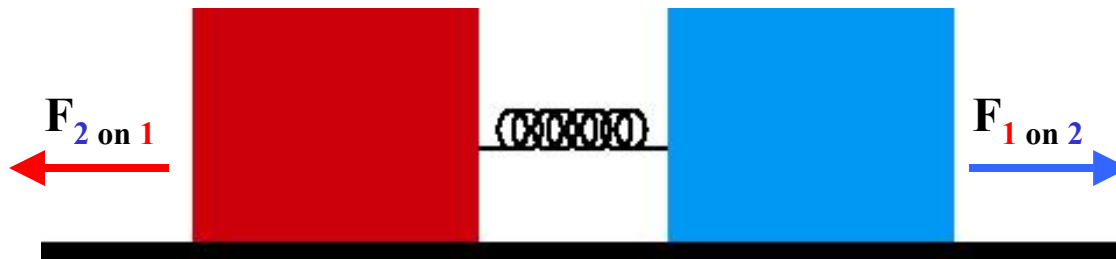


One of many ways: Find time to top & then double. Vertically, $V_f=0$ & $f_0=40$ m/s. We don't care about height, so use #2: $0 = 40 + (-10)*t_{1/2}$ $t_{1/2} = 2$ s 11
 So TOF= $2*t_{1/2} = 4$ s

Physics 2: Newton's Laws

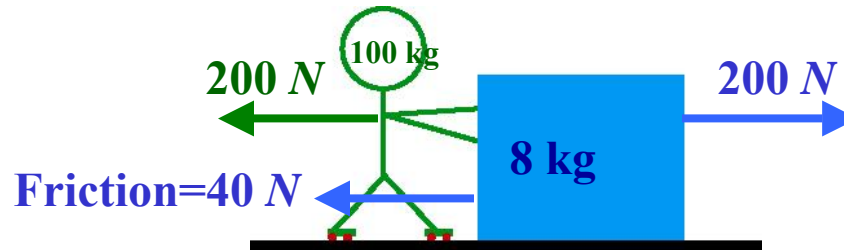


1. What's a force?: **A push or a pull *between* two objects resulting from an *interaction*.**
2. What's Newton's 1st Law?: **If an object at rest feels no *net* force, then it will stay at rest; if a moving object feels no *net* force, then it will maintain the same velocity (speed & direction).**
3. What's Newton's 2nd Law?: **$F_{\text{NET}}=ma$. You must calculate *net* force from a diagram first. Then set the *net* force equal to “m*a.”**
4. What's Newton's 3rd Law? **If an object (1) exerts a force $F_{1 \text{ on } 2}$ on another object (2), then that other object (2) must exert a force $F_{2 \text{ on } 1} = -F_{1 \text{ on } 2}$ on the first object (1). These two forces are called “action-reaction” pairs.**



5. Why can't I sum action-reaction force pairs?: **Because action reaction pairs act on two *different* objects and you *only* sum forces that act on the *same* object.** 13

6. I weigh 100 kg and push a 8 kg crate with 200 N of force. The box feels 40 N of friction with the ground as it slides. I feel none because I'm wearing roller-skates.



- Forces I feel are green
- Forces the box feels are blue

a. What net force do I feel?

- A) 2 N C) 160 N
 B) 40 N D) **200 N**

I only feel one force: the reaction force from the box.

$$F_{NET} = 200 N$$

b. How fast does the box accelerate?

- A) **20 m/s²** C) 160 m/s²
 B) 25 m/s² D) 200 m/s²

$$F_{NET} = ma$$

$$200 - 40 = 8 * a$$

$$20 m/s^2 = a$$

c. If the box feels 40 N of friction, what feels the reaction force?

Earth feels it & it points to the right.

In which direction does the reaction force act?

d. Some one hands me a 50 kg bar-bell and I repeat the above pushing. Now

how fast do I accelerate?

Without extra weight, I feel $F_{NET} = ma$
 $200 N = 100 * a$
 $2 m/s^2 = a$

With the 50 kg weight, I still feel $F_{NET} = 200 N$
 But since my mass has increased by 3/2,
 my acceleration must decrease by 2/3: $a \rightarrow 2/3 * 2 m/s^2 = 1.33 m/s^2$

- A) 2.00 m/s² B) 0.66 m/s² C) 1.00 m/s² D) **1.33 m/s²**

7. A hockey puck of mass 250 g slides on ice with a constant 4 m/s. What is the force of friction?



Puck moves @ constant v , so $a=0$. Newton's 2nd tells us $F_{NET}=0$.

Friction is the only possible force, so it must be zero.

- A) 0 N B) 1 N C) 62.5 N D) 1000 N

8a. A crane lifts a 1000 kg crate with an acceleration of 0.7m/s².

What is the tension in the cable?

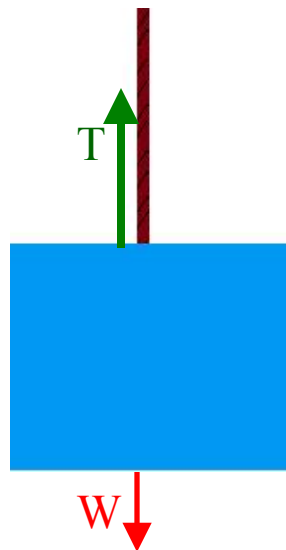
- A) 7000 N B) 9100 N C) 9800 N D) 10,500 N

$$F_{NET}=ma$$

$$T-W=ma$$

$$T-10,000=+0.7*1000$$

$$T=10,700 N$$



Notice how I've chosen the signs on acceleration in parts a& b!

8b. The crane's engine overheats and the crate begins to fall at 2m/s². Now what is the tension in the cable?

- A) 2000 N B) 7800 N C) 9800 N D) 11,800 N

$$F_{NET}=ma$$

$$T-10,000=-2*1000$$

$$T=8,000 N$$

9. What is Newton's Law of Gravitational attraction?

$$F_G = \frac{GMm}{r^2}$$

10. The sun collides with another star of the same mass and the as a result, the earth is blown to a distance from the sun that is twice as far as it was before. How does the force of gravity from the Earth on the Sun change?

- A) Doesn't change C) 1/2 times weaker
 B) 2 times stronger D) 1/4 times weaker

The mass of the star *doubles* while the earth's radius also *doubles*, so the gravitational force changes by:

$$F_g \propto \frac{M_{star}}{r^2} = \frac{2}{(2)^2} = \frac{1}{2}$$

10. Let's repeat problem 8, but this time on the moon, which is made of the same stuff as the Earth (rocks, dirt, cheese, etc) but has one half the radius. Now what's the cable's tension?

- A) 3200 N B) 2500 N C) 5000 N **D) 5700 N**

The radius of the moon is half of the earth's. Since both Earth and moon are spheres of same density, $V \sim r^3$ and so the moon's mass is $(1/2)^3 = 1/8$ that of Earth's

$$g_{Moon} \propto \frac{M}{R^2} \propto \frac{(1/2)^3}{(1/2)^2} = \frac{1}{2} g_{Earth} = 5m/s^2$$

So $F_{NET} = m \cdot a$
 $T - 5,000 = 0.7 \cdot 1000$
 $T = 8,000 N$

Normal Force: keeps object from going thru a solid surface

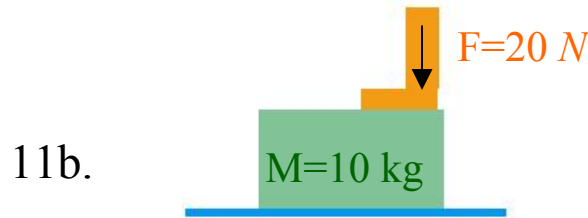


What's the normal force?

$F_{NET}=0$ cause it doesn't fall thru floor.

Since weight $mg=60\text{ N}$,

$N=60\text{ N}$

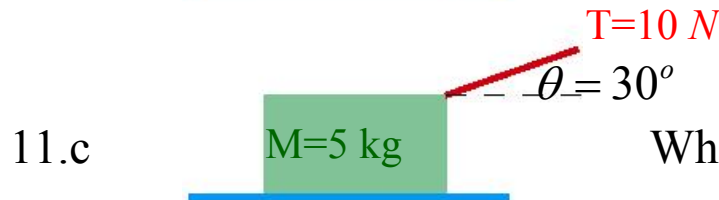


What's the normal force?

$F_{NET}=0$ still, but now my foot pushes down with 20 N .

So floor must push 20 N harder than weight $mg=100\text{ N}$

$N=120\text{ N}$



What's the normal force?

Now the rope helps out with a vertical component of tension = 5 N

So floor doesn't have to push as hard: $N=mg-T_{vert}=50-5$

$N=45\text{ N}$

12. If I hold a 40 N block against the ceiling with 50 N of force, what's the normal force?

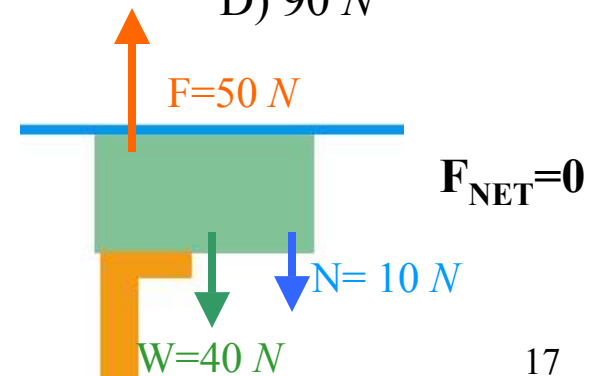
A) 0 N

B) 10 N

C) 50 N

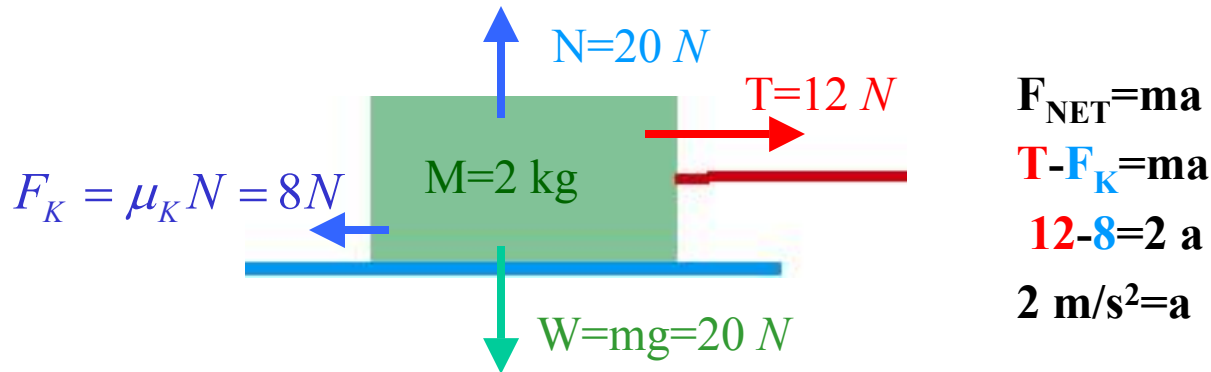
D) 90 N

Kinetic Friction	Static Friction
$F_K = \mu_K N $	$F_{MAX} = \mu_S N $
you pick direction of Force	only the max possible force. You pick value to hold object fixed
	You still have to pick direction too
$\mu_S > \mu_K$	



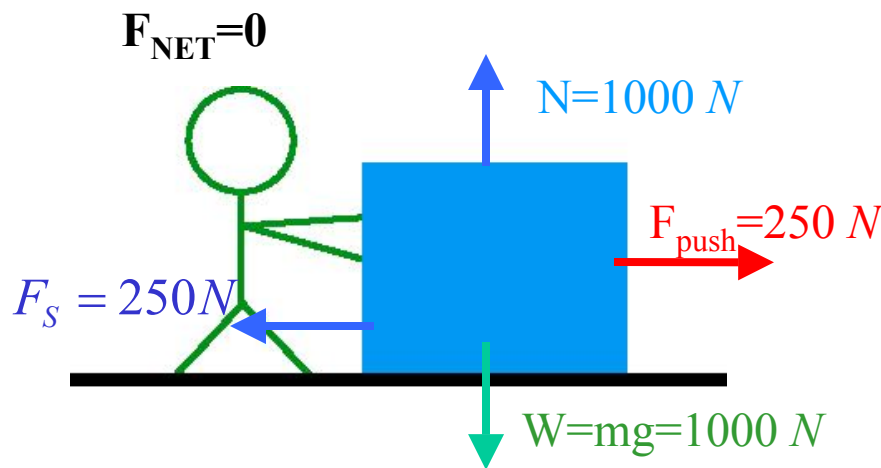
12. A 2 kg block slides along a level floor because a rope pulls horizontally with 12 N. If $\mu_K = 0.4$, what's the block's acceleration?

- A) **2 m/s²** B) 4 m/s² C) 6 m/s² D) 8 m/s²



13. A 100 kg rests on the floor with $\mu_S = 0.4$. As I push with 250 N to the left, what frictional force force does the crate feel from the floor?

- A) 0 N B) 150 N C) **250 N** D) 400 N



$$F_{s,MAX} = \mu_S |N|$$

$$= 0.4 * 1000 = 400$$

But we don't need that much friction to hold it still since we're only pushing with 250 N. So the force of $F_s = 250 \text{ N}$

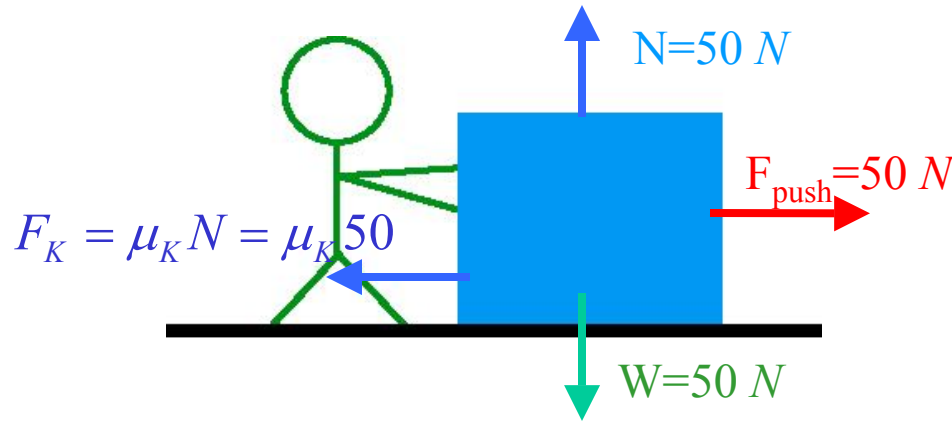
14. A 50 N force pushes a 5 kg crate, propelling it forward with an acceleration of 8 m/s². What is the μ_K ?

A) 1/5

B) 4/5

C) 9/5

D) Can't tell



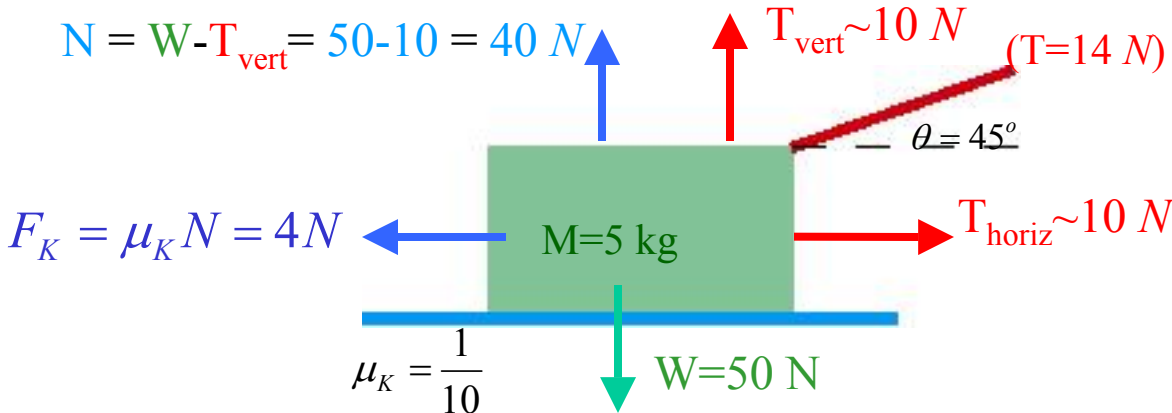
$$F_{\text{NET}} = ma$$

$$F_{\text{push}} - F_K = ma$$

$$50 - 50\mu_K = 5 * 8$$

$$\mu_K = \frac{1}{5}$$

14. What is this block's acceleration?

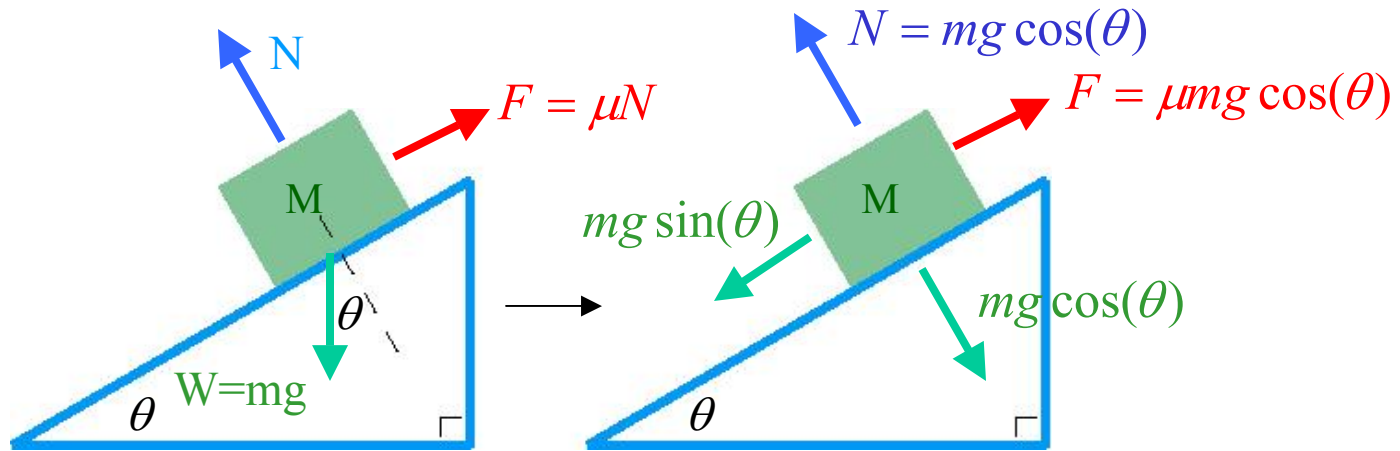


$$F_{\text{NET}} = ma$$

$$T_{\text{horiz}} - F_K = ma$$

$$50 - 4 = 5 * a$$

$$a = 5/6 \text{ m/s}^2$$



MEMORIZE THIS FIGURE

15a. 3 blocks of the same shape and material but different masses (1 kg, 2, kg, 3, kg) rest on a board with one end resting on the ground as the other end is lifted. Which object will slide 1st?

- A) 1 kg B) 2 kg C) 3 kg **D) all the same**

$$\mathbf{F}_{\text{NET}} = \mathbf{m} \mathbf{a}, \text{ applied when forces just barely balance}$$

$$mg \sin(\theta) - \mu_s mg \cos(\theta) = m a$$

Mass cancels; they all slide @ same angle (time)

15b. If we stop lifting when the first one begins to slide, which of those 3 blocks will reach the bottom first?

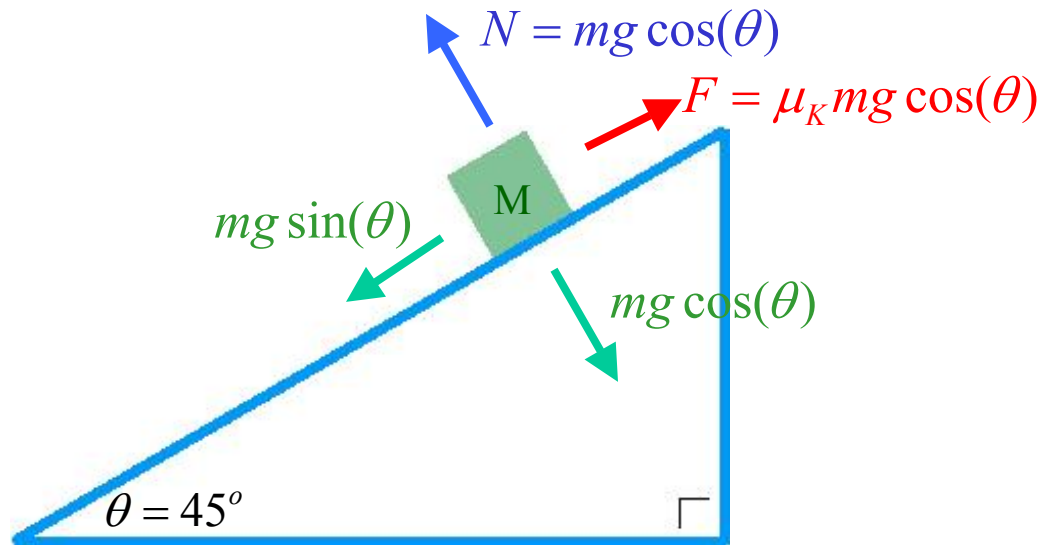
- A) 1 kg B) 2 kg C) 3 kg **D) all the same**

$$\mathbf{F}_{\text{NET}} = \mathbf{m} \mathbf{a}$$

Mass cancels; they all slide @ same speed

$$mg \sin(\theta) - \mu_k mg \cos(\theta) = m a$$

they all get to end @ same time



16. A block slides down a 45° inclined plane with $\mu_K = \underline{2/3}$. After sliding from rest down $\underline{1/3}$ the length of the ramp, it reaches a speed v . If we then oil up the ramp ($\mu_K = 0$) and let the block slide all the way down, how fast will it be moving?

- A) v B) $\sqrt{3}v$ C) $3v$ D) $9v$

$$\mathbf{F_{NET} = m a}$$

$$\cancel{mg \sin(\theta)} - \cancel{\mu_K mg \cos(\theta)} = \mathbf{m a}$$

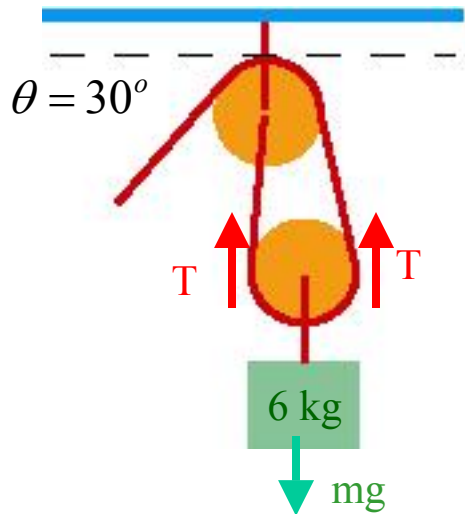
$$a = g \frac{\sqrt{2}}{2} (1 - \mu_K) \propto (1 - \mu_K)$$

So the acceleration equation @ left says that we're tripling the acceleration when we oil up the plane. We're also tripling the distance it travels. So using the 5th Kinematics Eqn with $v_0 = 0$:

$$v^2 = v_0^2 + 2a*d$$

So the final speed must be triple that of the first run

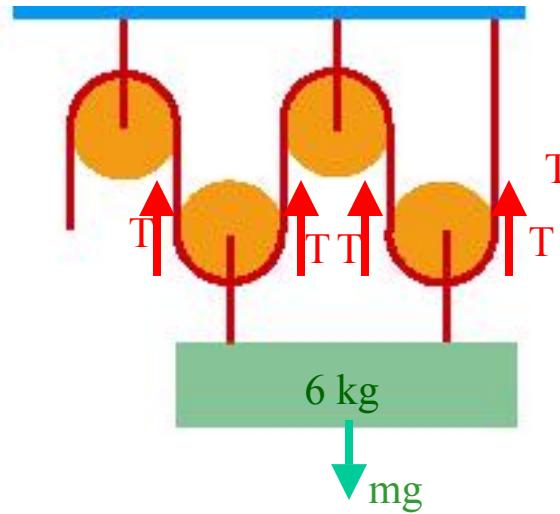
T must be constant along the rope, regardless of what direction it turns.



$F_{NET} = 0$
 $2 * T = mg$
 $T = 6 * 10 / 2 = 30 N$

17. What Tension is needed to hold the block?

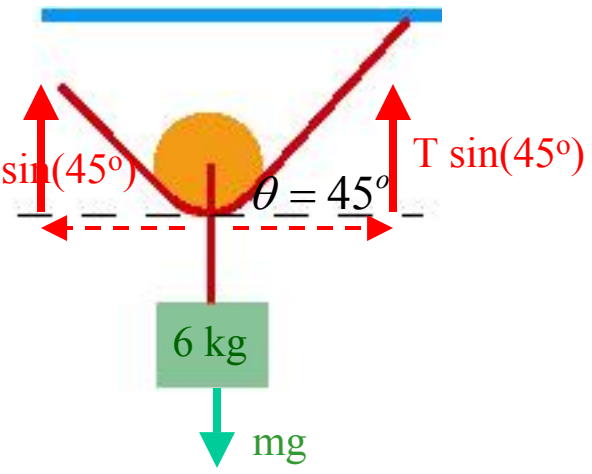
- A) 15 N C) 30 N
- B) 20 N D) 60 N



$F_{NET} = 0$
 $4 * T = mg$
 $T = 6 * 10 / 4 = 15 N$

18. What Tension is needed to hold the block?

- A) 12 N C) 20 N
- B) 15 N D) 30 N



$F_{NET} = 0$
 $2 * T * \sin(45^\circ) = mg$
 $T = 6 * 10 / 1.4$
 $\sim 60 / 1.5 = 40 N$

19. What Tension is needed to hold the block?

- A) 15 N C) 60 N
- B) 40 N D) 120 N

Physics 3: Center of Mass & Centripetal Motion

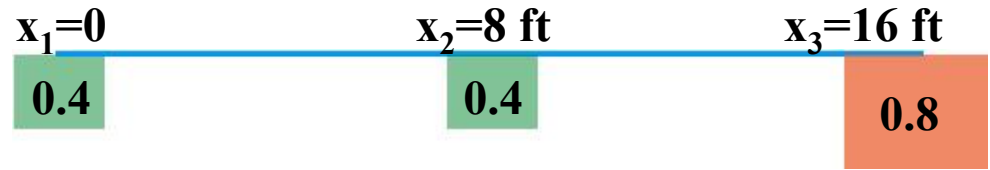


1. Three blocks hang from a massless 16 ft long rod:

$m_1 = 0.4$ lbs and is at left end

$m_2 = 0.4$ lbs and is at center of rod

$m_3 = 0.8$ lbs and is at right end



How far from the left end of the rod is the center of mass (COM)?

A) 6 ft

B) 9 ft

C) 10 ft

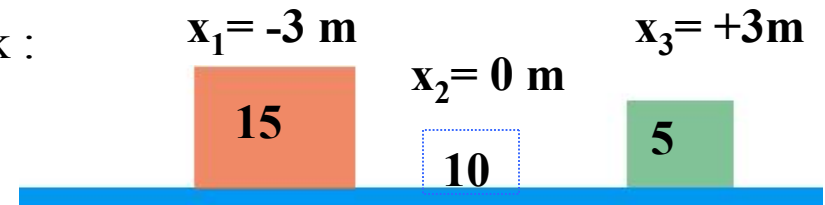
D) 12 ft

$$cm = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{0.4 * 0 + 0.4 * 8 + 0.8 * 16}{0.4 + 0.4 + 1.2} = \frac{8 + 2 * 16}{1 + 1 + 2} = 10 \text{ ft}$$

2. Two masses rest on a 8 m long 10 kg plank :

$m_1 = 15$ kg and rests 2 m left of the center

$m_2 = 5$ kg and rests 3 m right of the center



How far from the left end of the center of the plank is the center of mass?

A) $\frac{1}{2}$ m left of center

C) $\frac{1}{2}$ m right of center

B) $1\frac{1}{2}$ m left of center

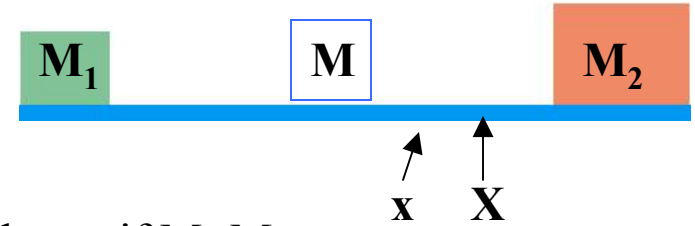
D) $1\frac{1}{2}$ m right of center

$$cm = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{15 * (-2) + 10 * 0 + 5 * (+3)}{15 + 10 + 5} = \frac{-15}{30} = -0.5m$$

3. Two masses $M_1 < M_2$ rest on left and right ends of a plank

Let x be the center of mass if plank is massless

Let X be the center of mass if plank has mass \underline{M} .



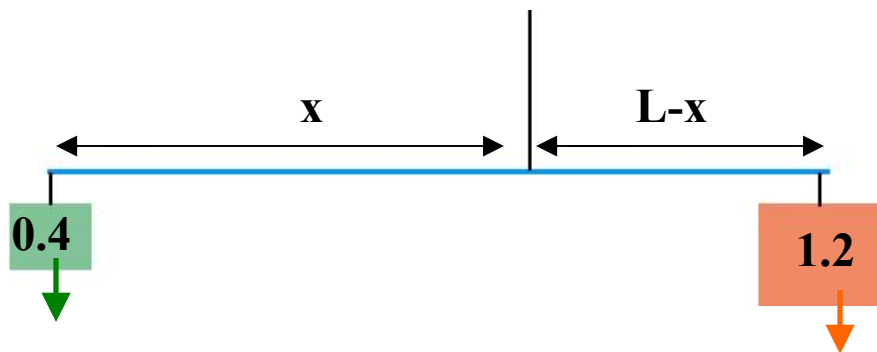
Which statement is true?

- A) $X=x$ C) X is closer to M_2 than x if $M > M_1$.
 B) X is closer to M_1 than x D) X is closer to M_2 than x if $M > M_2$.

Since $M_2 > M_1$, we expect the original center of mass (CM) X to be towards the right side. Giving the plank a mass M means that we effectively place a mass M at the plank's center, thus moving the CM x closer to the center regardless of how large M is compared to the original two masses

4. While building a mobile, I hang a 0.4 lbs weight on the left end of a massless rod of length L and a 1.2 lbs weight on the right end. How far from the left should I suspend the mobile such that it hangs balanced horizontally?

- A) $\frac{1}{4}L$ B) $\frac{2}{3}L$ C) $\frac{3}{4}L$ D) $\frac{5}{6}L$



Balance the two torques:

$$\begin{aligned} \tau_{0.4lbs} &= \tau_{1.2lbs} \\ 0.4 * x &= 1.2(L - x) \\ x &= 3(L - x) \\ x &= \frac{3}{4}L \end{aligned}$$

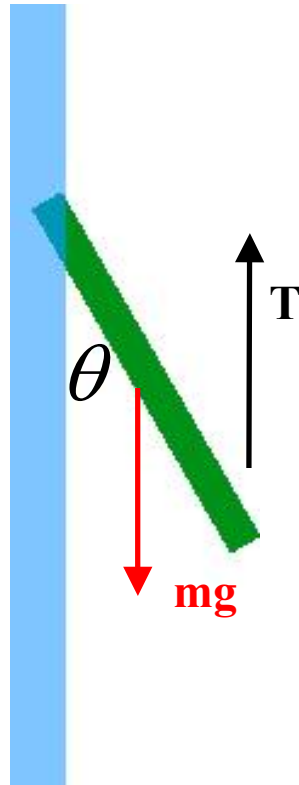
5. A uniform plank of mass M and length L has one end anchored in a wall and the other end supported by a cable with tension T . If the plank hangs at an angle θ from the wall, what is the tension in the cable?

A) $MG/2$

C) MG

B) $MG \sin(\theta) / 2$

D) $MG \sin(\theta)$



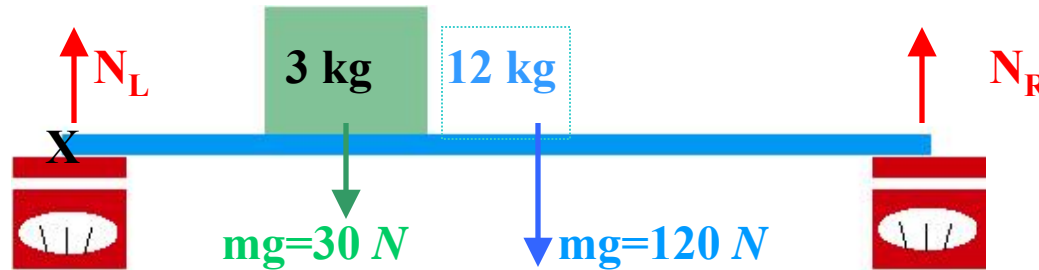
Balance the torques with the wall as the pivot point:

$$\tau_T = \tau_{mg}$$

$$T * L * \sin(\theta) = mg * \frac{L}{2} * \sin(\theta)$$

$$T = \frac{mg}{2}$$

6a. A uniform 12 kg plank of length L rests horizontally with each end supported by a spring scale. I place a 3kg box L/3 from the left end.



What does the scale on the right read?

- A) 50 N B) 70 N C) 80 N D) 90 N

We don't care about the left scale, so use it as a pivot point; this way it will produce zero torque and we can ignore it. Then just balance the torque from the right scale with those from the weights. $\tau_{N_R} = \tau_{mg_box} + \tau_{mg_plank}$

$$N_R * L = 30 * \frac{L}{3} + 120 * \frac{L}{2}$$

$$N_R = 10 + 60 = 70\text{ N}$$

6b. What does the scale on the left read?

- A) 60 N B) 70 N C) 80 N D) 150 N

Now balance all the forces: $F_{NET}=0$

$$N_R = m_{box}g + m_{plank}g - N_L = 30 + 120 - 70 = 80\text{ N}$$

THE TRICK: *NEVER* draw $m \frac{v^2}{r}$ in a force diagram.

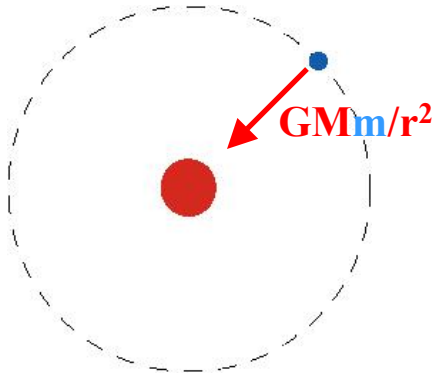
7. If the earth orbits the sun at radius r and speed v , then what is mass of sun?

A) $\frac{G}{v^2 r}$

B) $Gv^2 r$

C) $\frac{v^2}{Gr}$

D) $\frac{v^2 r}{G}$



$$F_{NET} = ma$$

$$GMm/r^2 = m * v^2/r$$

$$M = r * v^2 / G$$

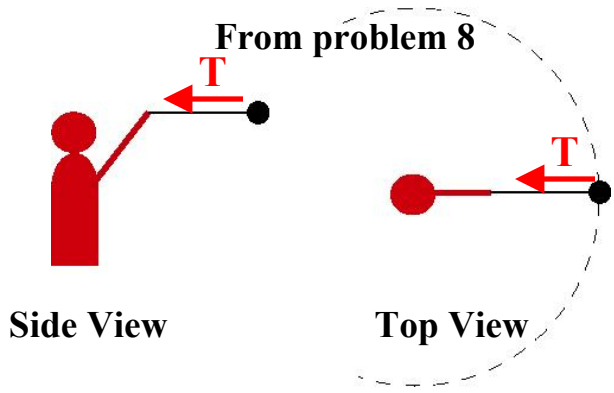
8. A 50 g stone is tied to the end of a 2 m long string and whirled in a horizontal circle at 20 m/s. Ignoring gravity, what is the tension in the string?

A) 5 N

B) 10 N

C) 100 N

D) 500 N



$$F_{NET} = ma$$

$$T = m * v^2 / r$$

$$= (50/1000) * (20)^2 / 2$$

$$= 10 \text{ N}$$

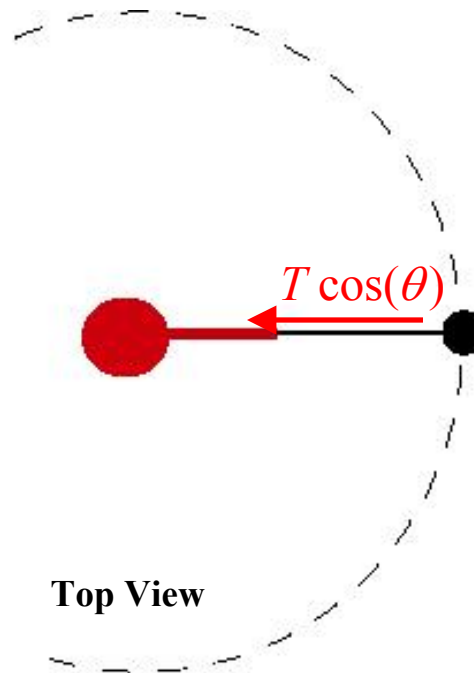
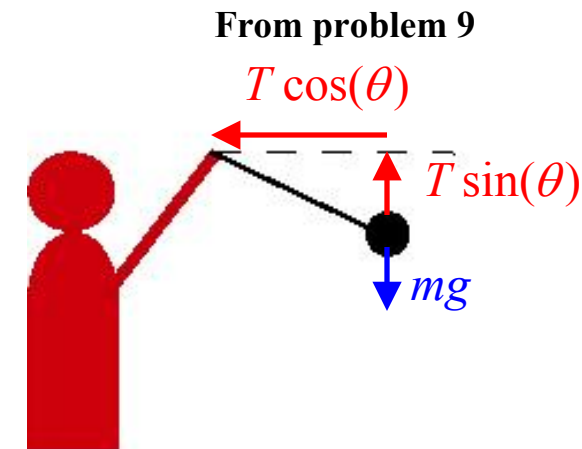
9. An object of mass m is attached to a string and whirled in a horizontal circle of radius r such that the rope is at an angle θ below horizontal. What is stone's speed?

A) $\sqrt{rg \tan(\theta)}$

C) $\sqrt{rg \csc(\theta)}$

B) $\sqrt{rg \cot(\theta)}$

D) \sqrt{rg}



Vertical:

$$F_{\text{NET}} = 0$$

$$T \sin(\theta) = mg$$

$$T = mg / \sin(\theta)$$

Horizontal:

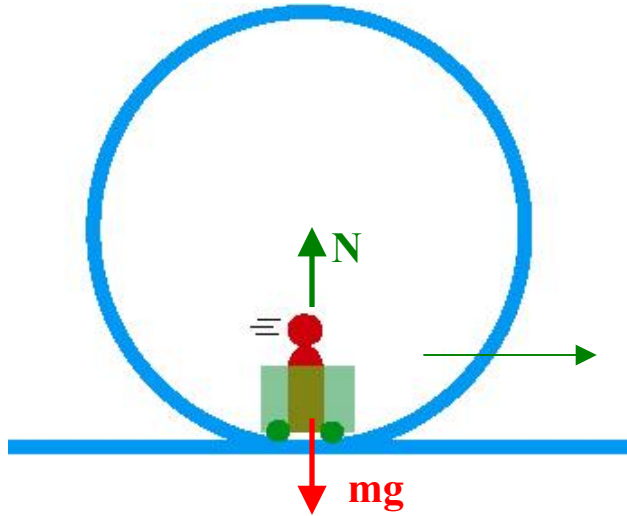
$$F_{\text{NET}} = ma$$

$$T \cos(\theta) = m \cdot v^2 / r$$

$$mg \cos(\theta) / \sin(\theta) = m \cdot v^2 / r$$

$$\boxed{r g \cot(\theta) = v^2}$$

10a. I weigh 100 kg and roll through the bottom of a 20 m roller coaster loop and feel an apparent weight of 1500 N. What is my speed there?



$$F_{\text{NET}} = ma$$

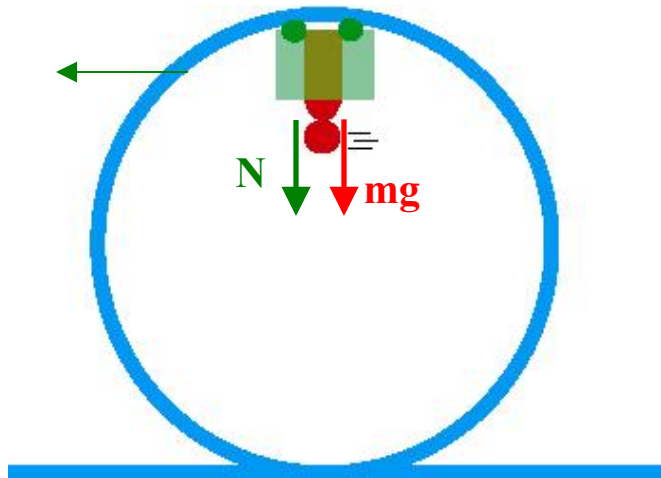
$$N - mg = m \cdot v^2 / r$$

$$1500 - 1000 = 100 \cdot v^2 / 20$$

$$500 = 5 \cdot v^2$$

$10 \text{ m/s} = v$

10b. If I go through the top of that loop with a speed of 16m/s, how heavy will I feel there?



$$F_{\text{NET}} = ma$$

$$N - mg = m \cdot v^2 / r$$

$$N - 1000 = 100 \cdot (16)^2 / 20$$

$$N - 1000 = 1280$$

$N = 280 \text{ N}$

11. Barrel of Fun:

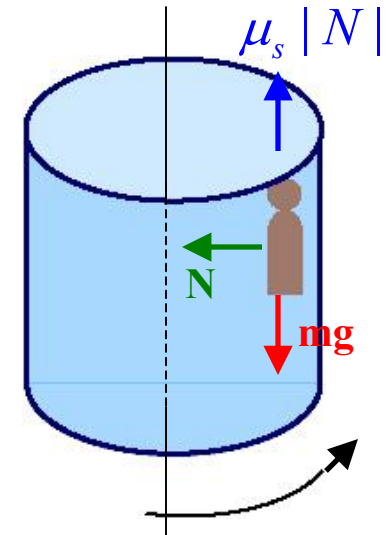
I weigh 100 kg and am centrifuged to the inside of a 10 m radius barrel with $\mu_s = \underline{0.4}$ between me and the wall. How fast must I be moving to ensure that I don't slide down the inner wall of the barrel?

A) 5m/s

C) 15 m/s

B) 10 m/s

D) **20 m/s**



Vertical:

We must have at least this much **N** to avoid sliding

$$F_{\text{NET}} = 0$$

$$\mu_s |N| = mg$$

$$|N| = mg / \mu_s$$

Horizontal:

$$F_{\text{NET}} = ma$$

$$N = m \cdot v^2 / r$$

$$mg / \mu_s = m \cdot v^2 / r$$

$$g r / \mu_s = v^2$$

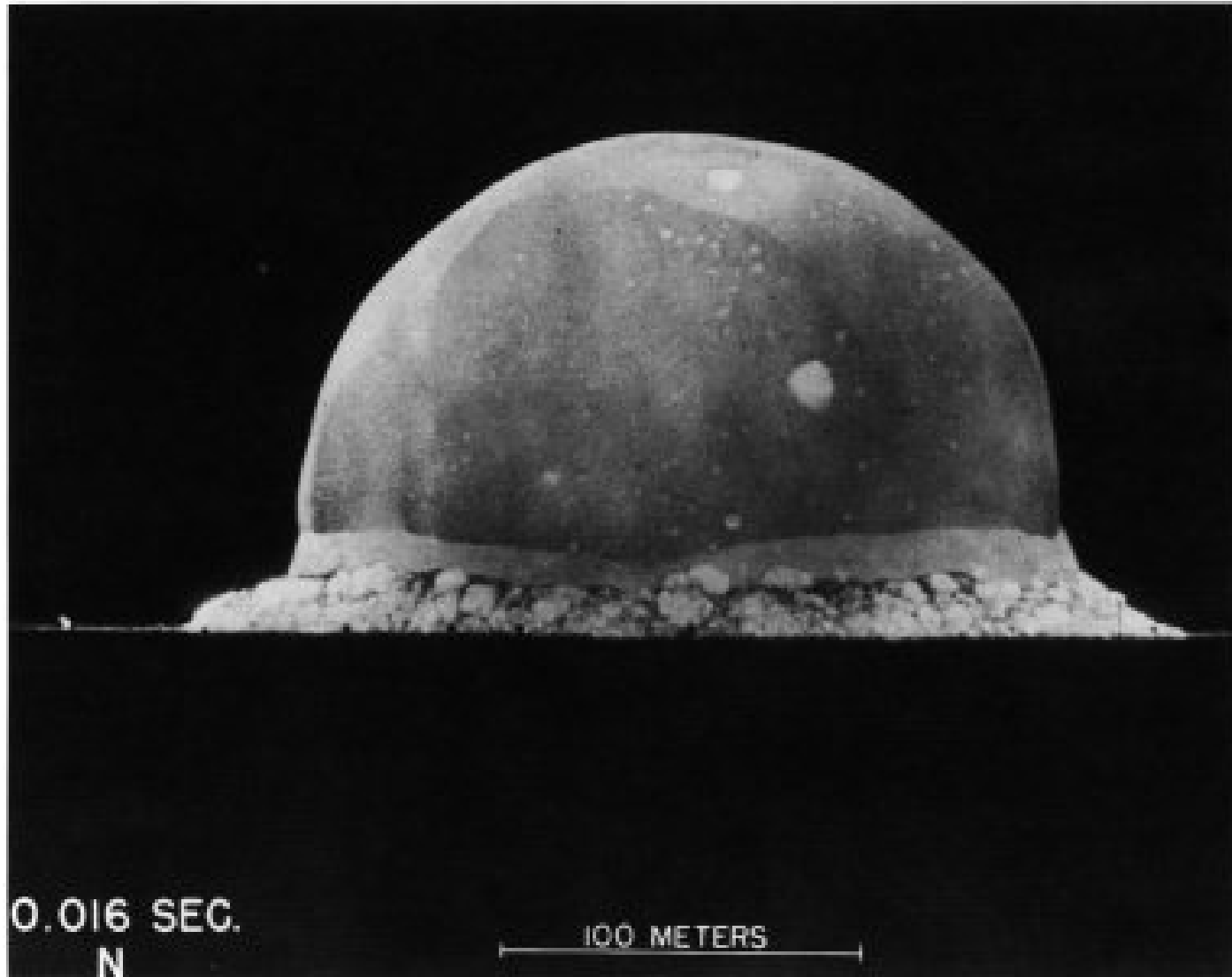
$$10 \cdot 10 / (0.4) = v^2$$

$$400 = v^2$$

$$\boxed{20 \text{ m/s} = v}$$

My mass cancels; that's good. It means people can ride this regardless of their weight!

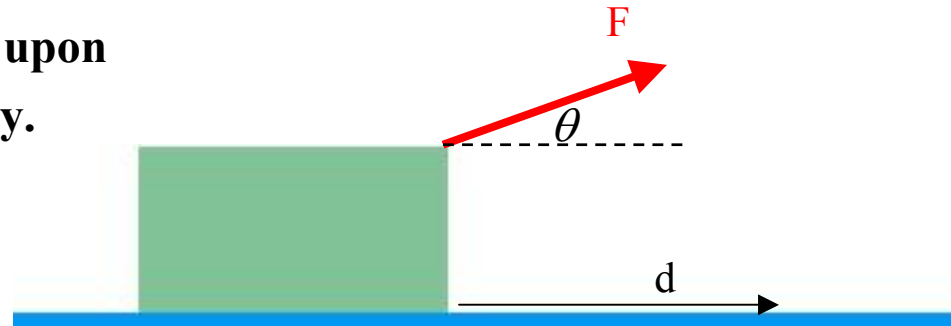
Physics 4: Energy & Momentum



1. What is Energy?: **A dynamic scalar possessed by an object that is transformed as forces Act upon that object.**

2. What is Work?: **Forces do work upon an object as they transform energy.**

$$W = Fd \cos(\theta)$$



3. An object is pulled along the ground by a 50 N force 45° above the horizontal. How much work does that force do over 8 m?

- A) 100 J **B) 280 N** C) 400 N D) 620 N

Just plug in the numbers to the work Eqn & be aggressive about approximations.

4. How much work does gravity do as a 10 kg object is lifted from a height of 1 m to 3 m above ground?

- A) -300 J **B) -200 J** C) 200 J D) 300 J

$d=2$ & $\theta=180^\circ$ since gravity is down & displacement is up. So the work is negative

5a. A 10 kg crate slides down a 2 m long inclined plane at an incline angle of 30°. μ_K is 0.4. How much work is done by friction?

- A) **-68 J** B) -39 J C) -34 J D) -20 J

Again, $\theta=180^\circ$ since friction is back up the plane while the displacement is down the plane.

5b. How much work is done by the normal force?

$W_{\text{normal}} = 0$

A normal force can never do any work on a sliding object since it ³³always acts perpendicular to the surface an object slides along. $\theta = 90^\circ$

6. How do we make Kinetic Energy of an object negative?

$$KE = \frac{1}{2}mv^2 \text{ can never be negative because } m > 0 \text{ \& } v^2 > 0$$

7a. What is the box's potential energy at point A?

$PE = mgh$ & the answer depends on where we pick $PE=0$.

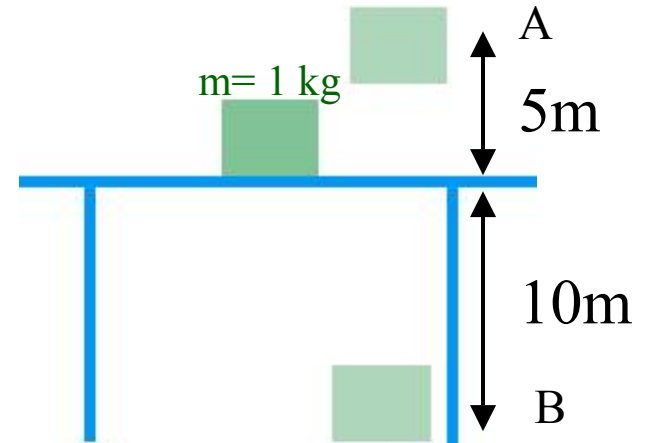
- If $PE=0$ at table top, then $PE(h=A) = 50 \text{ J}$.
- If $PE=0$ at ground, then $PE(h=A) = 150 \text{ J}$

7b. What is the box's potential energy at point B?

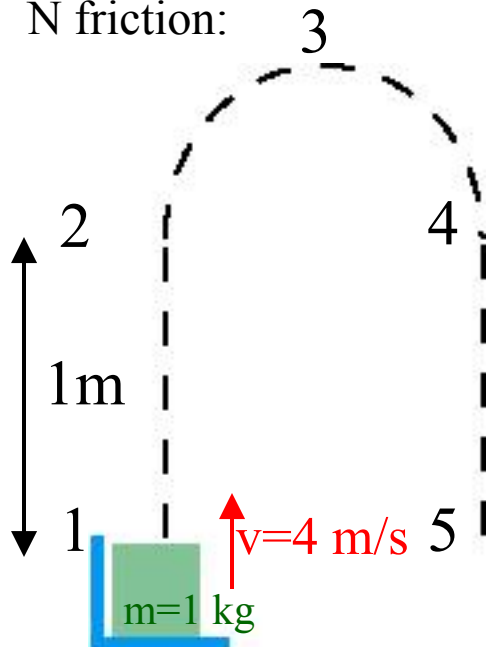
If $PE=0$ at table-top, then $PE(h=B) = -100 \text{ J}$

There is nothing wrong with having negative potential energy

Work Energy Theorem: $W = \Delta KE$



Keep track of energy & work as object is thrown. A parachute opens @ pt. 4, providing 5.5 N friction:



	$KE = (1/2)mv^2$	$PE = mgh$
$W_{mc} = +10 \text{ J}$	1 8J	$W_g = -10 \text{ J}$ 0J
	2 8J	$W_g = -8 \text{ J}$ 10J
	3 0J	$W_{mc} = +8 \text{ J}$ 18J
	4 8J	$W_{mc} = +10 \text{ J}$ 10J
$W_g = -5.5 \text{ J}$	5 12.5 J	0J

5.5 J heat

For real problems, don't do detailed book-keeping; just conserve energy.

8. A 10 kg mass is dropped from a height of 125 m. What's the impact speed with the ground?

- A) 20 m/s B) **50 m/s** C) 75 m/s D) 125 m/s

If we pick PE=0 @ bottom, then the PE @ top becomes all KE @ bottom. Note that m cancels out.

9. A 1 kg ball is dropped from a height of 6 m and feel a constant 3.3 N air resistance. What's the impact speed with the ground?

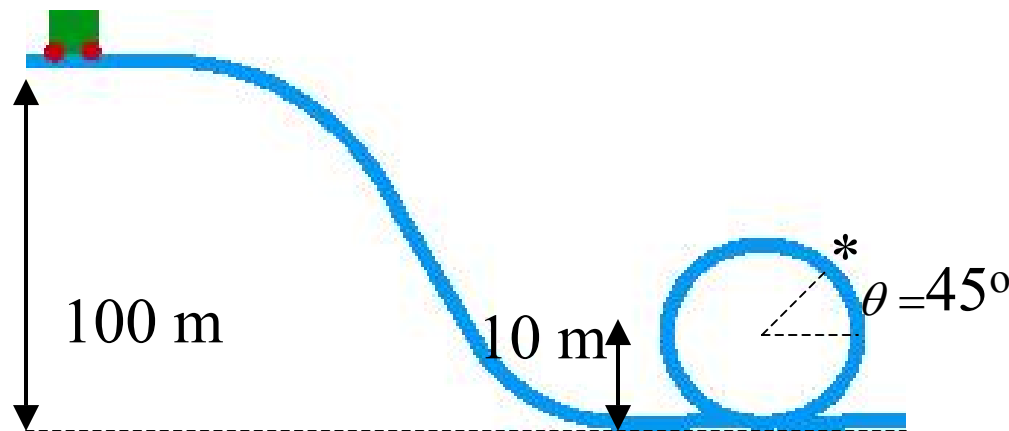
- A) **9.0 m/s** B) 10.0 m/s C) 10.6 m/s D) 11.1m/s

m won't just cancel out this time. Let's pick PE=0 @ top. Then total E=0. So 0=PE+KE+heat, where PE<0, heat>0 & KE>0

10. A 1 kg weight is attached to one end of a 5 m long massless rod. It is anchored to a pivot point on the other end, lifted to a horizontal orientation and released from rest. What is its maximum speed?

- A) **10 m/s** B) 20 m/s C) 30 m/s D) 40 m/s

m cancels again because there's no friction. Just set initial PE to final KE at bottom of pendulum's swing where KE (& hence v) is greatest.



11. The roller-coaster car at left starts from rest. How quickly is it moving at the point "*" on the loop?

The final height is ~17m above the dashed line. Just conserve energy, canceling out mass (no friction) & being consistent about the zero point for PE:

$$PE(h=100)=PE(h=17)+KE$$

$$m \cdot 10 \cdot 100 = m \cdot 10 \cdot 17 + 0.5 \cdot m \cdot v^2$$

$$v \sim 40 \text{ms}$$

12. If the earth has mass M and radius R , at least how quickly must we throw a mass m vertically such that it never falls back to the earth?

A) $\sqrt{\frac{2GM}{R}}$

B) $\sqrt{\frac{2GMm}{R}}$

C) $\sqrt{\frac{GM}{2R}}$

D) $\sqrt{\frac{GMm}{2R}}$

PE = $-GMm/r$ forces you to pick PE = 0 at $r = \text{infinity}$. If an object just reaches infinity, KE = 0 there. So total E = 0. So at the surface of the earth ($r = R$):

0 = KE + PE = $mv^2/2 - GMm/R$. Solve for v .

13. What power is needed to lift a 100 N object at a constant 5 m/s?

$P = F \cdot v = 100 \cdot 5 = 500 \text{ W}$

A) 0 W

B) 20 W

C) 200 W

D) 500 W

14. What power is needed to bring a 1000 kg car moving at 20 m/s to rest in 4 s?

The *magnitude* of work needed to stop this car is :

$W = |0 - mv^2/2| = 0.5 \cdot 1000 \cdot (20)^2 = 200 \text{ kJ}$.

$P = W/t = 200/4 = 50 \text{ kW}$

A) 5 kW

C) 500 kW

B) 50 kW

D) 5000 kW

15. Which requires the strongest engine?

i.e which represents the most power?

A) 20 J in 10 minutes

C) 200 J in 10 min

Answer: the one with the greatest ratio of W/t ; That with the most work and least time.

B) 100 J in 20 minutes

D) 10 J in 20 min

16. A fork-lift consumes 500 W over 10 s as it lifts a 1 kg crate. The box is then pushed off the lift and falls freely to the ground. What is the speed of impact?

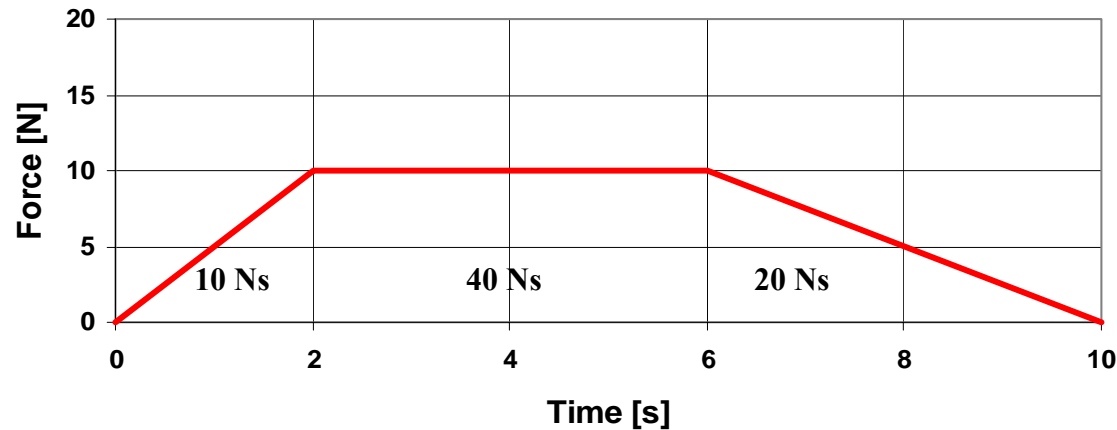
The box is given potential energy of $W = P \cdot t = 500 \cdot 10 = 5000$. If this energy gets converted into kinetic energy, then $5000 = mv^2/2 = 0.5 \cdot 1 \cdot v^2$. So $v = 100 \text{ m/s}$

A) 10 m/s

B) 32 m/s

C) 50 m/s

D) 100 m/s



17. If a 2 kg object moving initially at 10 m/s is subjected to the force depicted in the graph above, how fast is it moving 10 s later?

Initial momentum $P_0 = m \cdot v_0 = 2 \cdot 10 = 20 \text{ Ns}$.

The impulse $J = F \cdot t = \text{area under that graph} = 70 \text{ Ns}$

So final momentum $P = P_0 + J = 90 \text{ Ns}$. So $v = 45 \text{ m/s}$ at end.

18. A cue-ball (255 g) moving at 0.5 m/s hits the 8-ball (200 g) originally at rest. After the collision, the cue-ball moves with 0.1 m/s in the same direction as before. How fast is the 8-ball moving now?

- A) 0.23 m/s B) **0.45 m/s** C) 0.68 m/s D) 0.90 m/s

Conserve Momentum: $P_0 = P$. DON'T try to conserve Energy

$$P_0 = m_{\text{cue}} \cdot v_{\text{cue}} + m_8 \cdot 0 \sim (0.25) \cdot (0.5) = 1/8$$

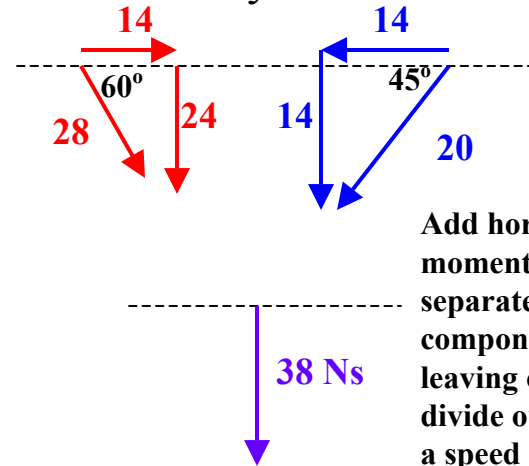
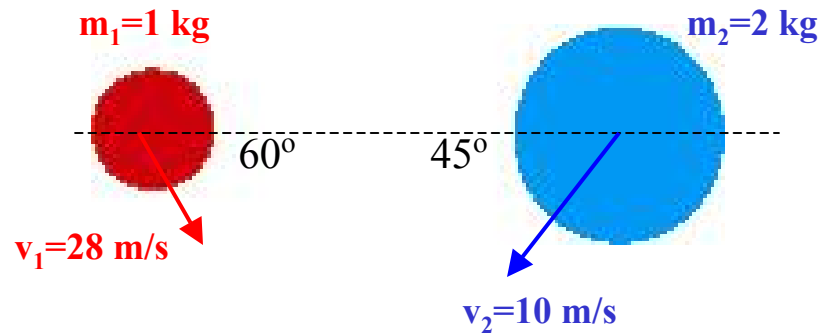
$$\text{So } 1/8 = P = m_{\text{cue}} \cdot v_{\text{cue}} + m_8 \cdot v_8 = (0.25) \cdot (0.10) + (0.20) \cdot v_8 = (1/4) \cdot (1/10) + (1/5) \cdot v_8 \quad \text{So } v_8 = 0.5 \text{ m/2}$$

19. A mass m traveling at a speed v collides with and sticks to an object with twice its mass sitting at rest, How quickly does the mass move after the collision?

- A) $v/4$ B) **$v/3$** C) $v/2$ D) $2v/3$

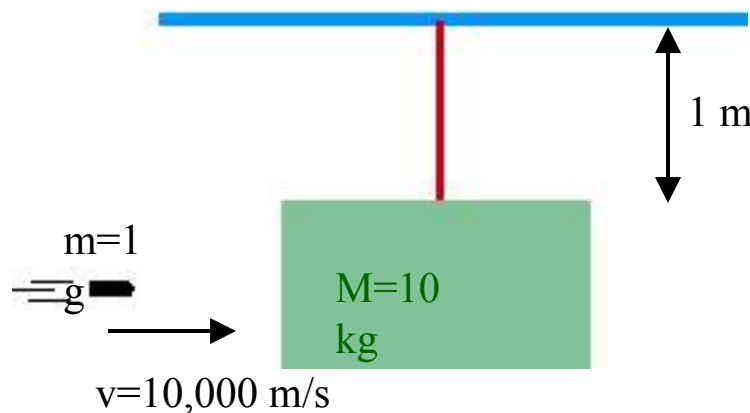
Conserve Momentum: $P_0 = P: \quad m \cdot v + (2m) \cdot 0 = (3m) \cdot v'$. So $v' = v/3$

19. These meteors collide and stick. What's their velocity after the collision?



Add horizontal & vertical momentum components separately. The horizontal components will cancel, leaving only vertical. Then divide out the 3 kg mass to get a speed of 13 m/s

20. A 1 g bullet traveling at 10,000 m/s hits a 10 kg block hanging at rest from a 1 m long string attached to the ceiling. The bullet gets lodged in the block.

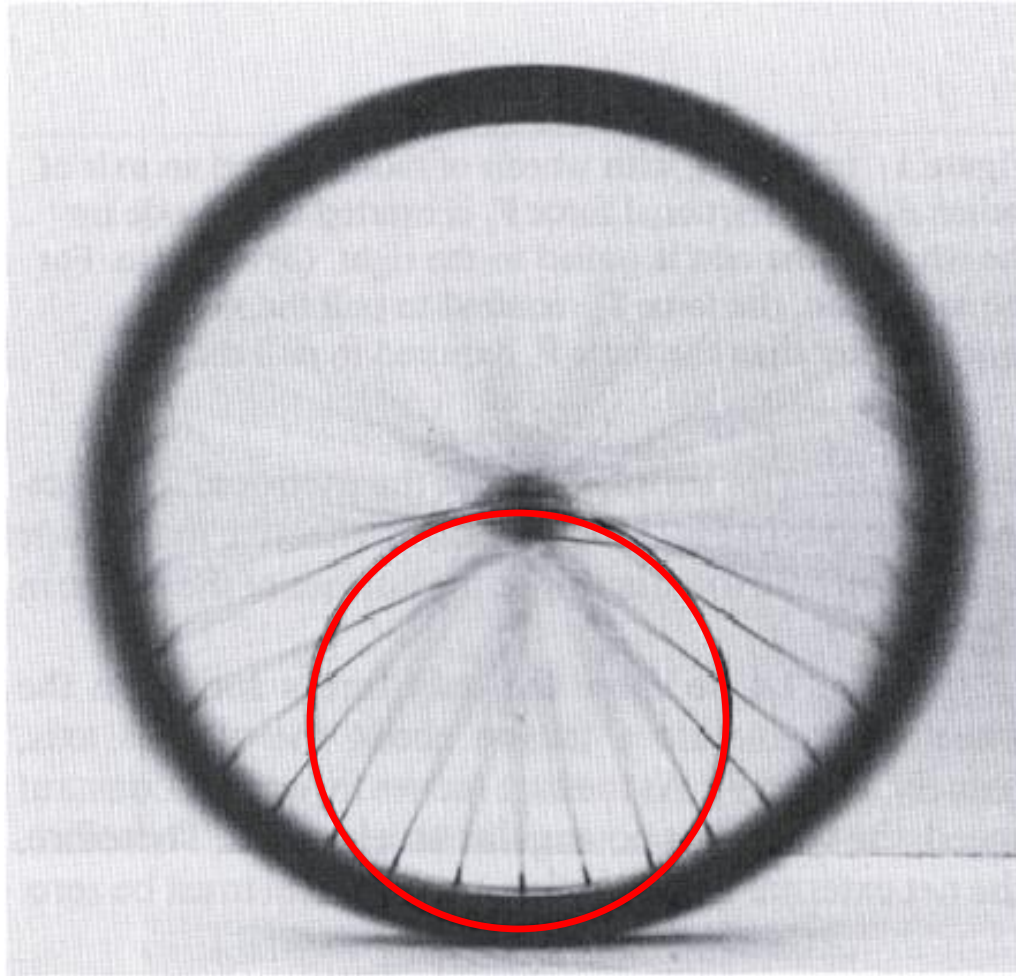


Conserve momentum during the collision and assume the final mass of block is ~10 kg. Then conserve energy as the block (and bullet) swing up.
 Momentum: $(1/1000) * (10,000) \sim (10) * v$
 so $v \sim 1 \text{ m/s}$
 Energy: $KE = PE$
 $0.5 * (10) * (1)^2 = (10) * (10) * h$
 so $h = 0.05 \text{ m}$

How high does it swing up?

- A) 5 cm B) 5 m C) 10 cm D) 10 m

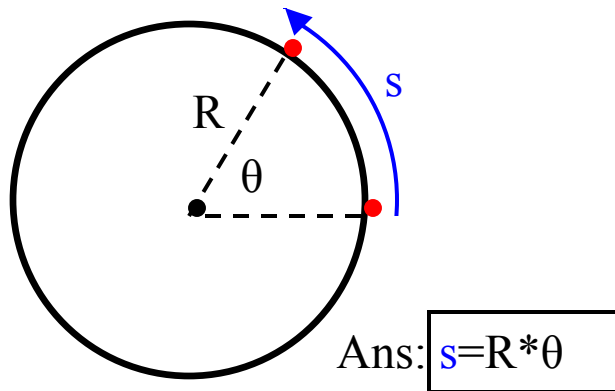
OH 1: Rotational Physics



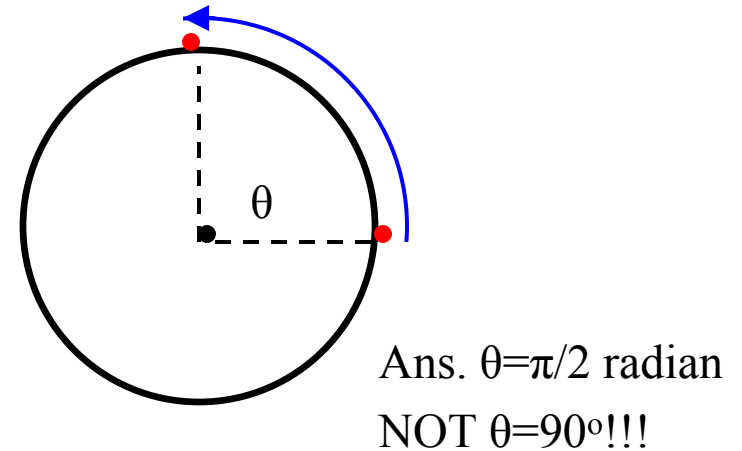
This stuff is extra; you probably don't need to know it in advance, but it may appear in a passage.

Angle θ and Angular Speed ω

How far (s) does the red dot move as the wheel rotates θ ?



What value of θ would I use here?



After one revolution of a 4m radius wheel, how far has the red dot moved?

The dot moves the circumference of the wheel: $2\pi*R=8\pi\sim 25\text{m}$

OR we can say it rotates $\theta = 1 \text{ rev} = 2\pi$, so $s=R*\theta = 4*(2\pi)=8\pi\sim 25\text{m}$

If during a small time Δt , the wheel rotates a small angle $\Delta\theta$, how far has the dot moved?

$$\Delta s = R * \Delta\theta$$

What's the speed of this dot?

$$v = \frac{\Delta s}{\Delta t} = R * \frac{\Delta\theta}{\Delta t} = R * \omega$$

$$v = R * \omega \quad \text{where } [\omega] = \text{rad/s} = "1/\text{s}"$$

Angular Acceleration α

What's the difference between the frequencies ω and f ?

Ans. The difference is in the units: $[\omega]=\text{rad/s} = "1/\text{s}"$ and $[f]=\text{cycles/s} = "Hz"$

f tells you how many *revolutions* the wheel rotates thru in a second

ω tells you how many *radians* the wheel rotates thru in a second

Since there are π radians in a revolution, $\omega = 2\pi * f$

If during a small time Δt , the wheel increases angular speed $\Delta\omega$, how much faster will the dot move?

$$\Delta v = R * \Delta\omega$$

What's the acceleration of this dot?

$$a = \frac{\Delta v}{\Delta t} = R * \frac{\Delta\omega}{\Delta t} = R * \alpha \quad \boxed{a = R * \alpha} \text{ where } [\alpha] = \text{rad/s}^2 = "1/\text{s}^2"$$

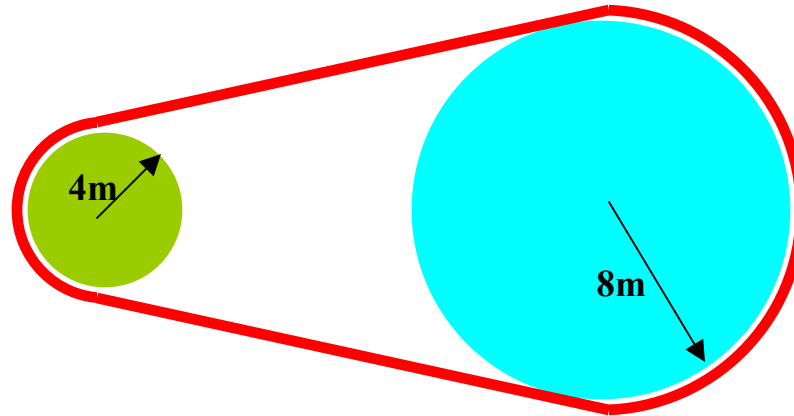
$$\boxed{\begin{array}{l} s = R * \theta \\ v = R * \omega \\ a = R * \alpha \end{array}}$$

Since these new angular kinematic variables are so similar to their Linear counterparts, they satisfy their own **"Big 5" Equations**

1. $\theta = \frac{1}{2}(\omega_0 + \omega)t$
2. $\omega = \omega_0 + \alpha t$
3. $\theta = \omega_0 t + \frac{1}{2} \alpha t^2$
4. $\theta = \omega t - \frac{1}{2} \alpha t^2$
5. $\omega^2 = \omega_0^2 + 2\alpha \theta$

Rotational Kinematics Problem

An engine accelerates a 4 m radius wheel at $1/3 \text{ s}^{-2}$ from rest. This wheel is attached by a belt to an 8m radius wheel. After 5 revolutions, how fast is the large one rotating?



A) 0.9 s^{-1}

B) 2.2 s^{-1}

C) 3.65 s^{-1}

D) 8.9 s^{-1}

Use the 5th equation to analyze the small wheel (the one which the engine drives directly):

$$\omega^2 = \omega_0^2 + 2\alpha\theta = 0^2 + 2(1/3)(2 \cdot 2\pi) \sim 20. \text{ So } \omega_{\text{small}} = 2 \cdot 5^{1/2}.$$

Now how fast is the large wheel moving at this time?

*The Belt forces the two wheels to have the same *tangential* speed*: $v_{\text{small}} = v_{\text{large}}$

From $v = R \cdot \omega$, we see that when we double R , we must half ω .

$$\text{So } \omega_{\text{large}} = 5^{1/2} \sim 2.2 \text{ s}^{-1}.$$

Angular *Dynamic* Variables: τ & L

Recall that **torque** is:

$$\tau = R * F * \sin(\varphi)$$

Dynamic variables bear a relationship to their linear counterparts that's opposite to those of the kinematic variables

$$s = R * \theta$$

$$v = R * \omega$$

$$a = R * \alpha$$

Angular Momentum is: $L = R * p * \sin(\varphi)$

φ : angle between R and p (or F) often = 90°

These new angular *dynamic* variables are similar to their Linear counterparts, So they satisfy their own:

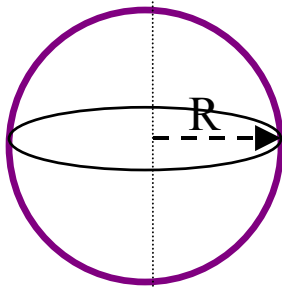
- **Newton's 2nd Law:**
- **Angular Momentum Conservation:**

$$\tau_{\text{NET}} = I * \alpha$$

$$L = I\omega \text{ is conserved when } \tau_{\text{net}} = 0$$

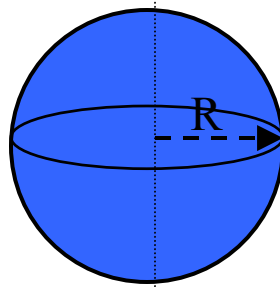
Moment of inertia I quantifies how heavy something is & how far it is from rotation axis:
Add up mr^2 for each chunk of mass.

Spherical Shell



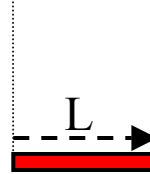
$$I = (2/3)MR^2$$

Solid Sphere



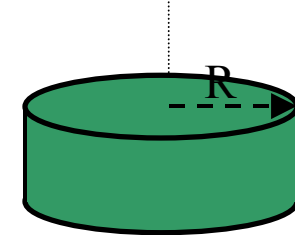
$$I = (2/5)MR^2$$

Thin rod



$$I = (1/3)ML^2$$

Solid Disk



$$I = (1/2)MR^2$$

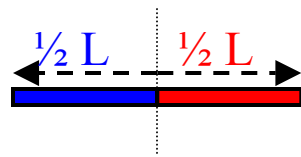
Sample Problems for Moment of Inertia

A thin rod with mass m and length L is rotated around an axis perpendicular to its length that passes through its midpoint (not at its end). What's the rotational inertia?

A) $I = (1/12)mL^2$ B) $I = (1/3)mL^2$ C) $I = (1/2)mL^2$ D) $I = (2/3)mL^2$

Ans1: The above table tells us that a rod rotating about its end must have $(1/3)mL^2$. But rotating about the center puts the mass, on average, closer to the axis. So we need something smaller. Hence A.

Ans2: This rod can be thought of as two rods with length $L/2$ and mass $m/2$ that are mounted at their ends. Then we can use the above formula and just add the moments:



$$I = I_{\text{left}} + I_{\text{right}} = (1/3) * (m/2) * (L/2)^2 + (1/3) * (m/2) * (L/2)^2 \\ = (1/24) * mL^2 + (1/24) * mL^2 = (1/12) * mL^2$$

As a hockey-puck rolls along, how many times bigger is its translational kinetic energy than its rotational kinetic energy?

A) $1/4$ B) $1/2$ C) same D) 2

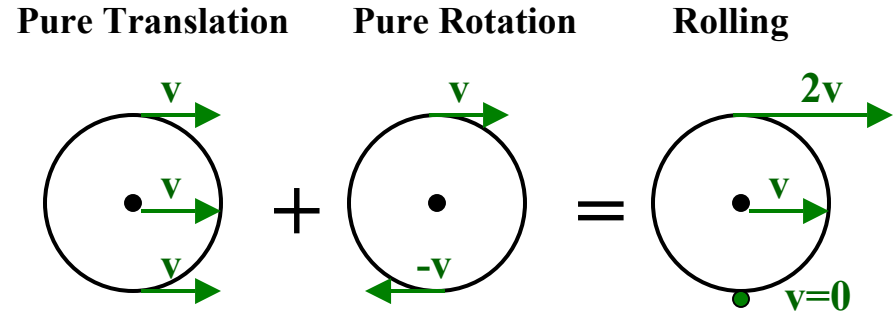
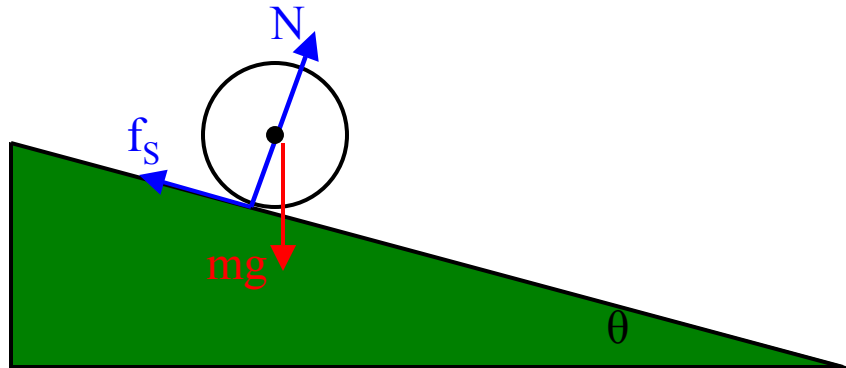
While Translational $KE_T = \frac{1}{2} m v^2$, Rotational $KE_R = \frac{1}{2} I \omega^2$

$$= \frac{1}{2} (\frac{1}{2} m R^2) (v/R)^2 \\ = \frac{1}{2} (\frac{1}{2} m v^2) \\ = \frac{1}{2} (KE_T)$$

Rolling objects

As the hockey puck rolls down an inclined plane, what force provides the torque to generate rotation?

- A) Gravity B) Normal Force C) **Static Friction** D) Kinetic Friction



Since bottom is still, it's *static* friction, not kinetic!

$$F_{\text{NET}} = m \cdot a: F_{\text{NET}} = mg \sin(\theta) - f_s$$

$$\tau_{\text{NET}} = I \cdot \alpha: R f_s = I \cdot (a/R)$$

eliminate f_s \longrightarrow

$$F_{\text{NET}} = mg \sin(\theta) - I \cdot a/R^2$$

Which of these will reach bottom first if they all have the same mass?

- A) Hollow Sphere B) **Solid Sphere** C) Cylindrical Disk D) All the same

The object with *smallest* I will have least reduction in F_{NET} , and hence the greatest acceleration.

From the table, that's the Solid Sphere.

Which of these will have the lowest translational speed at bottom?

- A) **Hollow Sphere** B) Solid Sphere C) Cylindrical Disk D) All the same

Energy Conservation: $PE = KE_T + KE_R$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

45

Since mgh is fixed, the object with *greatest* I will have the lowest v . That's the hollow sphere.

Conservation of Angular Momentum

I am spinning on a lazy-susan at 3 s^{-1} while holding two bar-bells 50 cm from me. I weigh very little compared to the bar-bells. If I pull them into 10 cm from me, how fast will I spin?

Recall that impulse was defined as $\Delta p = F_{\text{NET}} * t$.

If we multiply by the radius R, then: $\Delta(Rp) = R F_{\text{NET}} * t$

or $\Delta L = \tau_{\text{NET}} * t$

So if $\tau_{\text{NET}} = 0$, then L is conserved.

This L can be written as

$L = Rp = r * mv$

$= (mr^2) * (v/r)$

$L = I * \omega$

As we pull each barbell in, we decrease R by 5, which drops I by 25

Since L is conserved, ω must increase by 25. So $\omega = 75 \text{ s}^{-1}$

The clutch at right has two plates, the bottom of which is twice as massive and has twice the radius of the top. The bottom spins 4 times slower and in the opposite direction than the top. When we push the plates together, how fast will they rotate?

- A) $(3/8) \omega_0$ B) $(1/3) \omega_0$ C) $(1/8) \omega_0$ D) $(1/9) \omega_0$

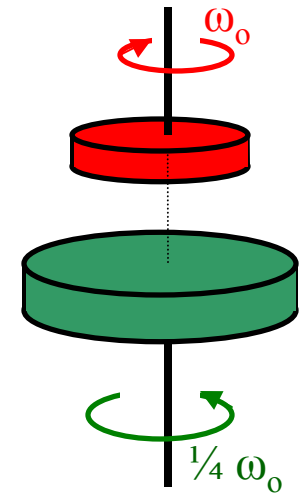
These are disks with (angular) inertia $\frac{1}{2}MR^2$, so $I_{\text{bottom}} = 8 * I_{\text{top}}$.

So if $L_{\text{top}} = I_{\text{top}} * \omega_0$, then $L_{\text{bottom}} = -(8 I_{\text{top}}) * (\omega_0/4) = -2 I_{\text{top}} * \omega_0$.

Together, total momentum is $-I_{\text{top}} * \omega_0$, and total inertia is $9I_{\text{top}}$.

Since L is conserved, and the collective inertia is 9 times larger,

the final rotation must be $(1/9) \omega_0$



Physics 5: Fluids



Archimedes Principle: Buoyant Force = weight of fluid displaced

1. An object of mass 4 kg floats on a fluid with specific gravity 0.8. What's the magnitude of the buoyant force?

- A) 6 N C) 50 N
 B) 32 N D) **40 N**

Since the object floats, $F_{NET}=0$ and so it's buoyant force must equal it's weight:

$$B = W = mg = 4 \cdot 10 = 40 \text{ N}$$

2. An object has specific gravity 4/5. How much of it's volume will float above water?

- A) **1/5** C) 1/2
 B) 1/4 D) 4/5

Once again, since the object floats, $F_{NET}=0$ and so it's buoyant force must equal it's weight:

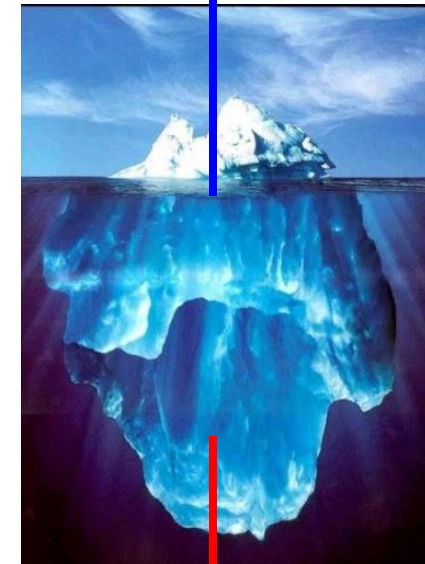
$$B = W$$

$$\rho_{water} g V_{submerged} = \rho_{object} g V_{Total}$$

$$\frac{V_{submerged}}{V_{total}} = \frac{\rho_{object}}{\rho_{water}} = \frac{4}{5}$$

So if 4/5 of the object's volume is underwater, then the remaining 1/5 must float above the water

$$B = \rho_{fluid} g V_{submerged}$$



$$W = \rho_{object} g V_{Total}$$

3. An person weighs 100 N in air but has an apparent weight of 60 N under-water. What is his specific gravity?

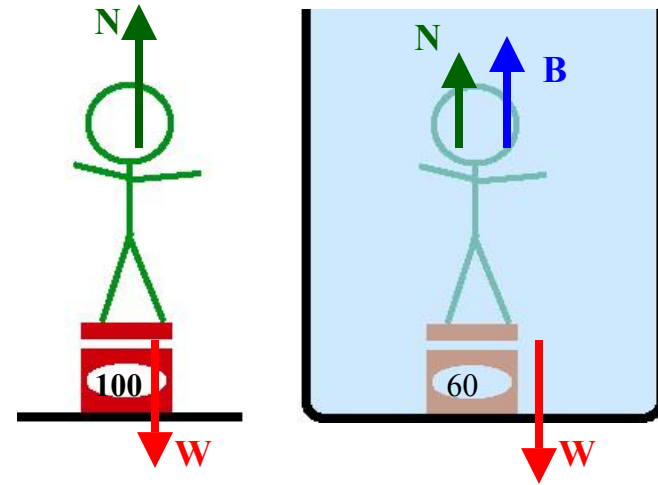
- A) 0.4 B) 0.6 C) 1.67 D) **2.5**

The specific gravity (spg) is nothing more than a sunken objects weight divided by buoyant force:

$$spg = \frac{\rho_{object}}{\rho_{water}} = \frac{\rho_{object} * gV}{\rho_{water} * gV} = \frac{W}{B}$$

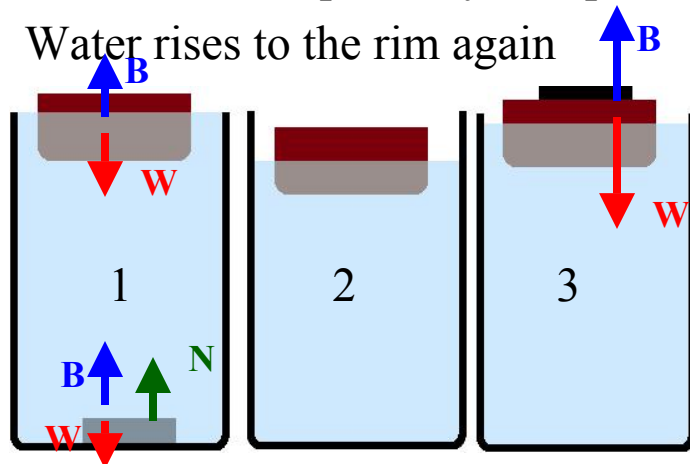
We know **W = 100 N**, and we can get the buoyant force by balancing forces ($F_{NET}=0$) on the submerged man once we recall that “apparent weight” is the normal force: **B = W - N = 100 - 60 = 40 N**

$$Spg = W / B = 100 / 40 = 2.5$$



4. A barge floats in a pool filled to the rim and with steel block at the bottom . If I lift the blocks out of the pool, the water level will obviously drop. If I place the steel on the barge, then:

- A) Water will rise part way to top C) **The water overflows**
 B) Water rises to the rim again D) We can't determine



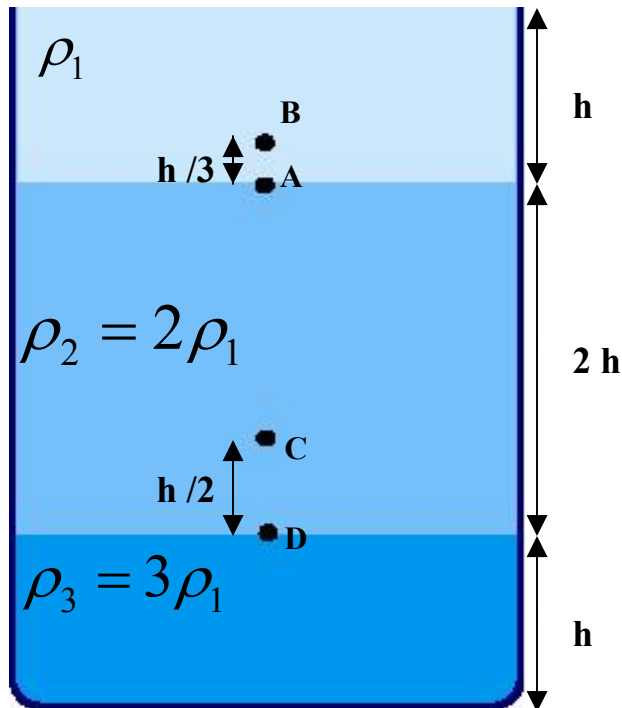
Originally, the water had to provide a buoyant force to balance the weight of the barge and partially balance the weight of the steel (*the normal force makes up the rest of the upward force for the steel*)

Once the steel is in the barge, the total buoyant force must support the barge *and all of the steel's* ⁴⁹weight. So more water must be displaced than before.

Gauge Pressure: $P = \rho gh$

5. A diver at point A reads a pressure P_0 more than at the surface. What does she read at:

- a. Point B: A) 0 B) $P_0/3$ **C) $2 P_0/3$** D) P_0
 b. Point D: A) $2 P_0$ B) $3 P_0$ C) $4 P_0$ **D) $5 P_0$**
 c. Point C: A) $3P_0/2$ B) $5P_0/2$ C) $3 P_0$ **D) $4 P_0$**



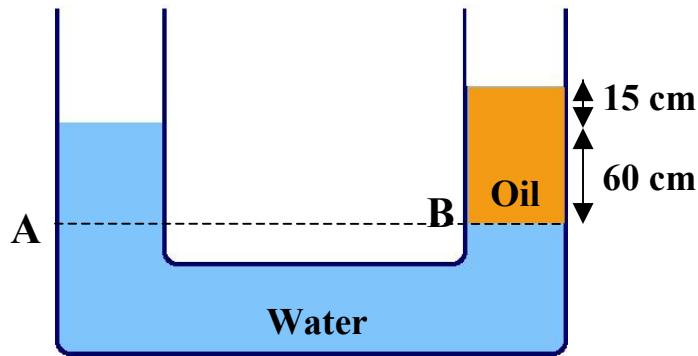
- a. **If the diver moves from A to B, his depth decreases by $2/3$. So will his gauge pressure.**
- b. **When she descends from A to D, she goes thru a layer that is twice as thick as the first and twice as dense. This will hence add a pressure of $4P_0$ on top of her original P_0 to give a total pressure of $5P_0$.**
- c. **Instead of adding the extra $4P_0$ from part b, we now only get thru a layer that is $3/2$ thicker than the top, but still twice as dense. So we add $3P_0$ to the original P_0 , yielding a total of $4P_0$.**

6. How far must I dive under the ocean ($\text{spg}=1.025$) to triple my pressure from the surface?

At the surface, my pressure is $1 \text{ atm} = 10^5 \text{ Pa}$. I can add 1 atm to my pressure by diving under 10 m of water ($\text{spg}=1$; you should check that this is true).

So to triple my pressure of 1 atm , I must add 2 atm , which means I must dive 20 m

- A) 2 m B) 20 m C) 25 m D) 30 m

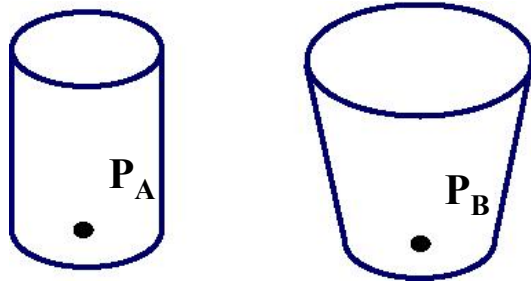


7. What is the specific gravity of the oil at left?

- A) $1/5$ C) $4/5$
 B) $1/4$ D) $3/2$

The pressure must be the same on both sides of the dashed line (points A and B); if pressure isn't balanced, these fluids will not be at rest.

$$\begin{aligned}
 P_A &= \rho_{\text{water}} gh & P_B &= \rho_{\text{oil}} gh \\
 &= \rho_{\text{water}} g(60\text{cm}) & &= \rho_{\text{oil}} g(75\text{cm}) \\
 60\rho_{\text{water}} &= 75\rho_{\text{oil}} \\
 \frac{\rho_{\text{oil}}}{\rho_{\text{water}}} &= \frac{60}{75} = \frac{4}{5}
 \end{aligned}$$

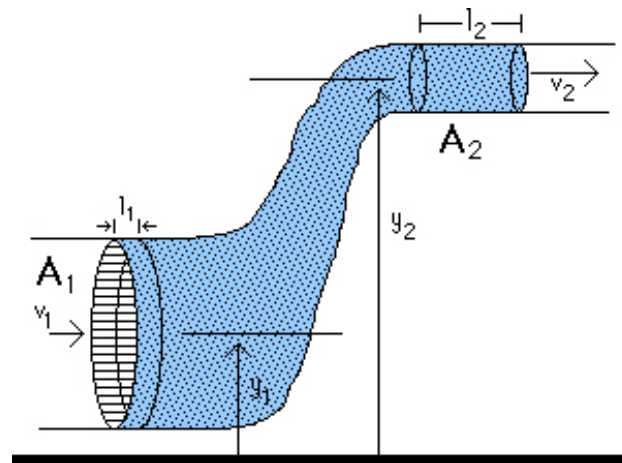


8. The cups at left are filled to the top with water. What is the ratio of pressures at their bases P_B/P_A ?

- A) $1/2$ B) $3/2$
 C) 1 D) 2

Pressure only depends on depth, not shape of container. So $P_A = P_B$

Continuity Equation



- This essentially says that mass is conserved as a fluid flows down a pipe (nothing gets lost or added)

- Flow rate = Av must be constant

$$A_1 v_1 = A_2 v_2$$

9. Water travels down a tube whose area doubles along the length. By what factor does the flow rate change?

A) $\frac{1}{4}$

B) $\frac{1}{2}$

C) 2

D) It Doesn't

Flow rate Av is constant along a pipe; flow speed can change. This is a classic MCAT gag; don't fall for it!

10. If water travels down a tube whose diameter increases from 2mm to 6mm, how will the flow speed change?

A) Decrease by $\frac{1}{9}$

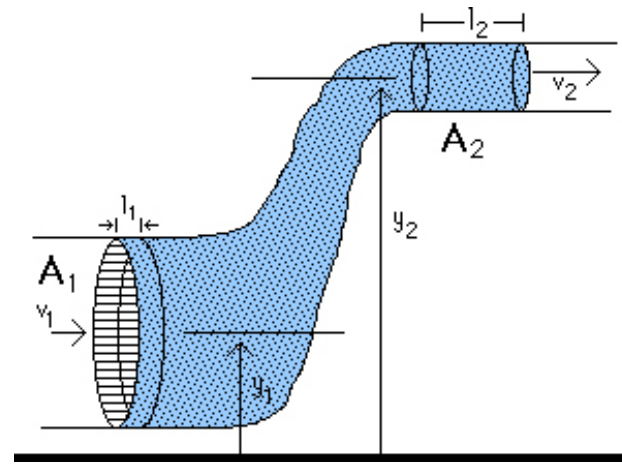
C) Increase by 3

B) Decrease by $\frac{1}{3}$

D) Increase by 9

If the diameter triples, the area increases by 9. But since the flow rate must be constant, the flow speed must fall by 9

Bernoulli Equation



• This says that energy (per unit volume) is conserved as a fluid flows down a pipe.

- $\frac{1}{2} \rho v^2$ is kinetic
- $\rho g y$ is potential
- P is work

$$\frac{1}{2} \rho v_1^2 + \rho g y_1 + P_1 = \frac{1}{2} \rho v_2^2 + \rho g y_2 + P_2$$

or $\frac{1}{2} \rho v^2 + \rho g y + P$ is constant along a pipe

11. A fluid with $\text{spg}=10$ accelerates from 7m/s to 5m/s as it falls 80cm . If the pressure at the top was 1 atm , how much greater will it be at the bottom?

A) 1.2 times greater

B) 2 times greater

C) 1.2 times greater

D) 3 times greater

$$P_2 - P_1 = \frac{1}{2} \rho (v_1^2 - v_2^2) + \rho g (y_1 - y_2)$$

$$= \frac{1}{2} (10^4) * (49 - 25) + (10^4) * (10) * (0.8)$$

$$= 1.2 * 10^5 \text{ Pa} + 0.8 * 10^5 \text{ Pa}$$

$$= 2 * 10^5 \text{ Pa} = 2 \text{ atm}$$

• $\text{Spg}=10$ tells us that $\rho = 10 * 10^3$

• Note that since the fluid falls,

$y_1 - y_2$ is positive

• So $P_2 = P_1 + 2 \text{ atm} = 3 \text{ atm}$

The pressure triples

12. A horizontal pipe with fluid of density ρ and entrance speed v has a cross-sectional area that narrows to $\frac{1}{3}$ of the entrance area.. How does the pressure change between the two ends?

A) Decreases by $3\rho v^2$

C) Increases by $3\rho v^2$

B) Decreases by $4\rho v^2$

D) Increases by $4\rho v^2$

- Please note that the continuity equation still applies, and it tells us that if the area decreases by 3, the flow speed must increase to $3v$.
- Since the pipe is horizontal, the Bernoulli equation simplifies to the “**Bernoulli effect**”

$$P_2 - P_1 = \frac{1}{2}\rho(v_1^2 - v_2^2) = \frac{1}{2}\rho(v^2 - (3v)^2) = -4\rho v^2$$

The negative sign means that the pressure drops

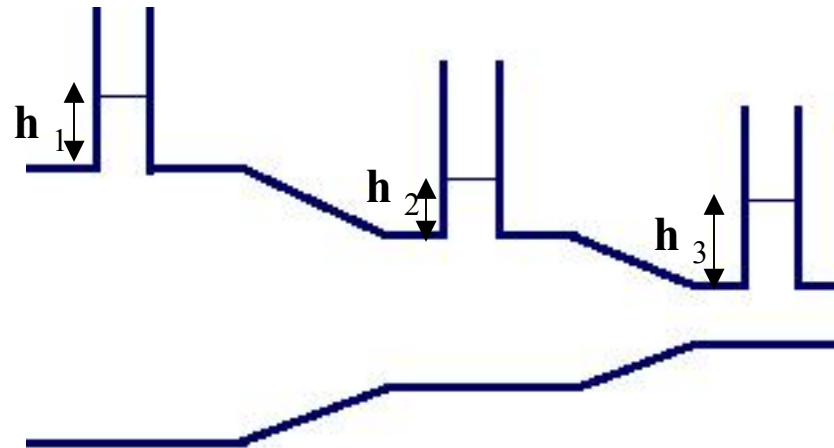
13. Which relationship is true?

A) $h_1 < h_2 < h_3$

B) $h_1 < h_2 = h_3$

C) $h_1 = h_2 < h_3$

D) $h_1 > h_2 > h_3$

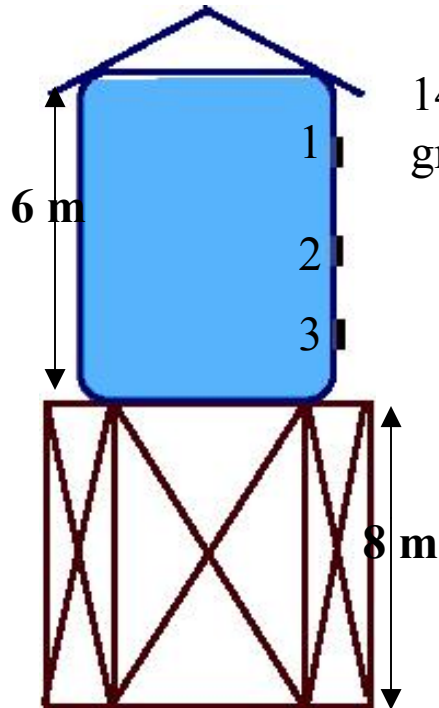


- The continuity Equation tells us that the speed increases as we go down the ever-narrowing pipe
- The Bernoulli Effect tells us that the pressure must fall as the fluid speeds up
- The lower pressure at the right will not be able to support as heavy (i.e. tall) water column as that on the right
- So we expect the water columns to be shorter on the right

An application of the continuity & Bernoulli equations (plus some clever approximations) gives *Torcelli's Result*:

A tank of fluid with depth h draining through a small hole has an exit speed of:

$$v = \sqrt{2gh}$$



14. A full water tower is 6 m deep and rests upon a tower 8 m off the ground.

a) which plug will release water with the greatest exit speed?

V increases as the depth does. So the deepest hole (#3) will have the greatest exit speed.

A) 1 B) 2 **C) 3** D) All the same

b) If plug 2 is 2 m above the tank's base, what's the exit speed?

Plug 2 will have a depth of 6 m - 2 m = 4 m. Put that into Torcelli's Result:

$$v = \sqrt{2gh} = \sqrt{2 * 10 * 4} = \sqrt{80} \approx 9 \text{ m/s}$$

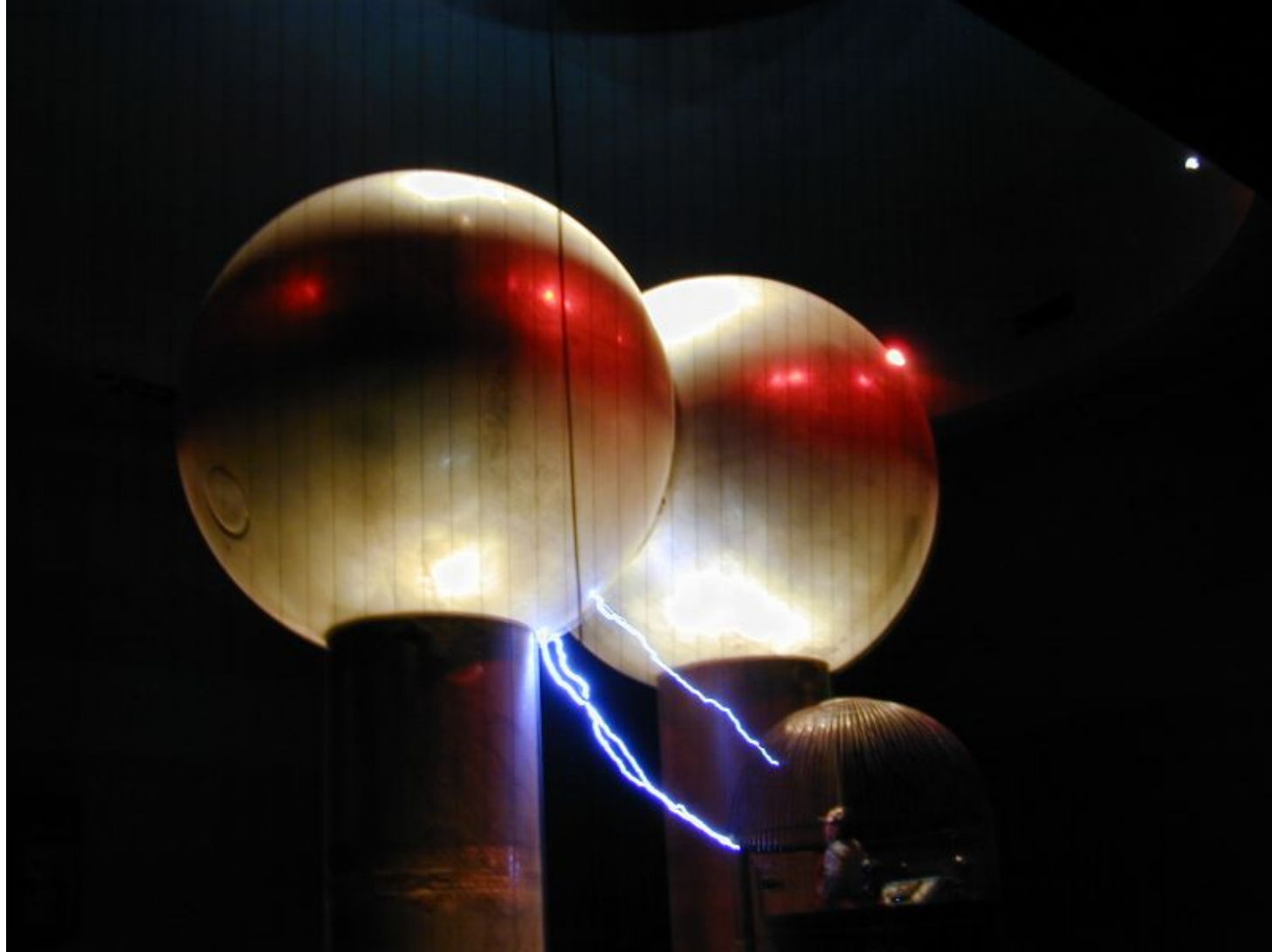
A) 4.5 m/s B) 6.3 m/s **C) 8.9 m/s** D) 10.7 m/s

c) If water gushes out of hole 3 with speed v , how fast would a tank full of oil (spg=0.5) flow from that same hole?

Torcelli's Result has nothing to do with fluid density; density cancels out in the derivation. So we expect the exit speeds to be the same for oil and water.

A) $v/2$ **B) v** C) $2v$ D) $4v$

Physics 6: Electricity



1a. What's the charge on an electron? = - e = -1.6*10⁻¹⁹ C

1b. What's the charge on a proton? = + e = +1.6*10⁻¹⁹ C

1c. What type of particle has a charge of 2e/3? No particle of concern on the MCAT does.

Newton's Gravity Law:

$$F_G = \frac{GMm}{r^2}$$

Coulomb's Electrostatic Law:

$$F_E = \frac{kQq}{r^2}$$

Similarities	Differences
both $\sim 1/r^2$	Gravity only attracts
$\sim M*m$ & $Q*q$	$G=6.67*10^{-11}$ $k=9*10^9$

2a. A positive and negative charge rest in close proximity. If we double the magnitude of each charge and the distance between them, then how does the electric force between them change? **Doubling each charge doubles the force twice (quadruples it) while doubling the distance between them quarters the force.**

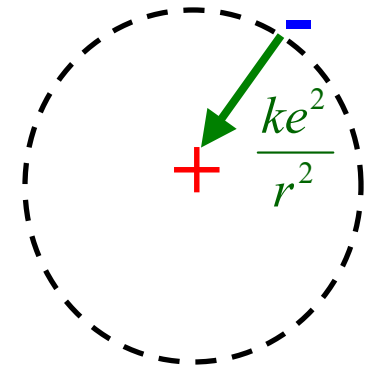
- A. 1/2 weaker **B. No change** C. 2 times stronger D. 4 times stronger

2b. If instead, we double the mass of the positive charge and double the magnitude of the charge on the negative, how does the electric force between them change? **Doubling one of the charges doubles the force while doubling the mass of the other does nothing for the electrostatic force**

- A. 1/2 weaker B. No change **C. 2 times stronger** D. 4 times stronger⁵⁷

3a. An electron with charge e and mass m orbits the nucleus of deuterium (1 proton & 1 neutron) with radius r . How fast does it move?

- A. $e\sqrt{\frac{mr}{k}}$ **B. $e\sqrt{\frac{k}{mr}}$** C. $e\sqrt{\frac{kr}{m}}$ D. $e\sqrt{\frac{m}{kr}}$



$$F_{\text{NET}} = ma$$

$$k e^* e / r^2 = mv^2 / r$$

$$v^2 = k e^* e / (m r)$$

3b. What is the kinetic energy of this electron?

- A. $\frac{ke}{2r}$ B. $\frac{ke}{2r^2}$ **C. $\frac{ke^2}{2r}$** D. $\frac{ke^2}{2r^2}$

$$\text{KE} = \frac{1}{2} mv^2$$

$$= \frac{1}{2} m (k e^* e / (m r)) = k e^2 / (2 r)$$

3c. If the the electron orbited at speed v_0 in part a, at what speed would it orbit if we add 5 protons and 5 neutrons to the nucleus and move the electron 4 times as far away as in part a? (i.e it's now ionized carbon)

- A. $3v_0$ B. $(3/2)v_0$ **C. $(3/2)^{1/2}v_0$** D. $(3/8)^{1/2}v_0$

Be careful here & don't just work blindly from the answer of part a:

- The radius increases by 4 and hence tries to quarter the v^2 .
- Meanwhile, the charge of the nucleus increases by a factor of 6, but not the electron charge, hence increasing the v^2 by 6.



$$v^2 \sim 6/4 = 3/2.$$

4. A charge $Q_1 = -3.2 \times 10^{-10} \text{ C}$ and another $Q_2 = 6.4 \times 10^{-9} \text{ C}$ rest 1 cm apart. Let F_1 be the force Q_1 feels and F_2 that which Q_2 feels. What is $\underline{F_1}/\underline{F_2}$?

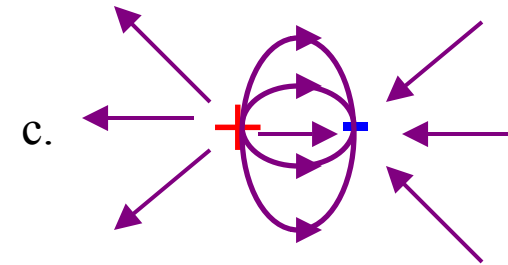
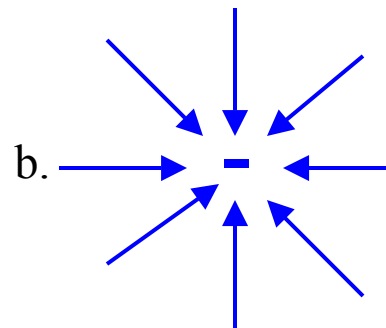
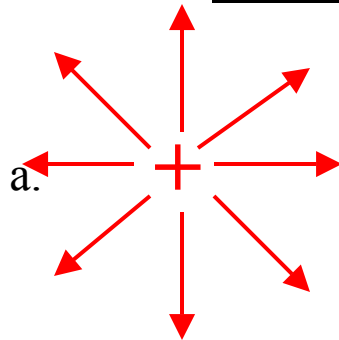
- A. 0.05 B. **1.00** C. 12.8 D. 20.0

Newton's 3rd Law
still applies

$$\vec{F} = q\vec{E}$$

We use the electric field in the same way that we used $g = 10 \text{ m/s}^2$ in gravity

5. Sketch the field lines on the charge configurations below:



6. An object of mass 1.25 kg and charge -1 C rests 2m above a large positively charged plate. If it feels a constant electric field of 5 N/C. Ignoring gravity, what is the charge's impact velocity?

$$F_{\text{NET}} = qE = ma$$

$$-1 \cdot 5 / 1.25 = a$$

$$-4 \text{ m/s}^2 = a$$

$$v_f^2 = v_o^2 + 2ad$$

$$= 0 + 2 \cdot 4 \cdot 2 = 16$$

$$v_f = 4 \text{ m/s}$$

A. 1 m/s

B. 2 m/s

C. **4 m/s**

D. 16 m/s 59

$$\left. \begin{aligned} F &= qE \\ F &= \frac{kQq}{r^2} \end{aligned} \right\} \Rightarrow E = \frac{kQ}{r^2} \quad \text{Only for the field produced by a single charge of value } Q$$

This also works for spheres of charge. Just draw an imaginary sphere concentric with the source and with the point of observation on its surface. Then Q = all charge inside that surface.

7a. The electric field has strength E_0 at the surface of an insulating sphere with a uniform distribution of charge. What is the field strength if we move 2 radii away?

Moving from 2 radii out from r_0 places us at a radius of $3r_0$

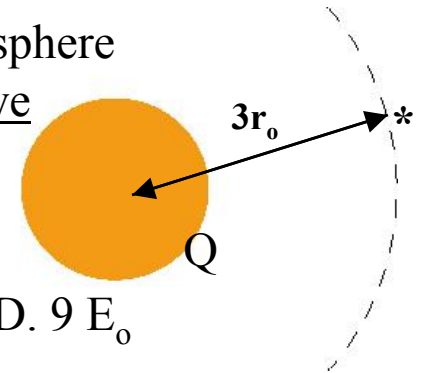
$$E \sim 1/r^2 = 1/3^2 = 1/9$$

A. $E_0/9$

B. $E_0/3$

C. $3 E_0$

D. $9 E_0$



7b. What is the field strength at one half the radius from the center?

Since $V \sim r^3$, only $(1/2)^3 = 1/8$ of the charge is inside the surface.

But we're also $1/2$ the distance from the center.

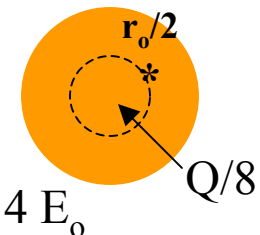
$$\text{So } E \sim Q/r^2 = (1/8)/(1/2)^2 = 4/8 = 1/2$$

A. $E_0/4$

B. $E_0/2$

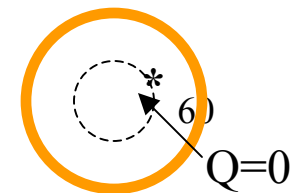
C. $2 E_0$

D. $4 E_0$



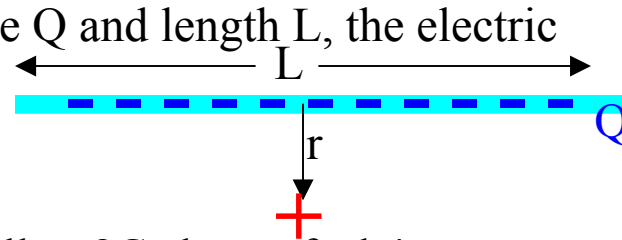
7b. What is the field strength at one half the radius from the center if it was a copper ball?

All the charge in a conductor moves to the outside. Since no charge is enclosed by the surface, $E=0$



8. At a distance r from a line with charge Q and length L , the electric field is:

$$E = \frac{2kQ}{Lr}$$



If $Q/L = -1/9 \times 10^{-9}$, then what force will a $-8C$ charge feel $4m$ away from the line?

$$|E| = 2 * (9 * 10^9) * ((1/9) * 10^{-9}) / 4$$

$$= 1/2 \text{ N/C}$$

and the field points towards the negatively charged line. But the $-8C$ will be **pushed away** with a force of:

$$F = qE = 8 * 1/2 = \boxed{4 \text{ N}}$$

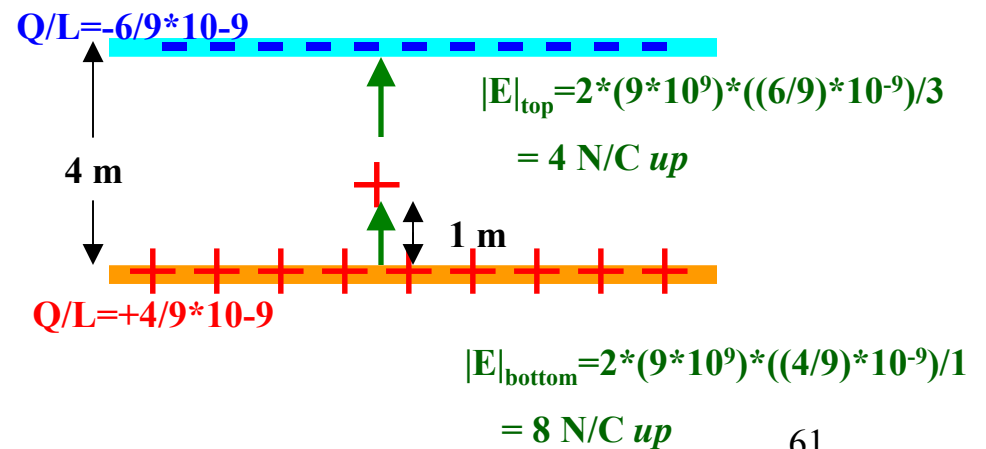
9. If a top line bears a charge $Q/L = -6/9 \times 10^{-9}$ while a parallel bottom line $4m$ away has $Q/L = 4/9 \times 10^{-9}$, then what force will a $+8C$ charge feel $1m$ above the bottom line?

Find the field from each source and then construct the superposition field.

$$E_{\text{top}} + E_{\text{bottom}} = 4 \text{ N/C up} + 8 \text{ N/C up} = 12 \text{ up}$$

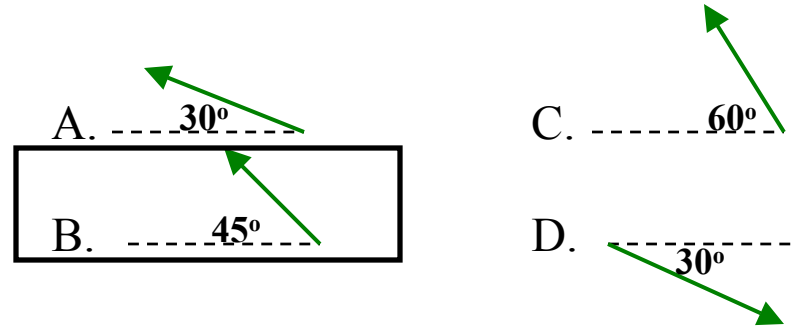
Finally, apply this field to the $8C$ charge to get the force

$$F = qE = (8C) * (12 \text{ N/C}) = \boxed{96 \text{ N}}$$

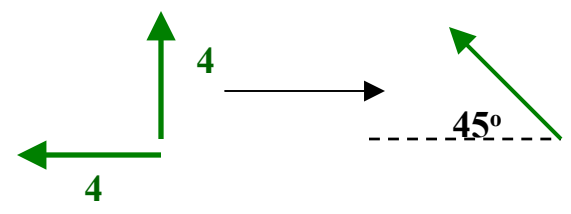


10. What's the direction of the field at point A?

$Q = -1/9 * 10^{-9} \text{ C}$
 $Q/L = +6/9 * 10^{-9} \text{ C}$
 50 cm
 300 cm
 $|E| = \frac{kQ}{r^2} = \frac{(9 * 10^9) * (1/9 * 10^{-9})}{(300 \text{ cm})^2} = 4 \text{ N/C}$
 $|E| = \frac{2k(Q/L)}{r} = \frac{2(9 * 10^9)(6/9 * 10^{-9})}{3} = 4 \text{ N/C}$



Find the field from each source and then construct the superposition field:



11. The top plate of a capacitor is positively charged while the bottom is negative. Which correctly depicts the electric field lines?



Field lines will start on positive charges (top plate) and terminate on negative charges (bottom plate).

$$PE = q\Phi \longrightarrow \Delta PE = qV$$

Where $V = \Delta\Phi$ is the **voltage**

12a. Will a proton move from low to high potential energy, or from **high to low**? If there are no other forces, an object will from high to low potential energy

12b. Will a proton move from low to high electric potential, or from **high to low**? If the proton is to decrease its potential energy, it must move to a place of low electric potential.

12c. Will an electron move from low to high potential energy, or from **high to low**? If there are no other forces, an object will from high to low potential energy

12d. Will an electron move from **low to high** electric potential, or from high to low? If the electron is to decrease its potential energy, it must move to a place of high electric potential. *This is opposite the proton because the electron has a negative charge.*

13. How much energy does a 20 C acquire as it passes through a 10 V potential drop? If it weighs 1 kg and starts from rest, then how quickly does it move after this potential drop?

$$\Delta PE = qV = 20 * 10 = \boxed{200J}$$

$$E = \frac{1}{2} mv^2, \text{ so } v = (2 * 200 / 1)^{1/2} = \boxed{20 \text{ m/s}}$$

14. How much energy does an electron acquire as it passes through a 10 V potential drop?

$$\Delta PE = qV = 1.6 * 10^{-19} * 10 = 1.6 * 10^{-18} J \equiv \boxed{10eV}$$

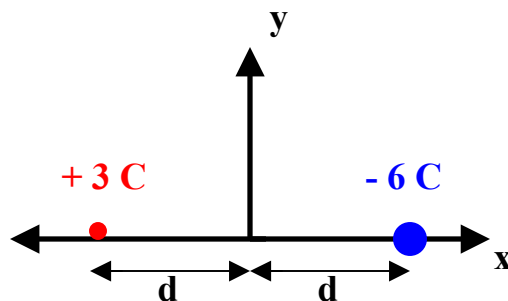
Electric potential looks a lot like gravitational potential energy:

$$\Phi = \frac{kQ}{r} \quad \text{Only for the potential produced by a single charge of value } Q$$

15. Where does the electric potential for a single point charge become zero?

$$\Phi \rightarrow 0 \quad \text{as} \quad r \rightarrow \infty$$

16. At what point along the x axis nearest the origin does the electric potential vanish?

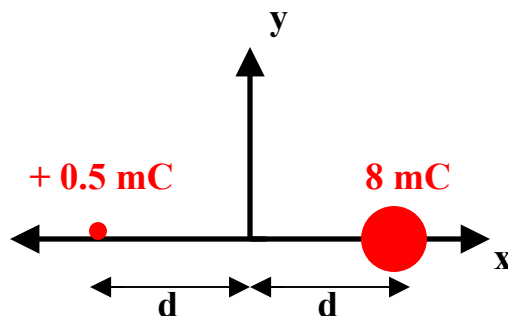


Since the charges have opposite signs, so will their potentials & they will cancel somewhere in between.

Since $\Phi \sim Q/r$ and the charge on the right has twice the magnitude as the left one, we want to stand twice as far from the right charge as we do from the left

- A. $d/3$ left of origin
- B. $2d/3$ left of origin
- C. $d/3$ right of origin
- D. $2d/3$ right of origin

17. At what point along the x axis nearest the origin does the electric field vanish?

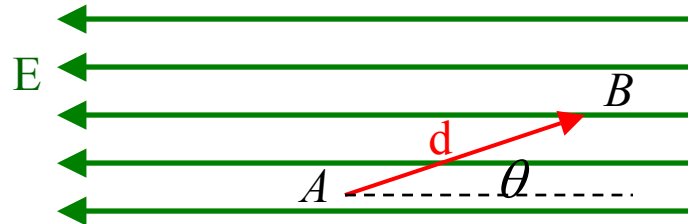


The field vectors will have opposite directions in between the charges and hence can cancel somewhere in between

Since $E \sim Q/r^2$ and the charge on the right has 16 times the magnitude as the left one, we want to stand *four* times as far from the right charge as we do from the left

- A. $3d/5$ left of origin
- B. $2d/5$ left of origin
- C. $2d/5$ right of origin
- D. $3d/5$ right of origin

18. What voltage change will a charge q experience as it moves a displacement d at an angle θ with a uniform electric field E ?



A. Ed

B. $Ed \cos(\theta)$

C. Edq

D. $Edq \cos(\theta)$

As we move from point A to B, the charge will feel a constant left-ward force of qE . So the electric field does $qE \cos(\theta)$ of negative work as it creates electric potential energy. This electric potential energy must be represented by the electric potential:

$$\Delta PE = qV$$

$$qV = qE \cos(\theta)$$

$$V = E \cos(\theta)$$

Physics 7: Magnetic Fields & Circuits



Warm-up: *Roughly* how many times more energy is released when a Uranium nucleus splits into two equal fragments than when a hydrogen molecule “burns” into two hydrogen atoms?

Assume that

- a hydrogen molecule has two protons 1 Å apart (10^{-10}m) while
- a Uranium nucleus is two balls of 100 protons each (and some neutrons) held 1 fm (10^{-15}) apart. The electric potential felt by one half of the nucleus from the other will be larger than the hydrogen proton feels from the other. In fact, from

$$\Phi = \frac{kQ}{r}$$

The electric potential is larger

- by a factor of 100 because there’s more charge creating the fields and also
- by a factor 10^5 of because the constituent parts are in closer proximity.

The total potential energy will be larger still by a factor of 100 because the other piece of the nucleus also has 100 times the charge of a single hydrogen proton.

So the total potential energy stored in the nucleus will be larger by a factor of: $100 \cdot 100 \cdot 10^5 = 10^9 = 1$ billion! We expect on order of a billion more times kinetic energy (and hence heat) per mol to be produced in a nuclear explosion

1a. How are potential energy and electric potential related? $PE = q\Phi$

1b. What is Voltage? $V = \Delta\Phi$

1c. What is Current?: $I = Q / t$

2. What is Ohm’s Law: $V = IR$ or $I = V / R$

3a. If we apply a battery with 9V across a 1 kΩ resistor, what current flows thru the circuit?

$$I = V/R = 9/1000 = 9\text{mA} \text{ (These are typical values that one might use in the lab)}$$

3b. If we apply a battery with 9V across a a wire instead of resistor, what current flows thru the circuit?

We can still use $V=IR$, but since R is small & $V=9\text{ V}$ the current must be HUGE. The battery gets drained really fast. Unless the wire's metal is very losy, this is a short circuit.

4. If we half the radius of a wire, by what factor will it's resistance change?

$$R = \frac{\rho L}{A} \quad \text{Halving the radius will quarter the cross sectional area. So the resistance quadruple.}$$

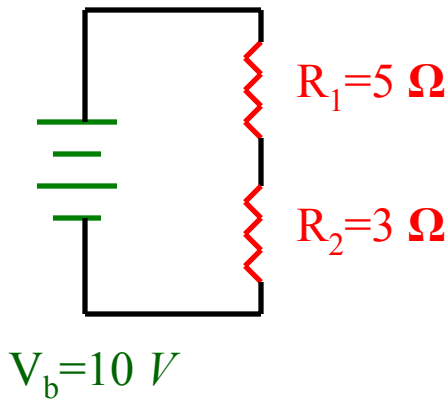
A. $\frac{1}{2}$ smaller B. 2 times larger **C. 4 times larger** D. No change

5. We place a battery directly across the above wire and then half the cross sectional area. What happens to the potential across the wire?

But the voltage is fixed by the battery and hence doesn't change. Sure, I'll double the resistance, but that will only cause the current through the wire to change.

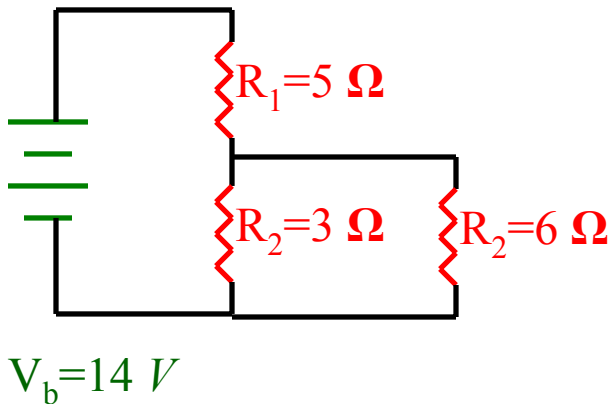
A. 2 times larger B. 4 times larger C. 8 times larger **D. No change**

5.



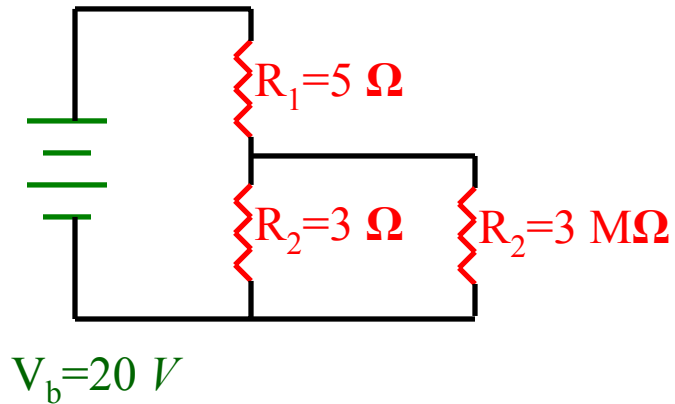
- a. What's the total resistance? $R_T = R_1 + R_2 = 5 + 3 = 8 \text{ } \Omega$
 - b. What's the current thru R_1 ?
 - c. What's the current thru R_2 ?
- } Current is the same if in series.
 $I = V_b / R_T = 10 / 8 = 1.25 \text{ A}$
- d. What's the potential across R_1 ? $V_1 = R_1 I = 5 * 1.25 = 6.25 \text{ V}$
 - e. What's the potential across R_2 ? $V_2 = R_2 I = 3 * 1.25 = 3.75 \text{ V}$
 or $V_2 = V_b - V_1 = 10 - 6.25 = 3.75 \text{ V}$

6.



- a. What's the total resistance? $R_p = 1 / (R_1^{-1} + R_2^{-1}) = 1 / (1/5 + 1/6) = 2 \text{ } \Omega$
 $R_T = R_1 + R_p = 5 + 2 = 7 \text{ } \Omega$
 - b. What's the current thru R_1 ? Same as drawn from battery:
 $I_1 = V_b / R_T = 14 / 7 = 2 \text{ A}$
 - c. What's the potential across V_1 ? $V_1 = R_1 I_1 = 5 * 2 = 10 \text{ V}$
 - d. What's the potential across R_2 ?
 - e. What's the potential across R_3 ?
- } Potential is the same if in parallel. $V_2 = V_3 = V_b - V_1 = 14 - 10 = 4 \text{ V}$
- f. What's the current thru R_2 ? $I_2 = V_2 / R_2 = 4 / 3 = 1.33 \text{ A}$
 - g. What's the current thru R_3 ? $I_3 = V_3 / R_3 = 4 / 6 = 0.67 \text{ A}$
 or $I_3 = I_1 - I_2 = 2 - 1.33 = 0.67 \text{ A}$

7.



- a. What's the total resistance? $R_p = \left(\frac{1}{R_2} + \frac{1}{R_3} \right)^{-1} = \left(\frac{1}{3} + \frac{1}{3 \cdot 10^6} \right)^{-1} \approx 3 \Omega$
 $R_T = R_1 + R_p = 5 + 3 = 8 \Omega$
- b. What's the current thru R_1 ? It's series now: $I_1 = V_T / R_T = 20 / 8 = 5 / 2 = 2.5 \text{ A}$
- c. What's the potential across V_1 ? $V_1 = R_1 I_1 = 5 \cdot 2.5 = 12.5 \text{ V}$
- d. What's the potential across R_2 ? } Potential is the same if in parallel. $V_2 = V_3 = V_b - V_1$
- e. What's the potential across R_3 ? } $= 20 - 12.5 = 7.5 \text{ V}$
- f. What's the current thru R_2 ? Same as R_1 : $I_2 = 2.5 \text{ A}$
- g. What's the current thru R_3 ? $I_3 = V_3 / R_3 = 7.5 / (3 \cdot 10^6) \sim 0 \text{ A}$

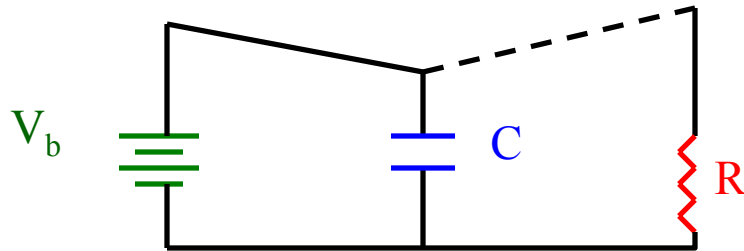
8. In the circuit from problem 6:

NOTE: I can use any of the first 3 equations to answer any of these 4 parts. I can only use the last equation once I've solved the other 3 parts.

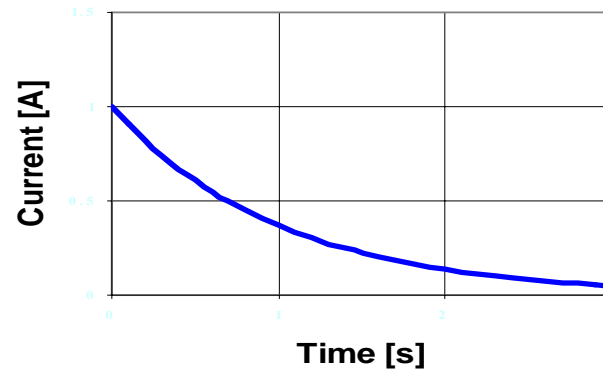
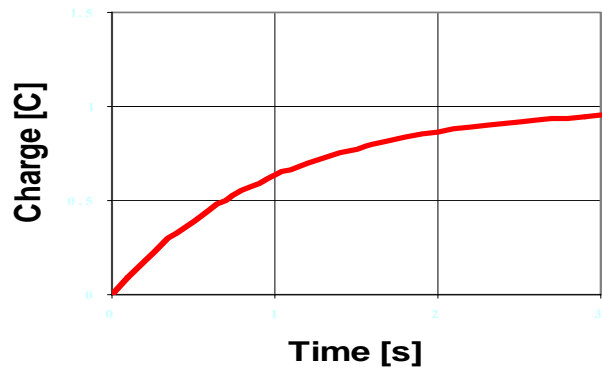
- a. What power is dissipated by R_1 ? $P_1 = R_1 I_1^2 = 5 \cdot 2^2 = 20 \text{ W}$
- b. What power is dissipated by R_2 ? $P_2 = V_2^2 / R_2 = 4^2 / 3 = 5.33 \text{ W}$
- c. What power is dissipated by R_3 ? $P_3 = I_3 V_3 = (2/3) \cdot 4 = 2.66 \text{ W}$
- d. What power is drawn from the battery?

$$P_b = V_1 + V_2 + V_3 = 20 + 5.33 + 2.6 = 28 \text{ W}$$

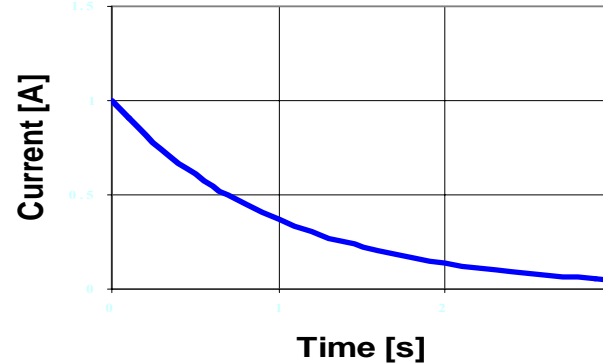
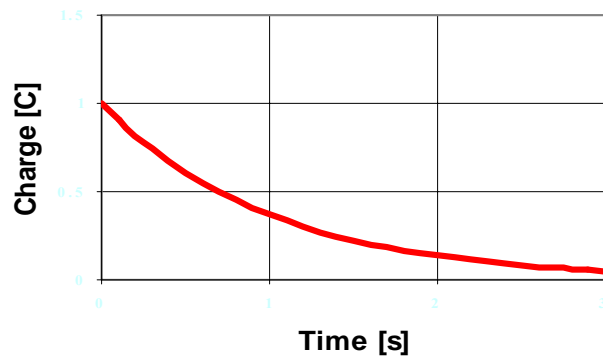
Capacitance: $Q=C*V$



9. As we charge an originally shorted capacitor with a battery, what happens to charge and current over time?



10. As we discharge a capacitor thru a resistor, what happens to the charge and current over time?



11. A capacitor has charge Q_0 & Voltage V_0 across it, creating a field E_0 between the plates and storing and energy U_0 . If I remove the battery & then double the plate area:

$C = \epsilon_0 A/d$ tells us that the capacitance doubles

- a. What's the charge now? A) $Q_0/2$ B) Q_0 C) $2Q_0$ D) $4Q_0$
With the circuit broken, the charge is stuck on the plates. So stored charge cannot change.
- b. What's the voltage now? A) $V_0/2$ B) V_0 C) $2V_0$ D) $4V_0$
So from $Q=CV$, the voltage must drop in half if Q is fixed because we are doubling C .
- c. What's the field strength now? A) $E_0/4$ B) $E_0/2$ C) E_0 D) $2E_0$
From $V=Ed$, the field must fall in half to keep up with the lowered voltage.
- d. What's the Energy now? A) $U_0/8$ B) $U_0/4$ C) $U_0/2$ D) U_0
From $U = \frac{1}{2}Q^2/C$, the energy must fall in half since the capacitance doubles.

12. A capacitor has charge Q_0 & Voltage V_0 across it, creating a field E_0 between the plates and storing and energy U_0 . The battery remains attached as I pull the plates twice as far apart:

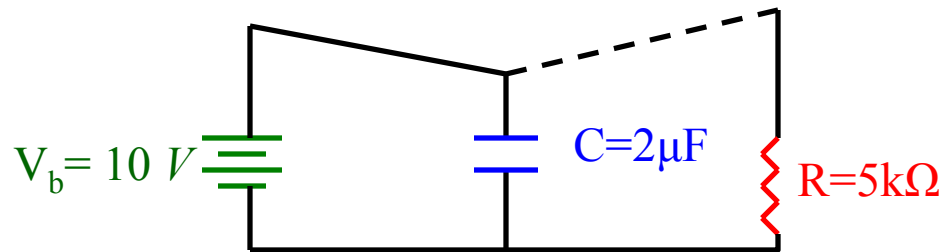
$C = \epsilon_0 A/d$ tells us that the capacitance falls in half

- a. What's the voltage now? A) $V_0/2$ B) V_0 C) $2V_0$ D) $4V_0$
The battery is still attached and still holds a voltage V_0 across the capacitor.
- b. What's the charge now? A) $Q_0/2$ B) Q_0 C) $2Q_0$ D) $4Q_0$
So from $Q=CV$, half the charge must return to the battery to hold V fixed since we are halving C .
- c. What's the field strength now? A) $E_0/4$ B) $E_0/2$ C) E_0 D) $2E_0$
From $V=Ed$, the field strength must fall in half to keep V fixed as I double d .
- d. What's the Energy now? A) $U_0/8$ B) $U_0/4$ C) $U_0/2$ D) U_0
From $U = \frac{1}{2}CV^2$, the energy must fall in half since the capacitance halves.

13. A capacitor has charge Q_0 & Voltage V_0 across it, creating a field E_0 between the plates and storing an energy U_0 . If the battery remains attached as I slip in a dielectric:
- What's happens to the voltage? **A) same** B) increases C) decreases D) can't determine
The battery is still attached and still holds a voltage V_0 across the capacitor.
 - What's happens to the field? **A) same** B) increases C) decreases D) can't determine
From $V=Ed$, the field must stay the same since V and d do not change.
 - What's happens to the charge? **A) same B) increases** C) decreases D) can't determine
The dielectric dipoles are trying to cancel the field, so the battery must send more charge to the plates to hold the field fixed. (From $Q=CV$, this means that the capacitance increases)
 - What's happens to the energy? **A) same B) increases** C) decreases D) can't determine
From $U= \frac{1}{2}CV^2$, the energy must increase since the capacitance increases.

With a dielectric of constant κ , $C= \epsilon_0 \kappa A/d$

14. A capacitor has charge Q_0 & Voltage V_0 across it, creating a field E_0 between the plates and storing an energy U_0 . If I remove the battery & then slip in a dielectric:
- What's happens to the charge? **A) same** B) increases C) decreases D) can't determine
With the circuit broken, the charge is stuck on the plates. So stored charge cannot change.
 - What's happens to the voltage? **A) same B) increases C) decreases** D) can't determine
The dielectric increases C . So from $Q=CV$, V must decrease since Q is fixed.
 - What's happens to the field? **A) same B) increases C) decreases** D) can't determine
From $V=Ed$, the field must decrease as the voltage does.
 - What's happens to the energy? **A) same B) increases C) decreases** D) can't determine
From $U= \frac{1}{2}Q^2/C$, the energy must decrease since C increased.



15. What's the initial current as this capacitor discharges?

The battery will charge the capacitor such that it has 10 V, which it still has when we flip the switch and it starts to discharge. 10 V across the $5\text{ k}\Omega$ resistor will create $I = V/R = \boxed{2\text{ mA}}$ current initially. The current and voltage will decay exponentially over time.

16. If $-4\ \mu\text{C}$ of charge is transferred from one side of an $8\ \mu\text{F}$ capacitor while charging, how much total work does the internal electric field do on these charges?

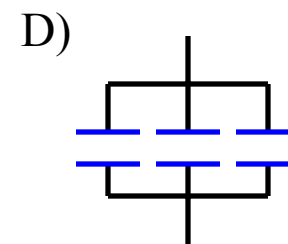
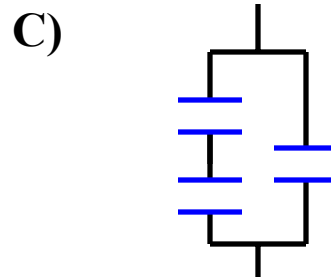
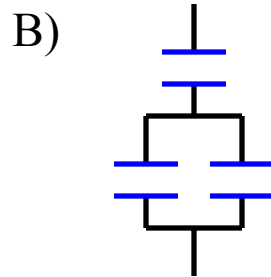
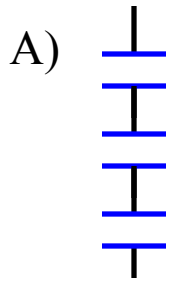
- A) $-4\ \mu\text{J}$ B) $-1\ \mu\text{J}$ C) $1\ \mu\text{J}$ D) $4\ \mu\text{J}$

Clearly, we're storing $PE = \frac{1}{2}Q^2/C = \frac{1}{2}(4)^2/(8) = 1\ \mu\text{J}$, and this energy was provided by the battery. So the battery did $1\ \mu\text{J}$ of work to make this happen.

If the internal electric fields had their way, they would move the charges such that each plate was neutral again; they'd put those $-4\ \mu\text{C}$ back. Such electric forces are opposite the charge's displacement, so the electric fields internal to the capacitor must have done negative work $\boxed{-1\ \mu\text{J}}$.

17. Using three $8 \mu\text{F}$ capacitors, which configuration will produce $12 \mu\text{F}$?

Recall that: $C_{series} = \left(\frac{1}{C_1} + \frac{1}{C_2} \right)^{-1}$ and $C_{parallel} = C_1 + C_2$



$$C_A = (C_1^{-1} + C_2^{-1} + C_3^{-1})^{-1}$$

$$= (1/8 + 1/8 + 1/8)^{-1} = 8/3 \mu\text{F}$$

$$C_B = (C_1^{-1} + (C_2 + C_3)^{-1})^{-1}$$

$$= (1/8 + 1/16)^{-1} = 16/3 \mu\text{F}$$

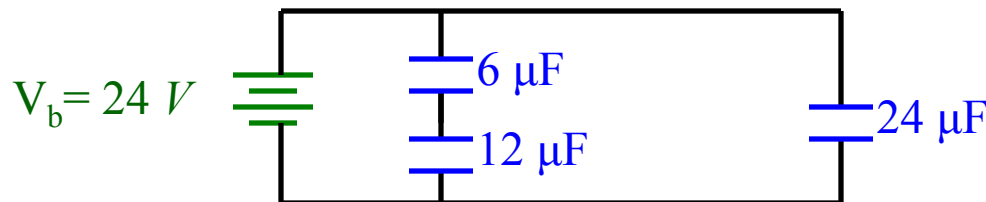
$$C_C = (C_1^{-1} + C_2^{-1})^{-1} + C_3$$

$$= (4 + 8)^{-1} = 12 \mu\text{F}$$

$$C_D = C_1 + C_2 + C_3$$

$$= 8 + 8 + 8 = 24 \mu\text{F}$$

18. After all of the below capacitors fully charge,



$$C_{series} = (C_1^{-1} + C_2^{-1})^{-1}$$

$$= (6^{-1} + 12^{-1})^{-1}$$

$$= 6 * (1 + 1/2)^{-1} = 4 \mu\text{F}$$

a. How much charge is stored on the $6 \mu\text{F}$ cap?

Since the $6 \mu\text{F}$ & $12 \mu\text{F}$ caps are in series, they store the same charge. $Q = C_{series} V = 4 * 24 = 96 \mu\text{C}$

b. What is the voltage across the $12 \mu\text{F}$ cap?

$Q = CV$ tells us that the voltage across the $12 \mu\text{F}$ cap is $V = 96/12 = 8 \text{ V}$

c. How much potential energy is stored in the $6 \mu\text{F}$ cap?

The battery provides 24 V , of which 8 V is lost over the $12 \mu\text{F}$ cap. So the voltage must fall 16 V over the $6 \mu\text{F}$ cap.

$$PE = \frac{1}{2} CV^2 = \frac{1}{2} (6 \mu\text{F})(16)^2 = 2 * 256 = 468 \mu\text{J}$$

19. A 1C charge moving 6 m/s to the right is subjected to a magnetic field of 2 T oriented out of this page.

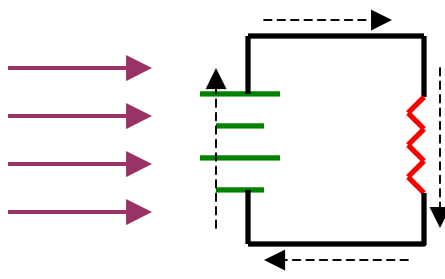
- a. How strong of a magnetic force will it feel? $F = q v B \sin(\theta) = (1)(6)(2) \sin(90^\circ) = \boxed{12 \text{ N}}$
- b. In what direction will the force be directed? **Use right hand rule**

- A) \rightarrow **B) \downarrow** C) \leftarrow D) \uparrow

20. A -2C charge moving 10 m/s to the left is subjected to a magnetic field of 12 T oriented to the right. In what direction will the force be directed?

- $\theta = 180^\circ$, so $F = q v B \sin(180) = 0$
- A) \uparrow B) \downarrow C) \otimes D) **None**

21. In Which way is this circuit torqued from the applied Magnetic field?



Use right hand rule on the dashed lines at left (representing the current). The top lines feel no force, while the side ones do. It spins it as per option B

- A)
- B)
- C)
- D)

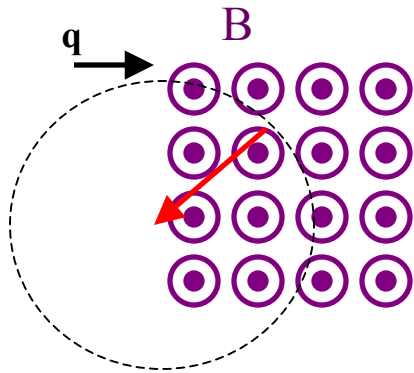
22. How much work does the magnetic field do on the charge below?

A) $qvB2\pi r$

B) $qvB2r$

C) $2qvB2\pi r^2$

D) 0



The right hand rule points the force perpendicular to the path. So it causes centripetal acceleration. It moves in a circle with radius:

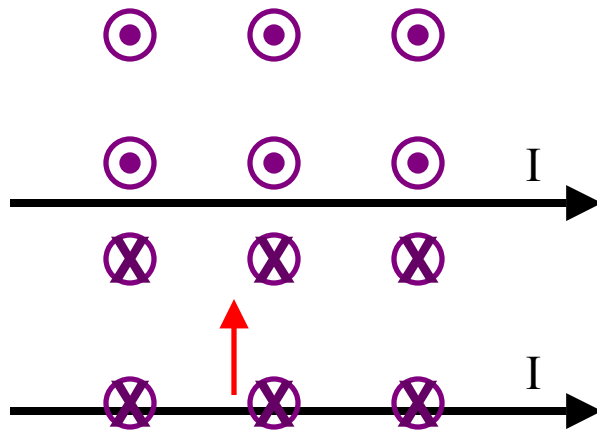
$$F_{NET} = ma$$

$$q v B = m v^2 / r$$

$$r = mv / (q B)$$

So it does no work since the force acts centripetally.

19. Do the wires below **pull together** or push apart?



Use the 2nd right hand rule to figure out the field from one of the wires (lets do the top one). I depict these at left in purple. Then use the first right hand rule to see how the field exerts a force on the other wire (red).

Worksheet on Right Hand Rule

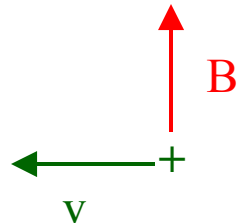


The only way to get comfortable with using the right hand rule (RHR) is to practice. Try each of these problems using different versions of the RHR to figure out which one you feel comfortable with. This will also give you plenty of practice.

The answers are listed here in a sufficiently annoying way to avoid spoiling the problems for you.

- A1. The force points into the page.
 A2. The field points to right (remember: it's negatively charged)
 A3: The force is into the page (recall that current is defined as a motion of positive charge)
 A4: This force cannot be caused by a magnetic field because magnetic forces are ALWAYS perpendicular to the velocity vector.
 A5. Field lines circle the wire pointing counterclockwise (as viewed from above the paper)
 A6. The r moves out of the page (don't forget that it's charge is negative)
 A7. For the magnetic force to push inward for clockwise motion, the charge must be negative. A positive charge would rotate counterclockwise
 A8. The current flowing out of the page will create counterclockwise B-fields around it. This field will be pointing up when it crosses the wire taking current into the page. That will pull the wire running into the page towards the one running out of the page. They pull together.
 A9. The force points into the page

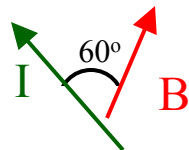
Q1. If a positively charged particle moves to the left in a Magnetic field that points up, which way will the force act?



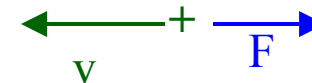
Q2. An electron feels a force down while it is moving out of the paper. Which way is the magnetic field pointing?



Q3. A wire carries a current as shown below subjected to a B-field. Which way does it feel a force?



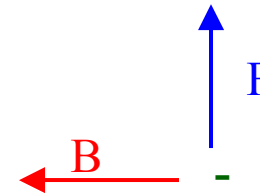
Q4. A positron moves to the left but feels a force to the right. Which way is the magnetic field oriented?



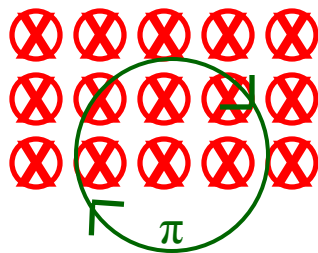
Q5. A wire carries current **out of the page**. Which way does the field created by this current point?



Q6. A τ particle (a cousin of the electron) feels **an upward force** from a **magnetic field pointing to the left**. Which way was the τ moving?



Q7. π mesons come in positive, negative, and neutral varieties. If we send a meson through a **uniform magnetic field pointing into the page** and it **moves in a clockwise circle**, what is the particle's sign?



Q8. If we send **current out of the page** in one wire but **into the page** in a neighboring wire, will these wires pull together or push apart?

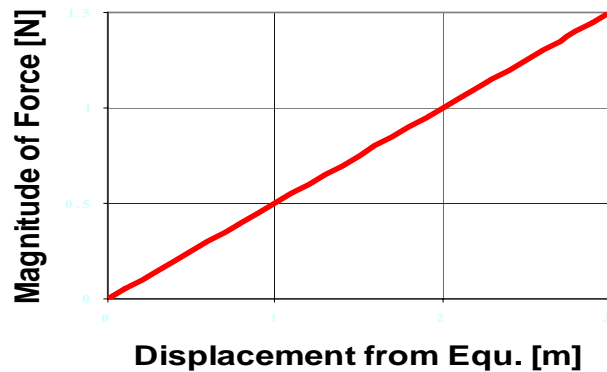
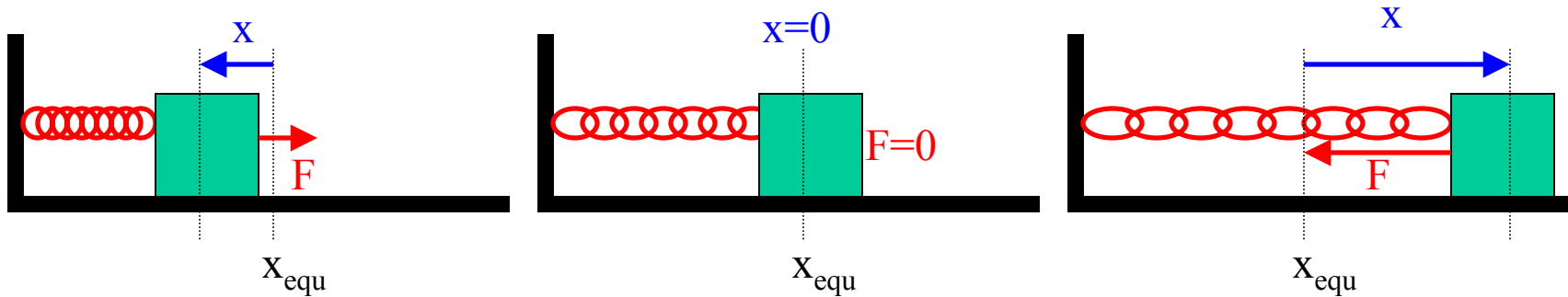


Physics 8: Waves & Oscillations



Hooke's Law: (The Spring Force)

$F = -k x$, where x = Displacement from equilibrium

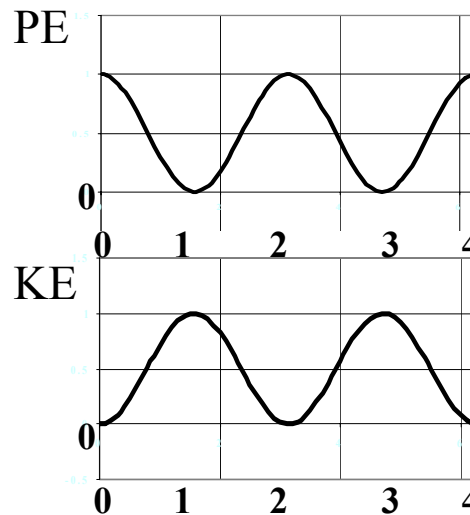
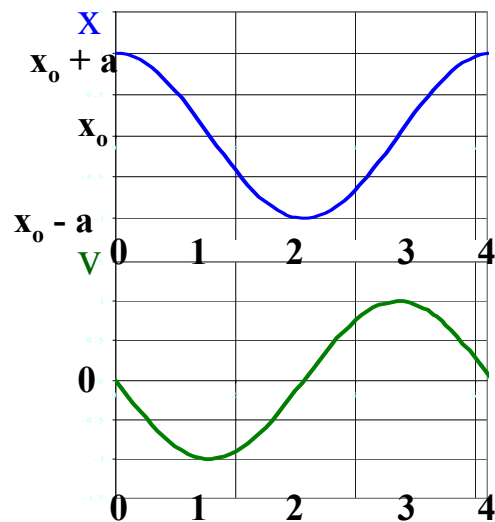


How much Potential Energy is stored in the spring when displaced to x :
(aka how much Work is required from us to pull or push it there):

$$W = \frac{1}{2} x |F| = \frac{1}{2} x |-k x| = \frac{1}{2} k x^2$$

$$\boxed{PE = \frac{1}{2} k x^2}$$

	$t = 0 \text{ s}$	$t = 1 \text{ s}$	$t = 2 \text{ s}$	$t = 3 \text{ s}$	$t = 4 \text{ s}$
	$x_0 - a$ x_0 $x_0 + a$	$x_0 - a$ x_0 $x_0 + a$	$x_0 - a$ x_0 $x_0 + a$	$x_0 - a$ x_0 $x_0 + a$	$x_0 - a$ x_0 $x_0 + a$
x	$+a$	0	$-a$	0	$+a$
F	$-ka$	0	$+ka$	0	$-ka$
a	$-ka/m$	0	$+ka/m$	0	$-ka/m$
PE	$\frac{1}{2} k a^2$	0	$\frac{1}{2} k a^2$	0	$\frac{1}{2} k a^2$
KE	0	$\frac{1}{2} m v^2 (= \frac{1}{2} k a^2)$	0	$\frac{1}{2} m v^2 (= \frac{1}{2} k a^2)$	0
v	0	$v = -(ka/m)^{1/2}$	0	$v = +(ka/m)^{1/2}$	0



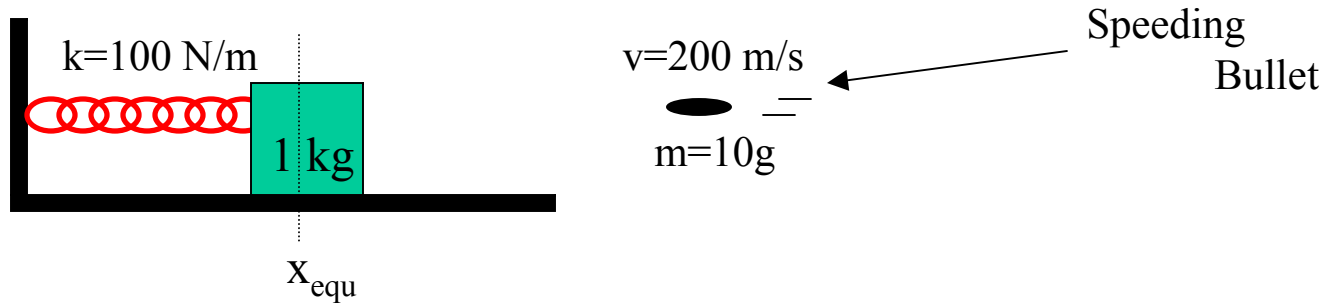
What is period of oscillation?

$$\mathbf{T=4s}$$

What is frequency of oscillation?

$$\mathbf{f=1/T=0.25 \text{ Hz}}$$

You can also use $f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$



A 10 g bullet traveling at 200 m/s becomes lodged in a 1 kg block, which then begins to oscillate horizontally with a spring of constant 100N/m

1. How can we change k so as to double the period?

$$f = (k/m)^{1/2} / (2\pi)$$

$$\text{So } T = 2\pi(m/k)^{1/2}$$

$$T \sim 1 / k^{1/2}$$

A) Quarter k B) Halve k C) Double D) Quadruple k

2. What is the frequency of oscillation?

$$f = (k/m)^{1/2} / (2\pi)$$

A) 0.02 Hz B) 0.6 Hz C) **1.6 Hz** D) 63 Hz $= (100/1)^{1/2} / (2*3) = 10/6 = 1.6 \text{ Hz}$

3. What is the amplitude of Oscillation?

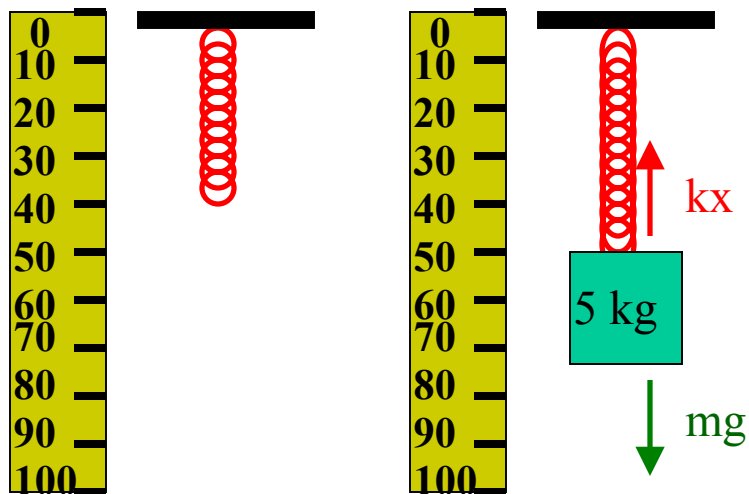
A) **20 cm** B) 40 cm C) 2 m D) 4 m

Conserve momentum in collision to find that $KE = PE$

$v = 20 \text{ m/s}$. Then convert the associated KE to PE: $\frac{1}{2} m v^2 = \frac{1}{2} k x^2$ So $x = v (m/k)^{1/2} = 2/10 = 20 \text{ cm}$

4. If a different bullet compressed the spring four times as much, how much more work would it have done on the block/spring?

A) 2 times B) 4 times C) 8 times D) **16 times** $W = \frac{1}{2} k x^2$ So $W \sim x^2$ ⁸⁴



We hang a massless spring from the roof and notice how long it is by measurement with a meter stick. When we hang an additional weight, it sags even further.

5. What is the spring's constant?

- A) 60 N/m B) 80 N/m C) 160 N/m **D) 250 N/m**

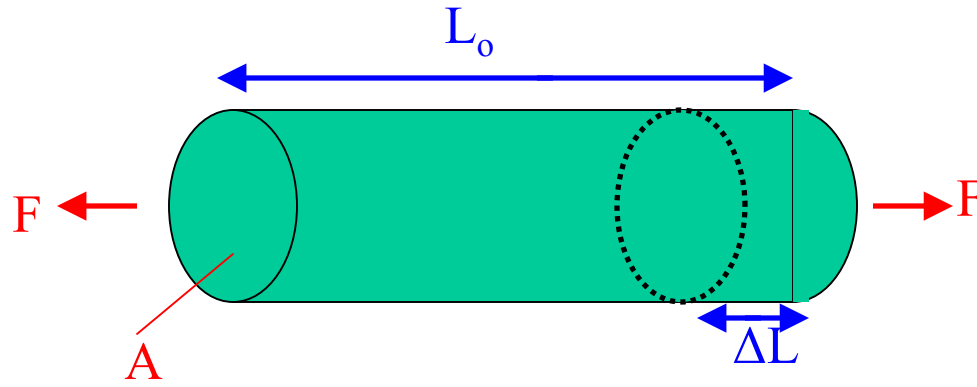
The spring drops $x=20 \text{ cm}=0.2\text{m}$ when subjected to a force of $mg=5*10=50 \text{ N}$.
So the spring constant $k=F/x=50/0.2=250 \text{ N/m}$

6. If the weight doesn't sit still, what's the frequency of oscillation?

- A) 0.63 Hz **B) 1.12 Hz** C) 4.00 Hz D) 7.07 Hz

$$f=(k/m)^{1/2}/(2\pi) = (50)^{1/2}/(2\pi) \sim (49)^{1/2}/(2*3) = 7/6 \sim 1.12 \text{ Hz}$$

Young's Modulus



Stress = F/A (“Stress” sound’s like “press”)
 Strain = $\Delta L / L_0$

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F L_0}{A \Delta L}$$

5. Which has the larger Young’s Modulus?

- A) Rubber **B) Steel** C) Same D) Y is not defined for elastics

If we apply a certain stress to steel, it will stretch less than rubber, i.e. have a smaller strain. So Stress/strain=Y is larger for steel than rubber.

$$F = (YA/L_0) * \Delta L$$

$$= k * \Delta L$$

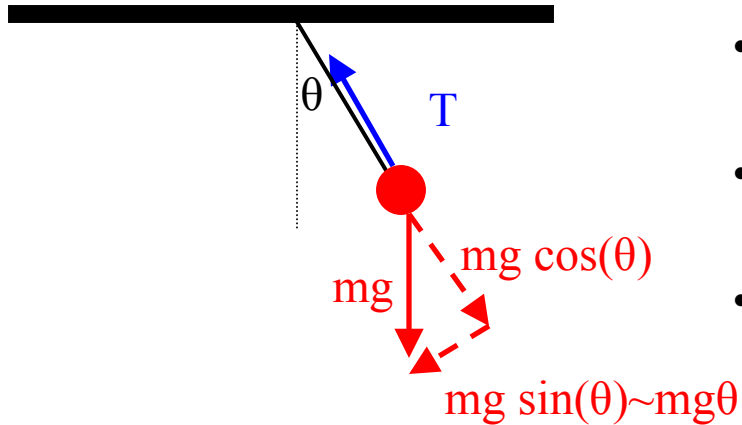
6. If I apply a stress of 10 N/m^2 to a piece of copper with $Y=2 \text{ N/m}^2$, how much energy is stored in the copper?

Stress*strain has units of J/m^3 (energy density). So this is like storing energy in a string with constant $k=Y*A/L_0$.

So Energy Density = $0.5 * \text{stress} * \text{strain} = 0.5 * \text{stress} * (\text{stress}/Y) = 0.5 * 10 * 10 / 2 = 25 \text{ J/m}^3$

- A) 12.5 J/m^3 **B) 25 J/m^3** C) 50 J/m^3 D) 100 J/m^3

The Pendulum (a lot like a Spring)



- the rope tension T will cancel the component of gravity $mg \cos(\theta)$.
- The other gravity component is the net force $mg \sin(\theta)$
- For small θ this force is $mg\theta$

Restoring Force is like a spring: $F = (mg)\theta$ where $\theta =$ angle from equilibrium.

Frequency of oscillation if rope has length l : $f = \frac{1}{2\pi} \sqrt{\frac{g}{l}}$

NOTE: Frequency is mass-independent.

The mass in the restoring force $mg\theta$ cancels the inertial mass on Newton's 2nd law: $F_{\text{net}} = ma$

7. If I increase mass of weight by 9 & length by 4, then by how much will frequency change?

Mass doesn't matter; only l does. If I quadruple l, then f decreases by $4^{1/2} = 2$.

A) 1/2 smaller B) 2/3 smaller C) 3/2 larger D) 2 times larger

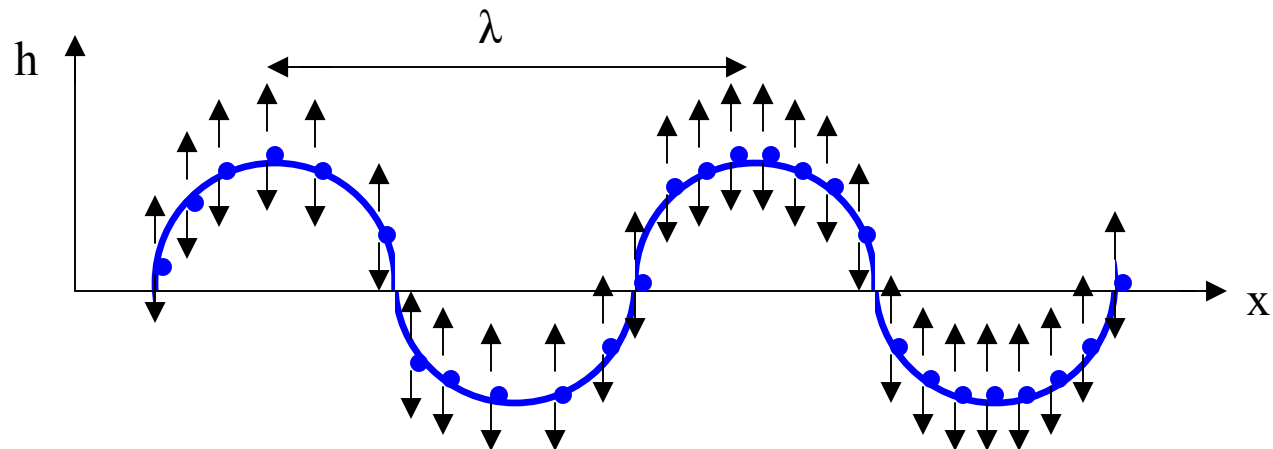
8. A 50 cm long pendulum on Mars has a period of 2.3 sec. What's g there?

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{l}} \quad \& \quad f = 1/T$$

These tell us that $g = l \cdot (2\pi/T)^2$, so $g \sim 0.5 \cdot (2 \cdot 3/2)^2 = 0.5 \cdot 9 = 4.5 \text{ m/s}^2$ Option B is closest.

A) 1.9 m/s² B) 3.7 m/s² C) 7.5 m/s² D) 13.8 m/s²

Transverse Wave on String



*This picture is a snap-shot of what a wave would look like at a specific time

- A String has lots of connected pieces (●) each oscillating *up & down* at a frequency f and with period T
- The distance between adjacent peaks is the wavelength λ
- So wave peaks move at speed $\boxed{v = \lambda / T = \lambda f}$

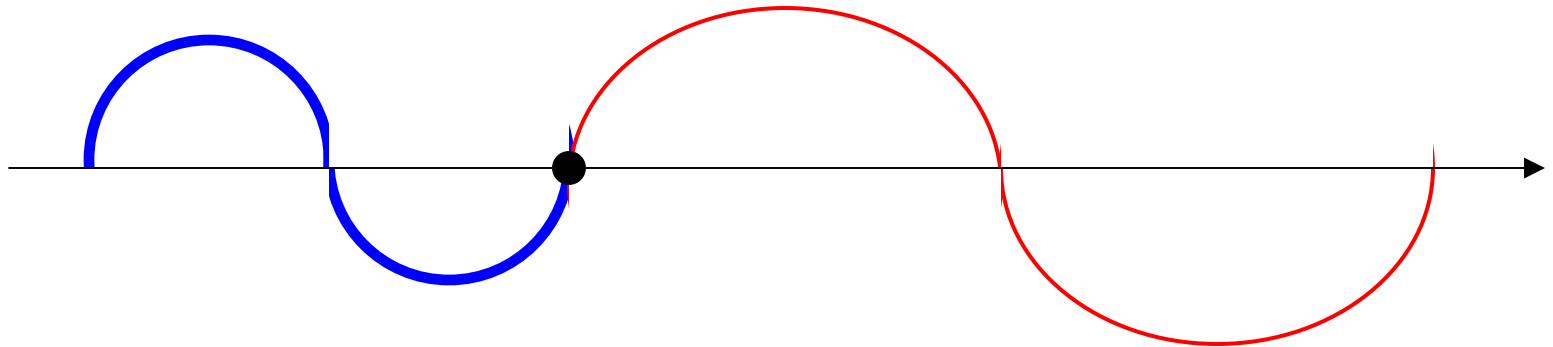
In particular, a string with tension T and weight per length m/L supports waves that all have the *same* speed

$$v = \sqrt{\frac{T}{(m/l)}}$$

When I shake a Rope:

- I pick the f by how fast I shake it up and down.
- The wave speed is set by how heavy & taught the rope is
- The size λ is selected by $v = \lambda f$.

9. A 2m wavelength wave travels down a steel cable at 10m/s. But this Cable is tied to a rope with $\frac{1}{4}$ the density. What happens to the wavelength when the wave enters the rope?



Waves in the rope are created because the steel cable shakes the knot between the two and hence creates waves along the rope too *with the same frequency*.

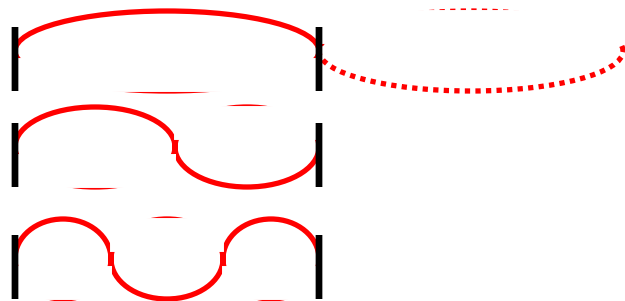
The tension in the rope and in the cable must be the same (else the knot would get pulled off to the side), but the rope is 4 times lighter. So waves will travel in the rope with twice the speed!

If v doubles, then by $v = \lambda f$, so must λ since f is the same in both parts.

So $\lambda = 4$ m

When I pluck a Guitar String:

- The ends are tied down and *the geometry* forces λ to be certain values as per the drawing.
- The wave speed is still set by how heavy & taught the rope is
- f is now set by $v = \lambda f$



$$\lambda_1 = 2L = (2L)/1$$

$$f_1 = v / \lambda_1 = v / (2L) * 1$$

$$\lambda_2 = L = (2L)/2$$

$$f_2 = v / \lambda_2 = v / (2L) * 2$$

$$\lambda_3 = (2/3)L = (2L)/3$$

$$f_3 = v / \lambda_3 = v / (2L) * 3$$

$$\lambda_n = (2L)/n$$

$$f_n = v / (2L) * n = n * f_1$$

10. A 10m long string fixed at both ends permits waves with $v=30$ m/s. Which frequencies will excite standing waves?

2 HZ **3Hz** 4Hz 5 Hz **6 Hz**

The fundamental mode has $\lambda_1 = 2 * L = 20$ m.

So Fundamental Frequency is $f_1 = v / \lambda_1 = 30 / 20 = 1.5$ Hz

This means that standing waves can have: 1.5 Hz, 3 Hz, 4.5 Hz, 6 Hz, 7.5 Hz, etc

11. A guitar string's 3rd harmonic is 4 Hz. What's its 9th harmonic?

All modes oscillate at a multiple of the fundamental harmonic frequency: $f_n = n * f_1$

So if I triple n , I triple f_n . Since $f_3 = 4$ Hz, $f_9 = 12$ Hz

Sound: A Pressure Wave



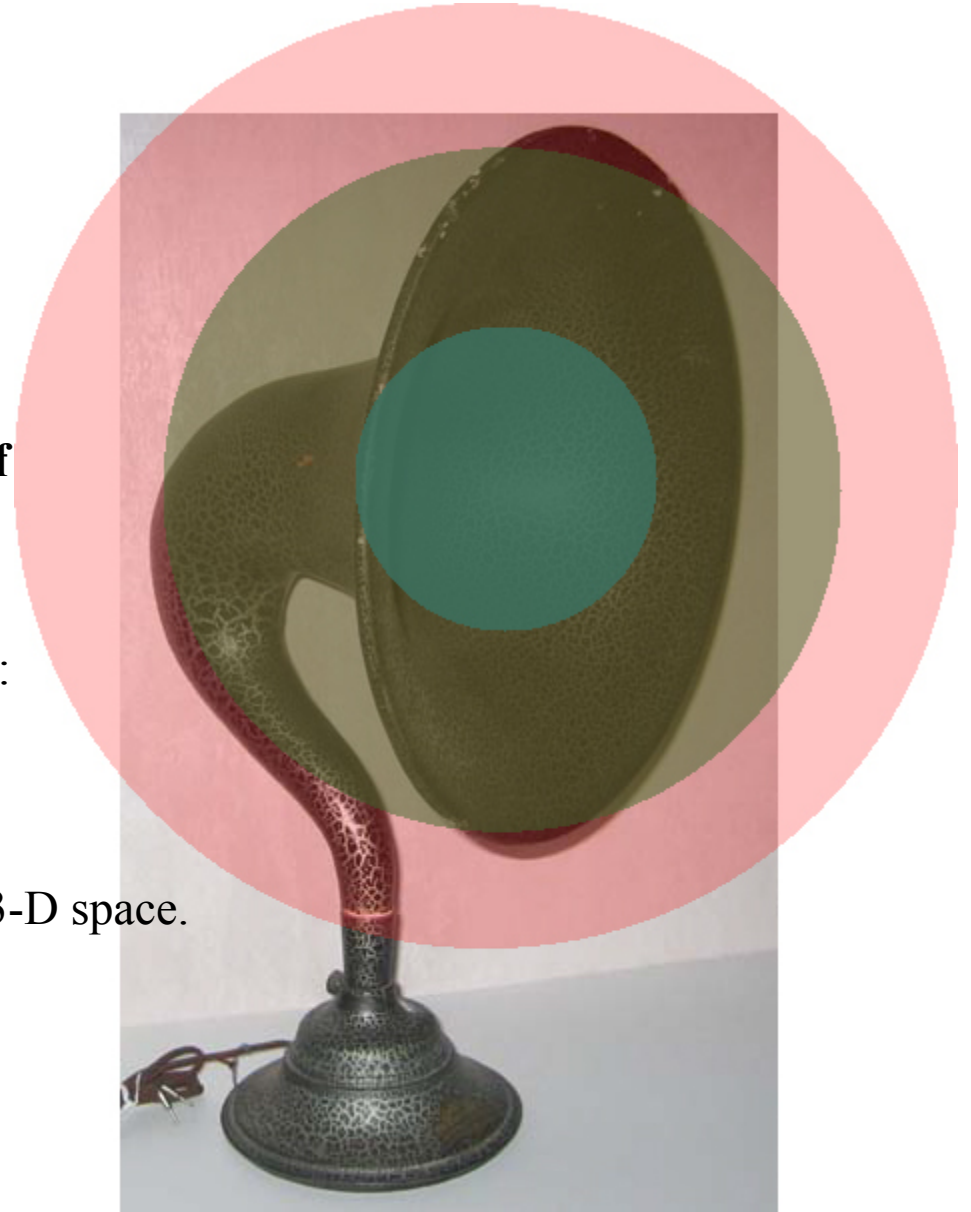
- We still describe this wave with λ and f and $v = \lambda f$
- Speed is set by air density ρ and air-springy-ness (Bulk Modulus) β :

$$v = \sqrt{\frac{\beta}{\rho}}$$

- But the sound's energy spreads out in 3-D space. So the intensity=Power/area

$$I = P / (4\pi r^2)$$

Intensity decreases as $1/r^2$



12. If I'm standing 10m away from a speaker but then move to 100m away from that speaker, how does the intensity that I feel change?

The distance from the source to me increases by a factor of 10. Since $I \sim 1/r^2$, the intensity falls by 100.

A) **1/100 x smaller** B) 1/10 x smaller C) 10 x larger D) 100 x larger

13. If I'm standing 18m away from a speaker but then move 12m towards that speaker, how does the intensity that I feel change?

The distance from the source to me drops by a factor of 3. Since $I \sim 1/r^2$, the intensity falls by 9.

A) 1/9 x smaller B) 1/3 x smaller C) 3 x larger D) **9 x larger**

Decibel Change: It's stupid to compare two intensities if one's so much smaller than the other. So instead, use:

$$dB_{12} = 10 \log_{10} \left(\frac{I_2}{I_1} \right)$$

14. When I move from a distance of 1m from the source to 100m away, what's the change in intensity in dB?

We know I will be smaller by $(100)^2=10^4$. $I/I_0 = 10^{-4} \dots dB = 10 * \log_{10}(10^{-4}) = -40$ dB. **Intensity falls by 40 dB**

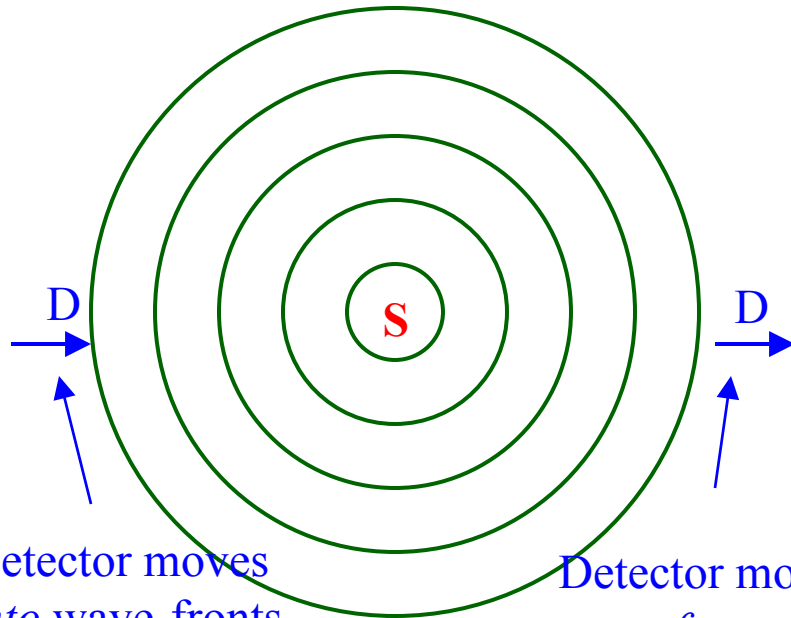
15. If a sound meter reads 30 dB as I stand 10m from a siren, how far would I have to stand from the siren to experience 50dB intensity?

In other-words, where do walk to *increase* intensity by 20dB? $20 = 10 * \log_{10}(I_{new}/I_{old})$. So I_{new} will be $10^2=100$ times greater than I_{old} . Since $I \sim 1/r^2$, we need to stand not 10m from the source, but 1m.

Doppler shift: Running into or away from waves

In these cartoons, **S** refers to the **source** while **D** refers to a **detector** (e.g. our ears)

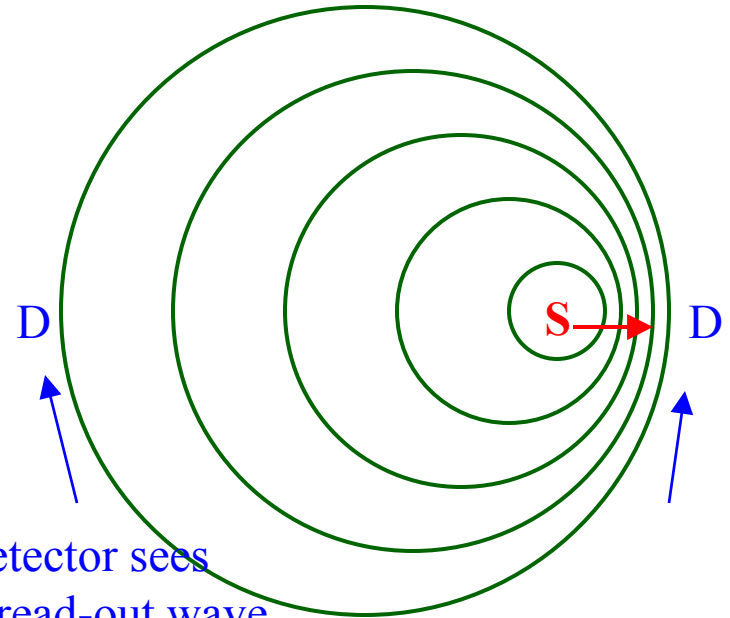
Source is stationary



Detector moves *into* wave-fronts so sees them more frequently

Detector moves *away from* wave-fronts so sees them less frequently

Source is in motion



Detector sees spread-out wave fronts if **Source** moves away

Detector sees compressed wave fronts if **Source** moves towards it

In general:

$$f_D = f_S \frac{c \pm v_D}{c \mp v_S}$$

where c =speed of sound=340 m/s

and just pick the signs to match cartoon intuition

Speed-trap!



$$f_D = f_S \frac{c \pm v_D}{c \mp v_S}$$

A cop sits on the side of the road with a “sonar-gun” (Radar guns use EM waves which have a different Doppler formula) watching you as you approach in your truck at a speed 1/10 that of sound (34m/s~75mph). His gun emits 1.0 MHz sound.

a) What frequency would you see in your truck with a speed-trap detector?

The source (the cop) is stationary, so $v_S=0$ and you (the detector) move at $v_D=c/10$. Since I'm moving into the waves, I expect to see a higher frequency, so pick the “+” sign in numerator.

$$f_D = 1.0 * (c + c/10) / (c + 0) = 1.0 * (1.1) = 1.1 \text{ MHz}$$

A) 0.9MHz B) 1.0MHz C) **1.1 MHz** D) 1.2 MHz

b) What frequency will the cop see when that wave reflects back to his car?

Use the truck as the source of reflected waves with $f_S=1.1$ MHz. The truck is moving at $v_S=c/10$ so as to compress the wave-fronts, so we expect the stationary detector (cop at $v_D=0$) to see a higher frequency yet. So pick the “-” sign in denominator:

$$f_D = 1.1 * (c + 0) / (c - c/10) = 1.1 * (1/0.9) \sim 1.1 * (1.1) \sim 1.2 \text{ MHz}$$

A) 1.1 MHz B) **1.2MHz** C) 1.3 MHz D) 1.4 MHz

c) What frequency will the cop see after you pass him and are moving away?

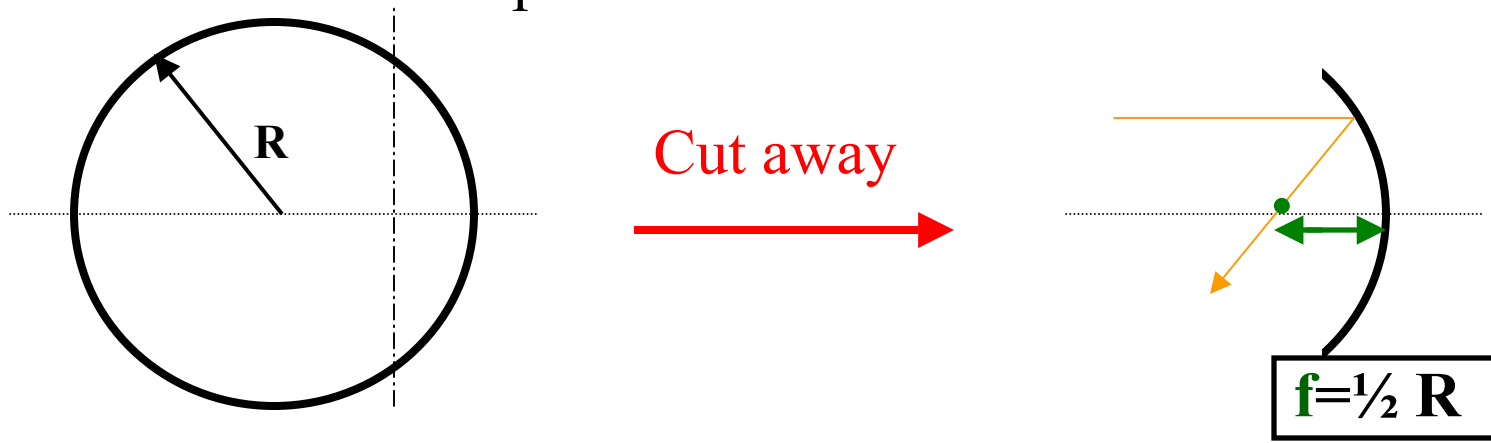
Repeat the reasoning behind a & b for the truck moving away. As the sound moves to the car, pick the “-” in the numerator. As it reflects back, pick the “+” sign in the denominator:

A) 0.7 MHz B) **0.8 MHz** C) 1.0 MHz D) 1.2 MHz

Physics 9: Optics

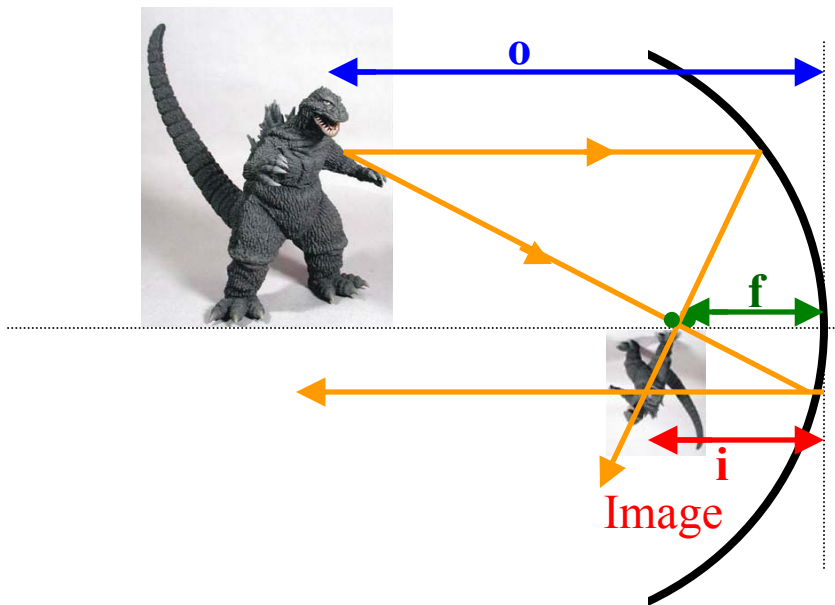


Spherical Mirrors



- All rays parallel to the central axis pass thru the focal point. •
- That focal point is a half radius from the mirror's center

Object



1. Where's the **image**?

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i}$$

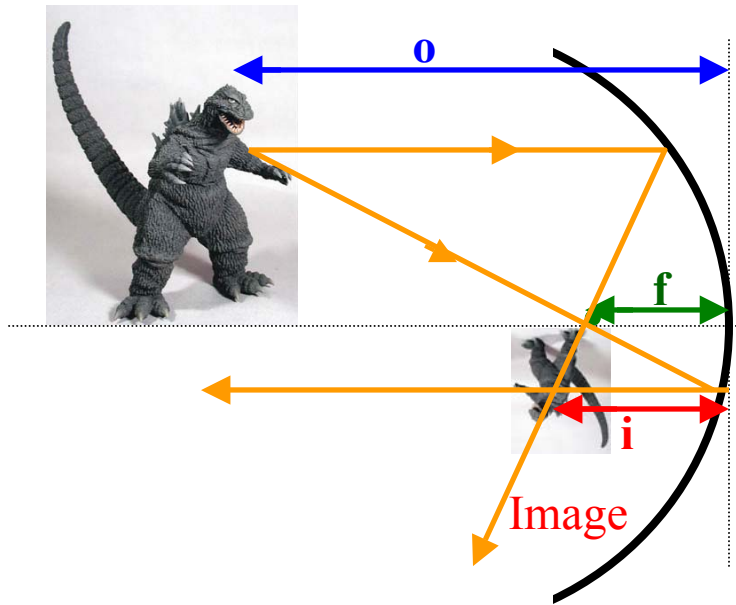
2. How magnified is the **image**?

$$m = - \frac{i}{o}$$

NOTE: You'll never need to draw ray-diagrams on the MCAT!

Concave Mirrors have $f > 0$

If the **Object** is outside of focal point •
Object



• Since $o > f$, i is positive:

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{o}$$

$$= \text{big} - \text{small} > 0$$

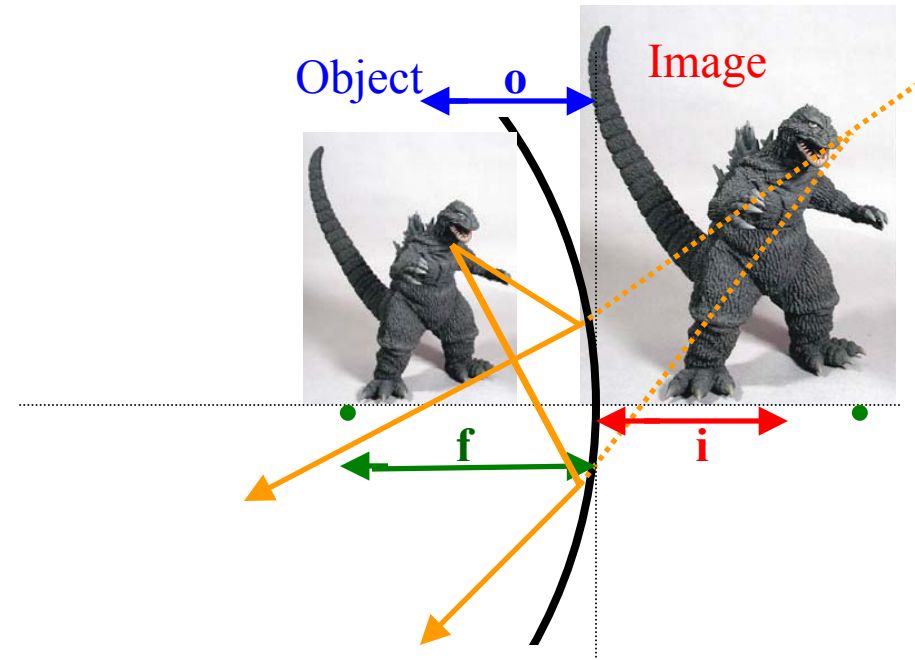
• Since $o > 0$ & $i > 0$, m is negative

$$m = -i/o < 0$$

Image will be *inverted*

• Since rays **cross** at **image**,
it's real

If the **Object** is inside of focal point •



• Now that $o < f$, i is negative:

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{o}$$

$$= \text{small} - \text{big} < 0$$

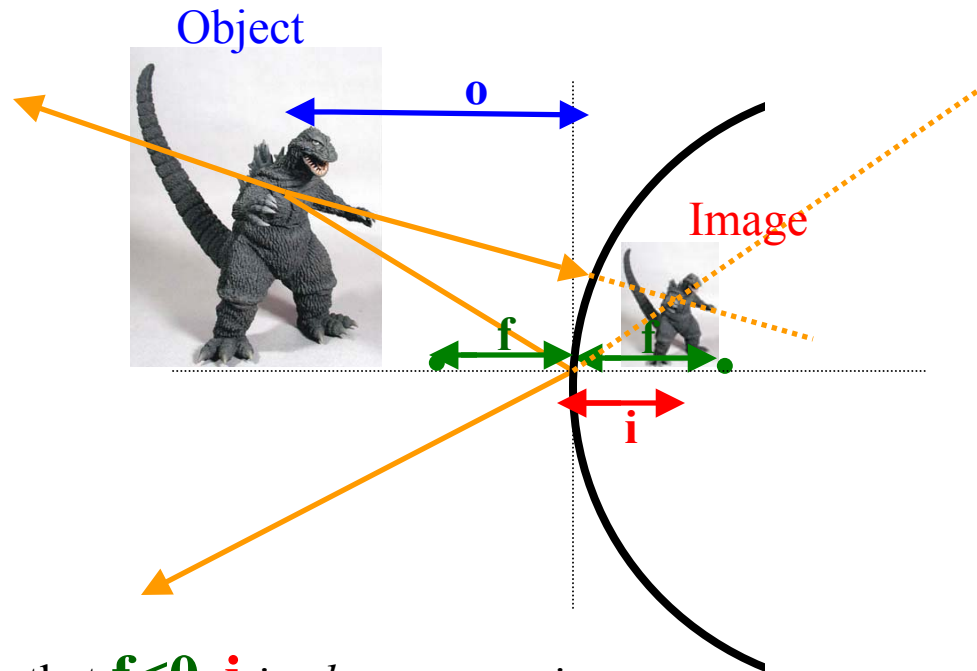
• Since $o > 0$ but $i < 0$, m is positive

$$m = -i/o > 0$$

Image will be *right-side-up*

• The rays don't cross at **image**, we
only think they do: *it's virtual*

Convex Mirrors have $f < 0$



- Now that $f < 0$, i is always negative:

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{o}$$

$$= \text{negative} - \text{something} < 0$$

- So the image is always on the right and always virtual
- Since $o > 0$ and $i < 0$ always

m is always positive: $m = -i/o > 0$

Image is always right-side up

Reflection from Convex and Concave Surfaces



Figure 3

The only reason you think spherical mirrors are hard is because you lack the proper intuition. So you should check these rules for spherical mirrors by looking at a metal spoon.

1. A convex mirror with radius of curvature 24 cm produces an image that is $\frac{1}{2}$ the size of the object. How far was the object from that mirror?

A) 4 cm B) 6 cm C) 12 cm D) 36 cm

$$m = \frac{1}{2} = -\frac{i}{o} \text{ (positive because it's convex) So } i = -\frac{o}{2}$$

$$f = \frac{r}{2} = -\frac{24}{2} = -12 \text{ cm (Negative bc it's convex)}$$

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$

$$\frac{1}{o} - \frac{2}{o} = -\frac{1}{12}$$

$$-\frac{1}{o} = -\frac{1}{12} \longrightarrow o = 12$$

2. An object is placed 10 cm from a concave mirror and results in an inverted image 3 times bigger than the object. What's the focal length?

A) 7.5 cm B) 15 cm C) 20 cm D) 30 cm

$$m = -3 = -\frac{i}{o} \text{ (negative because it's concave) So } i = 3 * o = 3 * 10 = 30 \text{ cm}$$

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i}$$

$$= \frac{1}{o} + \frac{1}{i} = \frac{1}{10} + \frac{1}{30} = \frac{4}{30}$$

$$f = \frac{30}{4} = 7.5 \text{ cm}$$

Spherical Lenses

Lenses are the same as mirrors *except* that:

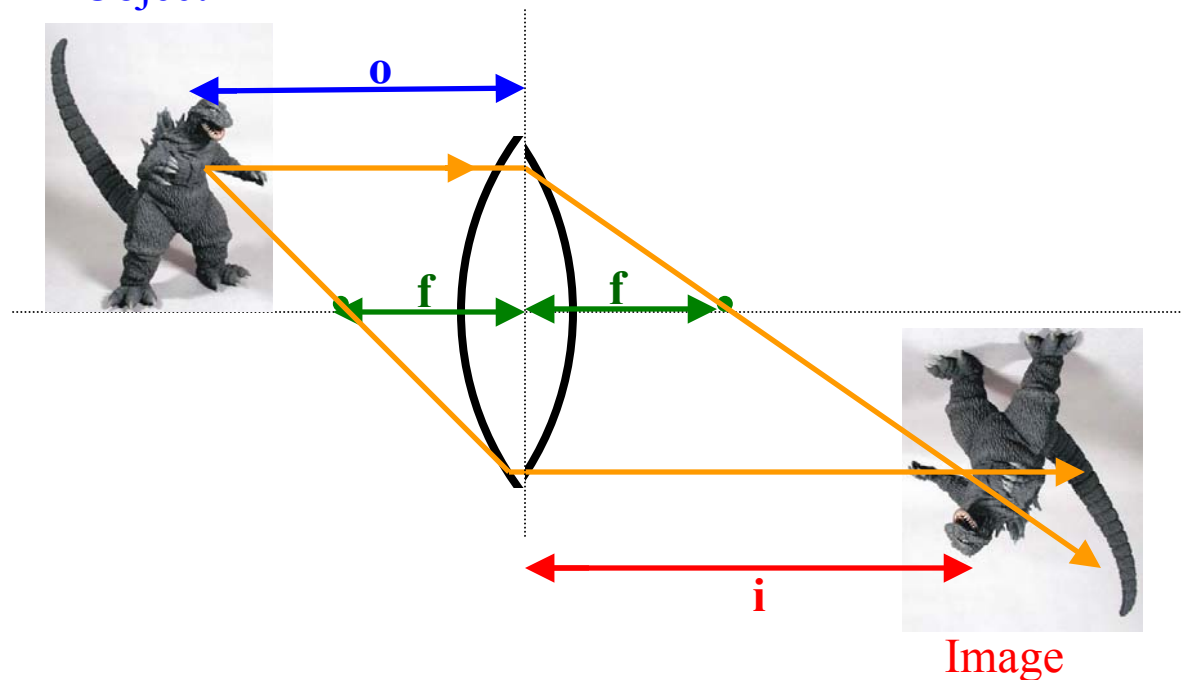
1. You *can't* use $f=R/2$ for lenses
2. *Real* images form on the *opposite* side of the lens, where $i>0$

You can still use these equations:

1. $1/f = 1/o + 1/i$

2. $m = -i/o$

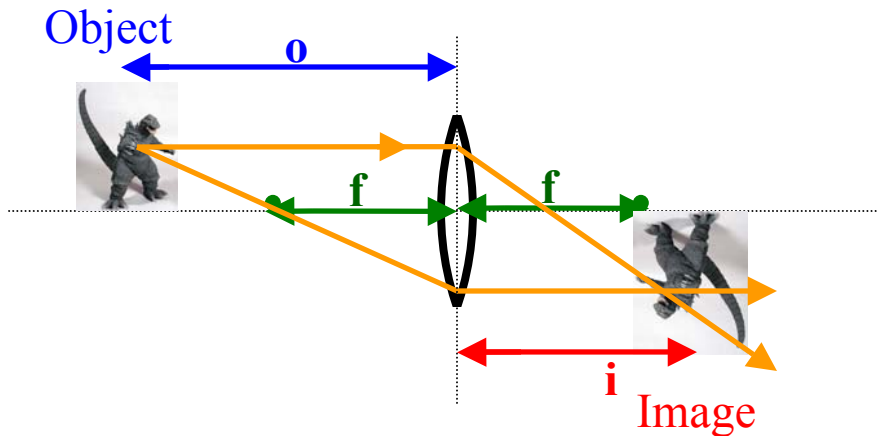
Object



Lens Power: $P=1/f$. $[P]=1/m=D$ “Diopter”

Converging Lenses have $f > 0$

If the **Object** is outside of focal point •



• Since $o > f$, i is positive:

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{o}$$

$$= \text{big} - \text{small} > 0$$

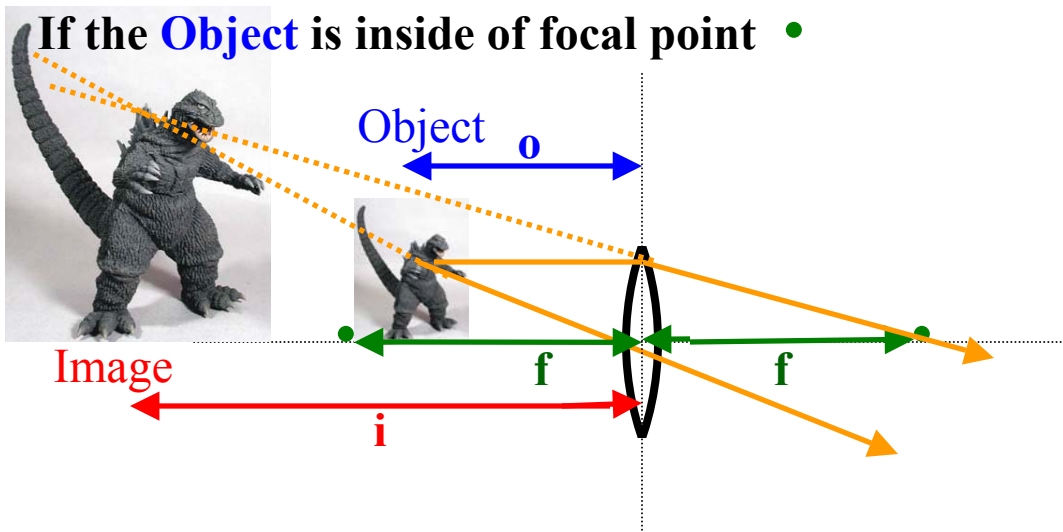
• Since $o > 0$ & $i > 0$, m is negative

$$m = - \frac{i}{o} < 0$$

Image will be inverted

• Since rays cross at image, it's real

If the **Object** is inside of focal point •



• Now that $o < f$, i is negative:

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{o}$$

$$= \text{small} - \text{big} < 0$$

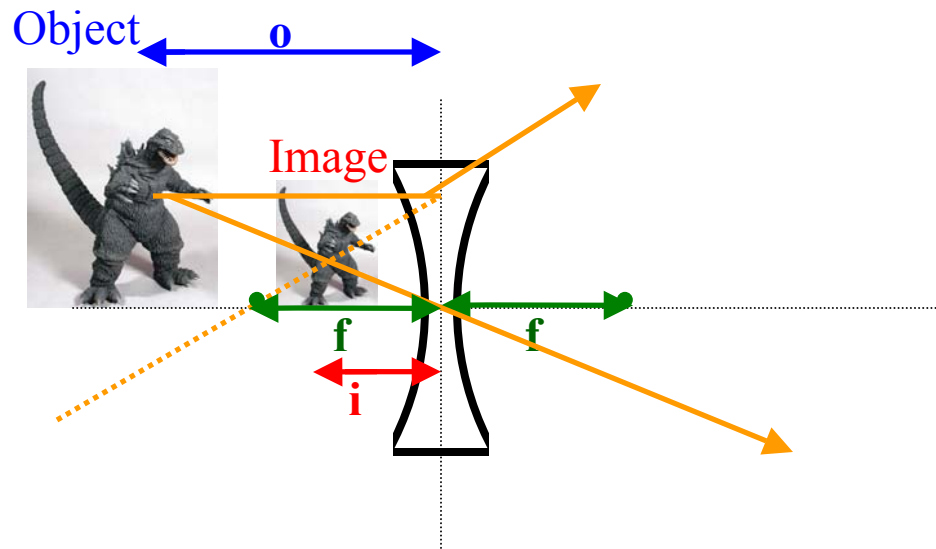
• Since $o > 0$ but $i < 0$, m is positive

$$m = - \frac{i}{o} > 0$$

Image will be right-side-up

• The rays don't cross at image; we only think they do: it's virtual

Diverging Lenses have $f < 0$



- Now that $f < 0$, i is always negative:

$$\begin{aligned} \frac{1}{i} &= \frac{1}{f} - \frac{1}{o} \\ &= \text{negative} - \text{something} < 0 \end{aligned}$$

- So the **image** is always on the left and *always virtual*
- Since $o > 0$ and $i < 0$ always

m is always positive: $m = -i/o > 0$

Image is always right-side up

3. I create a real image twice as large as the object with a 4D lens. How far is the *image* from the lens?

A) 6 cm B) 12 cm C) 37.5 cm **D) 75 cm**

$D=4$, so $f=1/4$ \therefore $f=25$ cm.

$m = -2$ (negative because the image is *real*). So $o=i/2$

$$1/o + 1/i = 1/f$$

$$2/i + 1/i = 1/25$$

$$3/i = 1/25 \longrightarrow i = 75 \text{ cm}$$

4. I use a converging lens to produce an image that appears at the same position as the object. Where would I put the object?

A) $f/2$ B) f C) $2f$ **D) Can't be done**

If the image and object are at the same position, the image must be on the same side as the object. So it's virtual and: $i=-o$

$$1/f = 1/o + 1/i$$

$$= 1/o + 1/(-o) = 0 \quad 1/f=0 \text{ has no solution. } \underline{\text{This can't be done!}}$$

When we place an object at the focal point of a converging lens, the rays are focused to $i=\text{infinity!}$

5. I place a 10 cm lens and 20 cm lens side-by-side. What is their total power?

A) 10 D B) **15 D** C) 20 D D) 30 D

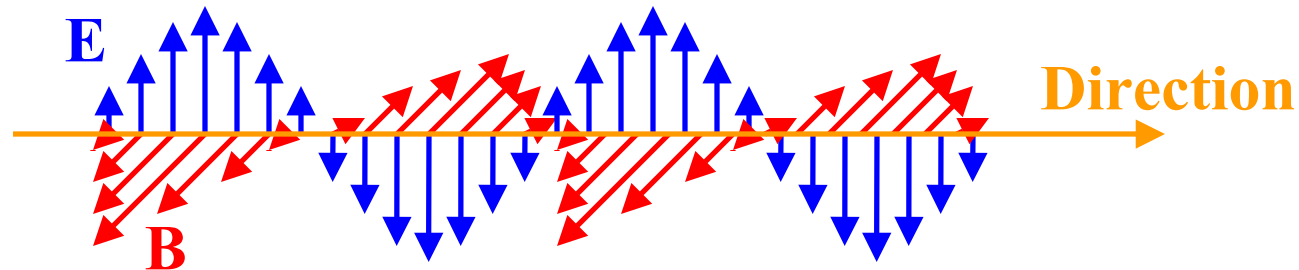
When we cascade lenses, just add their powers!

$$P_1 = 1/f_1 = 1/10 \text{ cm}^{-1} = 10 \text{ m}^{-1} = 10 \text{ D}$$

$$P_2 = 1/f_2 = 1/20 \text{ cm}^{-1} = 5 \text{ m}^{-1} = 5 \text{ D}$$

$$P_1 + P_2 = 10 \text{ D} + 5 \text{ D} = 15 \text{ D}$$

Light is an Electromagnetic Wave



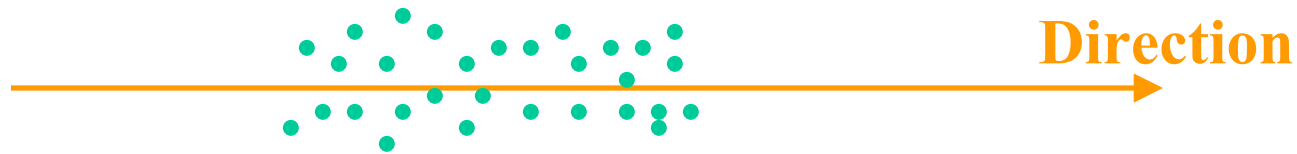
- **E** = **B** Fields are in phase
- **E** & **B** are perpendicular to each other and to Direction of Propagation
- The plane that **E** is in is called the polarization
- The greatest magnitude of **E** is related to Light's intensity
- The frequency of the wave determines the color.
- The wave speed is given by:

$$v=c/n$$

$$c=3*10^8 \text{ m/s}$$

$$n\text{- index of refraction } \geq 1$$

Light is also a swarm of Photons



- Photons travel at 3×10^8 m/s because they have no mass
- The intensity of the light is related to the number of photons
- The color is related to the energy of each photon:

$$E=hf$$

$$h=6.67 \times 10^{-34} \text{ Js}$$

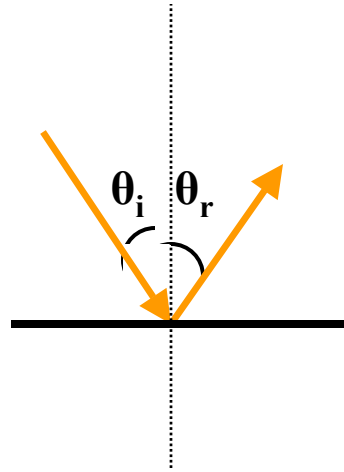
6. Light with wavelength 200 nm travels through a material with index of refraction 1.5. What is the energy of three photons?

The speed of light in this material is $v=c/n=3 \times 10^8(3/2)=2 \times 10^8$ m/s.

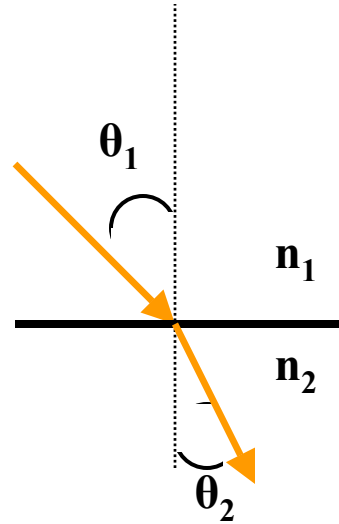
So the frequency is $f=c/\lambda=(2 \times 10^8)/(600 \times 10^{-9})=(1/3) \times 10^{15}$ Hz=330 THz

So the energy is $E=hf=(6.6 \times 10^{-34})((1/3) \times 10^{15})=2.2 \times 10^{-19} \sim \underline{1 \text{ eV}}$

Reflection & Refraction



$$\theta_i = \theta_r$$



$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

Or $n \sin(\theta)$ is constant

ALL angles are measured relative to the normal (dashed line), not the plane

7. A ray strikes glass ($n=3^{1/2}$) from air ($n=1$) at an angle 30° above the surface. What's the angle of reflection

- A) $\arcsin(1/(2 * 3^{1/2}))$ C) $90^\circ - \arcsin(1/(2 * 3^{1/2}))$
B) 30° D) 60°

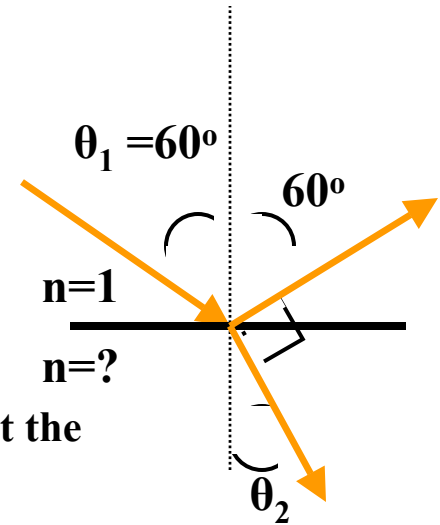
8. A ray strikes fluid from air ($n=1$) at an angle of incidence of 60° . The (partially) reflected and refracted rays are perpendicular. What's the fluid's index of refraction?

A) $2/3^{1/2}$

C) $3 * 3^{1/2} / 2$

B) $3^{1/2}$

D) $2 * 3^{1/2}$



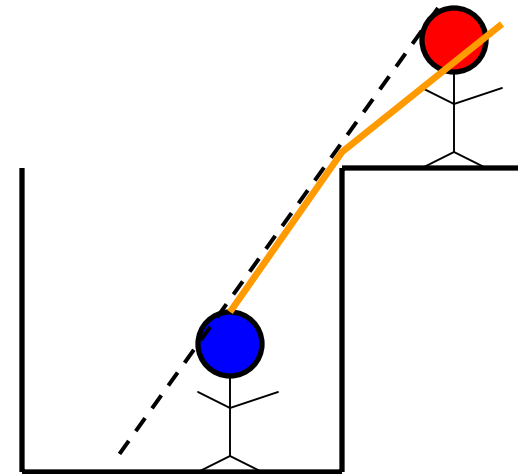
Since reflected & refracted rays are 90° apart, we see from diagram that the refracted ray is 30° from the normal. Now use Snell's law:

$$1 * \sin(60^\circ) = n * \sin(30^\circ)$$

$$n = 3^{1/2}$$

9. A swimmer stands on the bottom of the pool while a lifeguard stands near the edge. If the pool had no water in it, they would just barely see each other (dashed line). What happens when we fill the pool up to the rim with water?

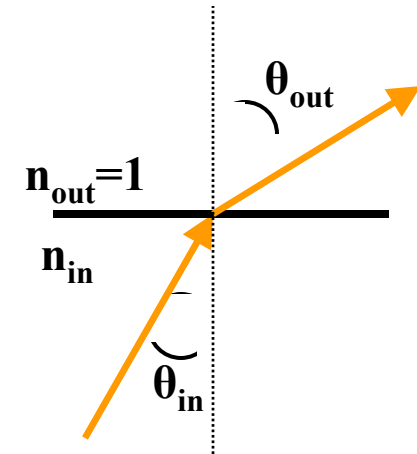
- A. Swimmer & Lifeguard both see each other
- B. Neither sees each other
- C. Only the swimmer can see
- D. Only the lifeguard can see



Snell's Law tells us that $n * \sin(\theta)$ is the same on both sides of the water for a ray. Imagine a ray of light leaves the submerged swimmer's head and then just grazes the edge of the pool as it leaves the water. Since n drops to 1 once it reaches air, θ must increase. In other words, the ray bends away from zenith and *towards the lifeguard*. We reach a similar conclusion¹⁰⁸ following rays from the lifeguard to the swimmer.

Total Internal Reflection

Imagine that a ray originates *within* a piece of glass ($n_{in} > 1$) and passes through the surface to air ($n_{out} = 1$). Will θ_{out} be greater or smaller than θ_{in} ? (*Ans: larger*)



If we keep making θ_{in} larger and larger (grazing the surface), then eventually, the ray will refract to $\theta_{out} = 90^\circ$, at which point, the light doesn't really leave the glass at all. This is total internal reflection and the inside angle θ_{in} is called θ_{crit} :

$$\sin(\theta_{crit}) = n_{out}/n_{in}$$

NOTE: $n_{in} > n_{out}$ for there to be a critical angle. The light must start "inside."

10. A fiber-optic cable has a glass core with $n=2$ and a protective cladding with $n=1.73$. What is the greatest angle of incidence that light in the core can strike the interface and still not leak out of the cable?

A) 60°

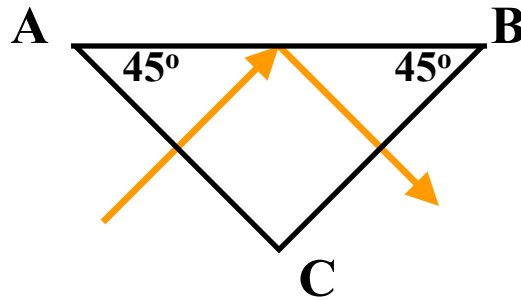
C) 53°

$$\sin(\theta_{crit}) = n_{out}/n_{in} = 1.73/2 = 0.85$$

B) 45°

D) 60°

$$\text{So } \theta_{crit} = 60^\circ$$



11. The prism has 45° angles at points A and B. If a beam enters normal to face AC from air and experiences total internal reflection at face AB, what values of index of refraction can the prism have?

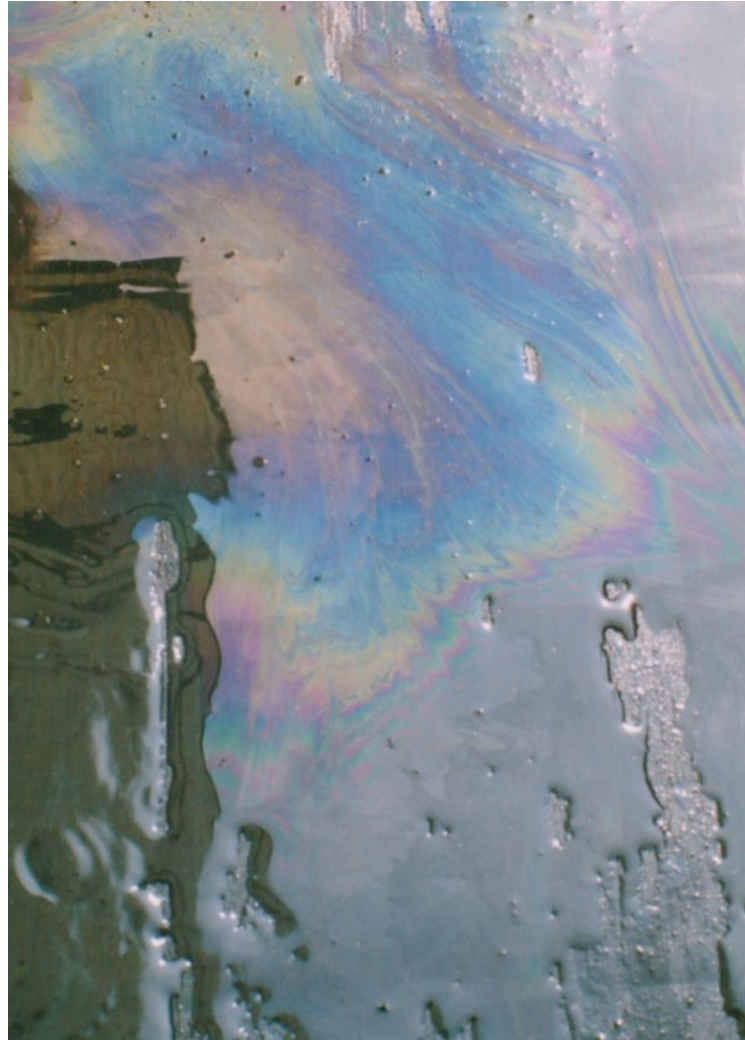
- A. $1/2^{1/2} < n < 1$
- B. $1/2^{1/2} < n < 2^{1/2}$
- C. $1 < n < 2^{1/2}$
- D. $2^{1/2} < n$**

From the figure, θ_{crit} must be no greater than 45° (we *could* be beyond the critical angle): $\theta_{\text{crit}} < 45^\circ$

$$1/n = \sin(\theta_{\text{crit}}) < \sin(45^\circ) = 1/2^{1/2}$$

$$\text{So } n > 2^{1/2}$$

OH 2: Interference

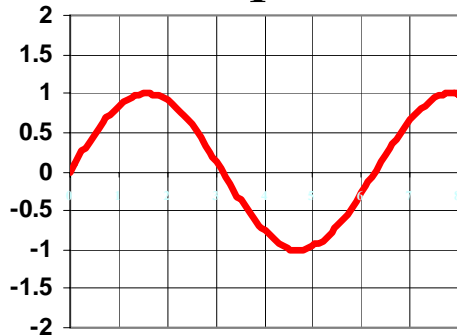


This stuff is extra; you probably don't need to know it in advance, but it may appear in a passage.

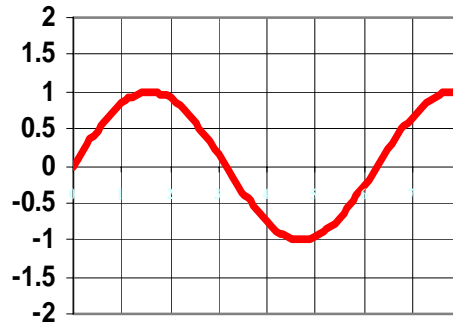
Interference of two String Waves

When we put two waves on the same string, we add up the heights of the two waves to create a combined wave:

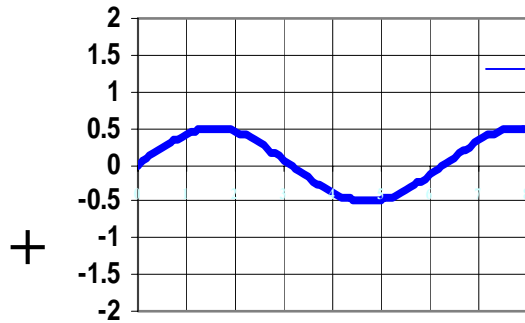
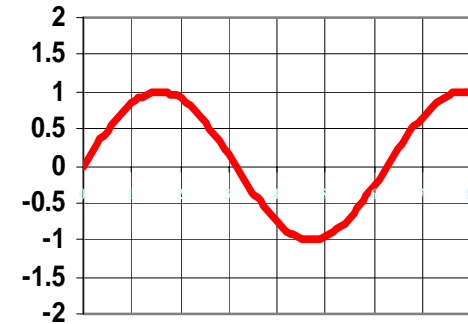
Example 1



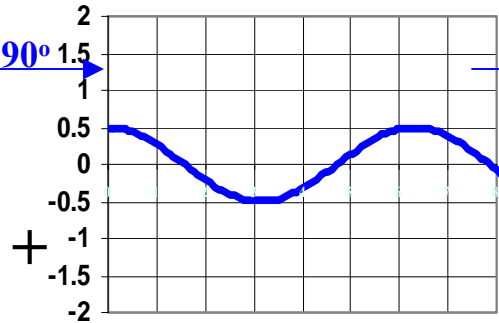
Example 2



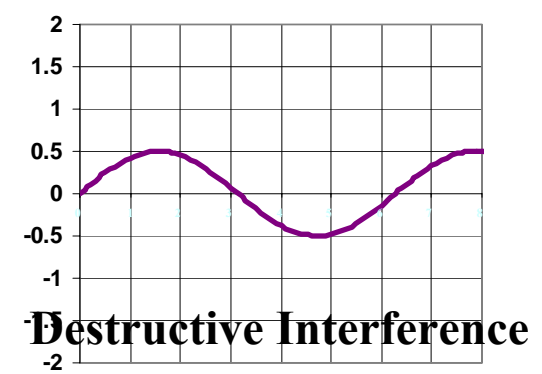
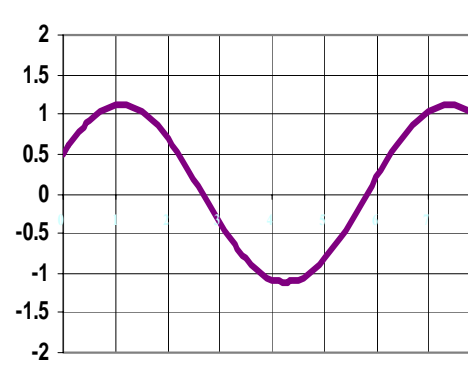
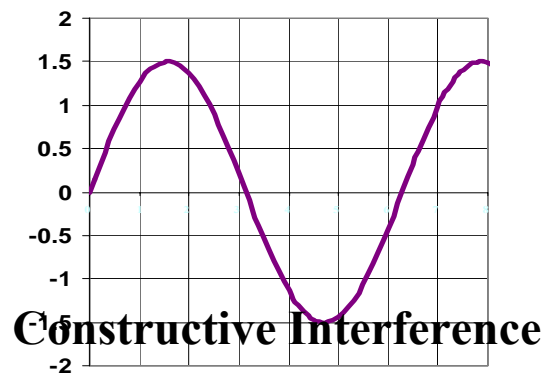
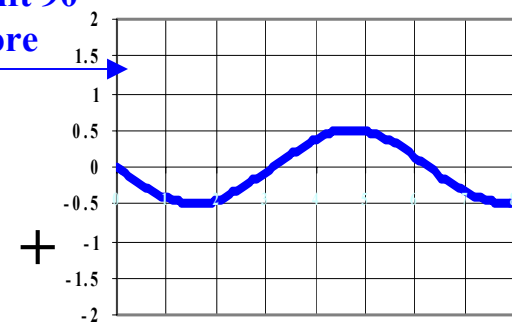
Example 3



Shift 90°

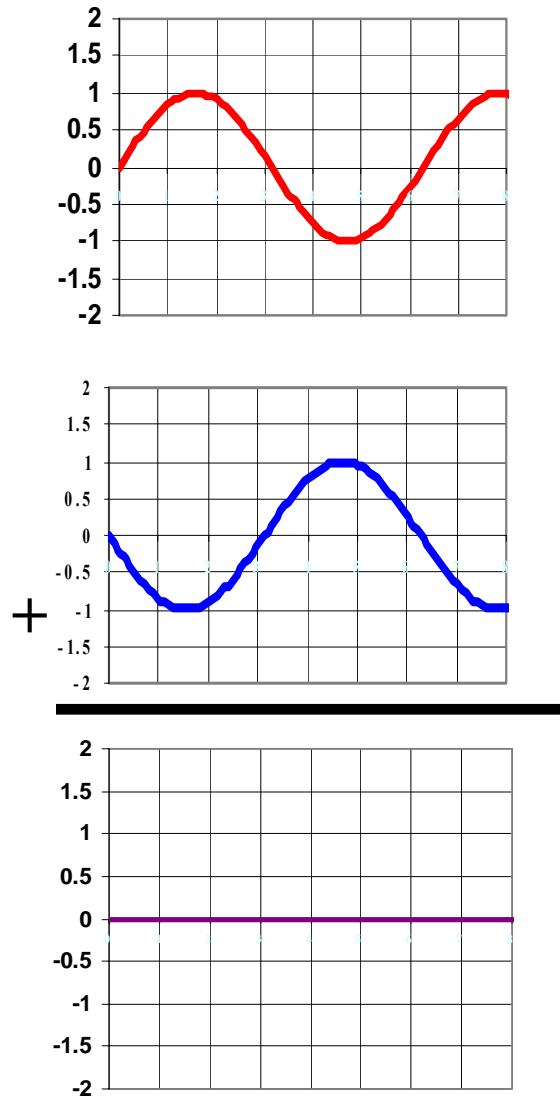


Shift 90°
more



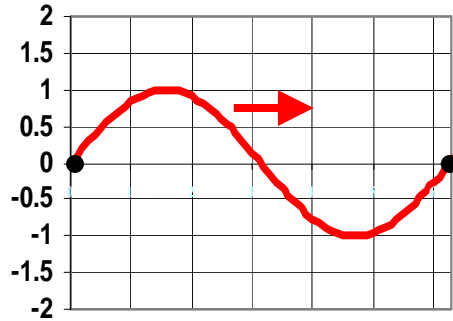
Complete Destructive Interference

If the waves have the same amplitude and are 180° out of phase, then they *cancel!*

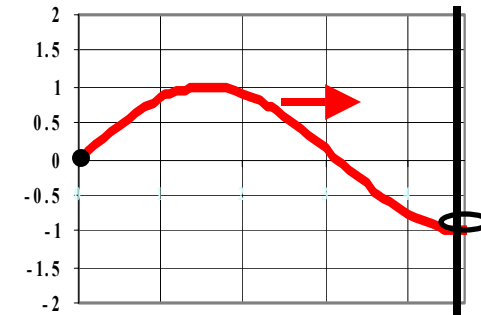


Reflected Traveling waves interfere to create Standing Waves

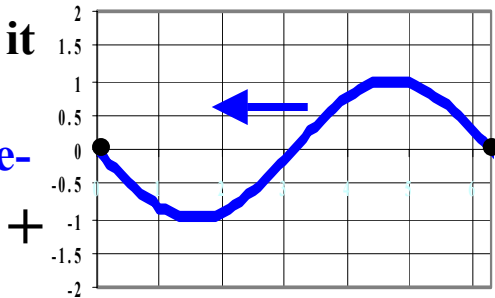
Right end is tied down



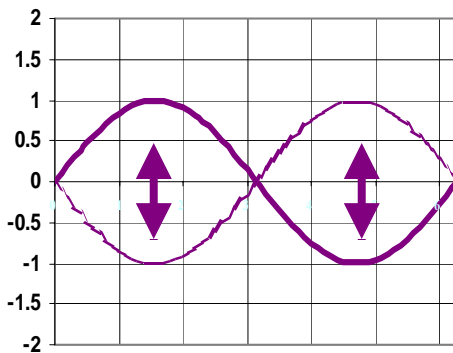
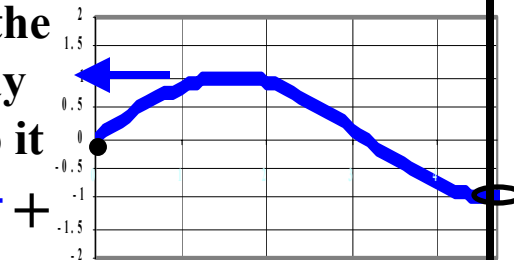
Right end on sliding loop



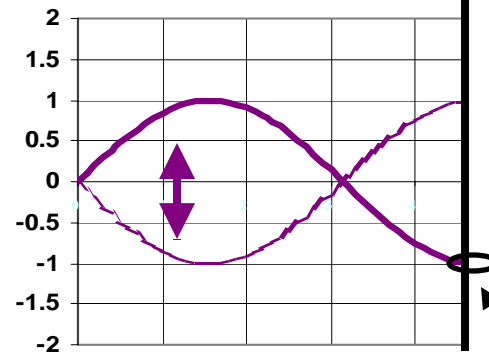
When the **incident** wave hits the end, it reflects and turns **up-side-down**



When the **incident** wave hits the end, the rope must stay horizontal, so it reflects **right-side-up**.

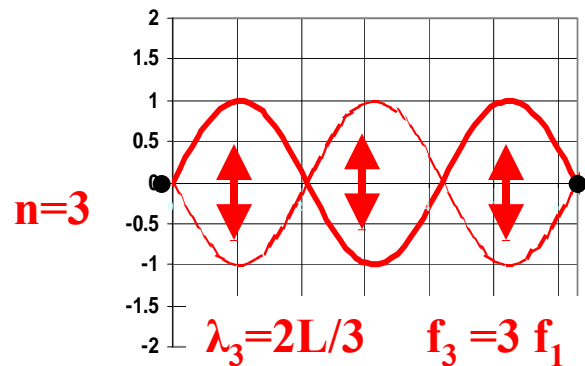
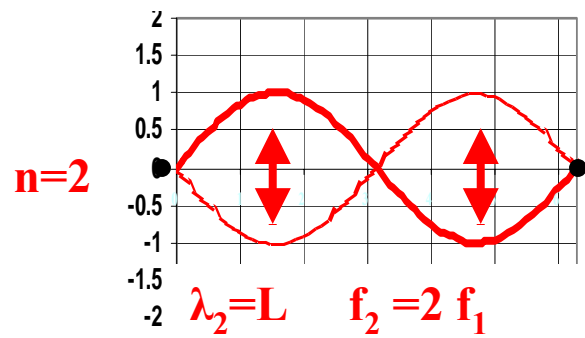
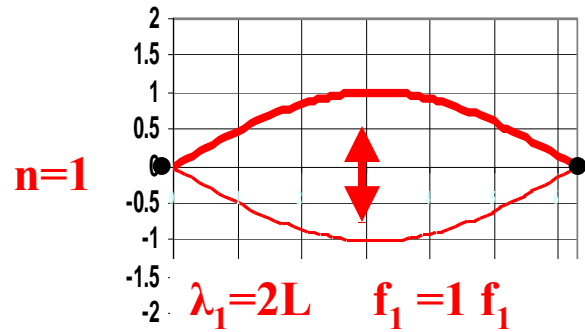


Right end is node!

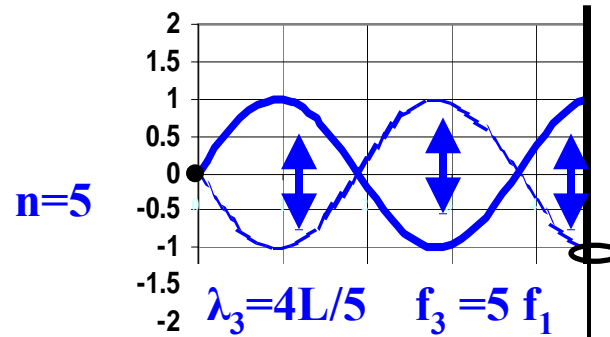
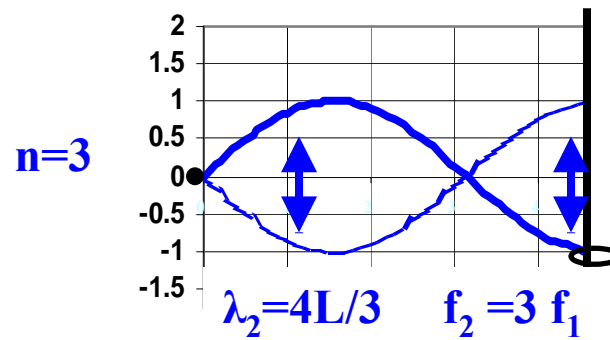
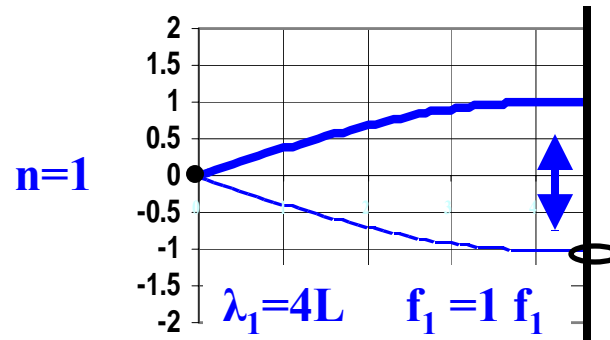


Right end is *anti-node*!

Standing Wave Frequencies



$\lambda_n = 2L/n$ $f_n = n f_1$ $n=1,2,3,\dots$

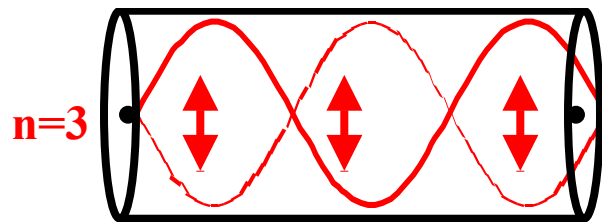
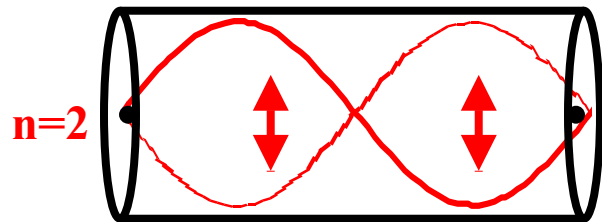
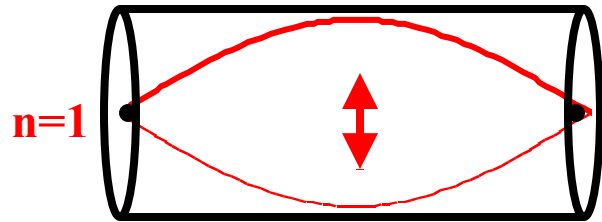


$\lambda_n = 4L/n$ $f_n = n f_1$ $n=1,3,5,\dots$

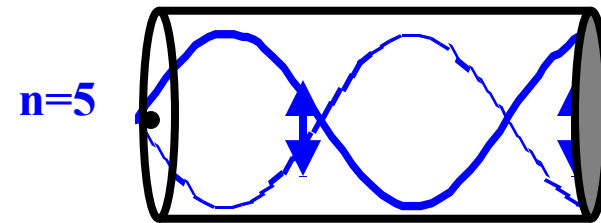
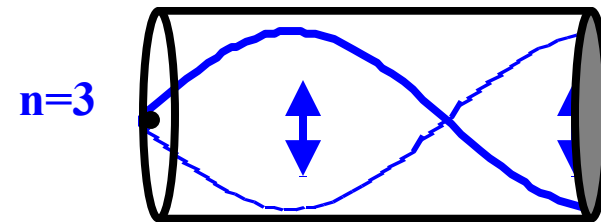
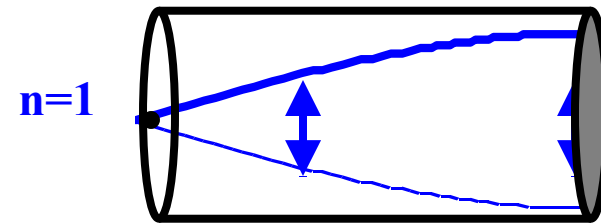
Because of the ring, only odd modes allowed!

Sound in a flute

- Open end pins pressure at 1 atm; **it's a node**
- Closed end builds up pressure to a maximum; **it's an antinode**



Both ends open allows all modes:
n=1,2,3,...



One end closed allows only odd modes:
modes: **n=1,2,3,...**

Problems with Standing waves in tubes

In a 2m long organ pipe open at both ends, how many times greater is the 3rd harmonic frequency than the fundamental?

- A) 1.5 B) 2 C) 3 D) 6 $f_n = n f_1$ and in this case $n=3$

Consecutive harmonic wavelengths in an organ pipe open at both ends are 2m and 1.3m. How long is this pipe?

- A) 1m B) 2m C) 4m D) 8m

You can write two equations with $\lambda=2L/n$ then eliminate n to solve for L . Instead, let's do it by inspection:

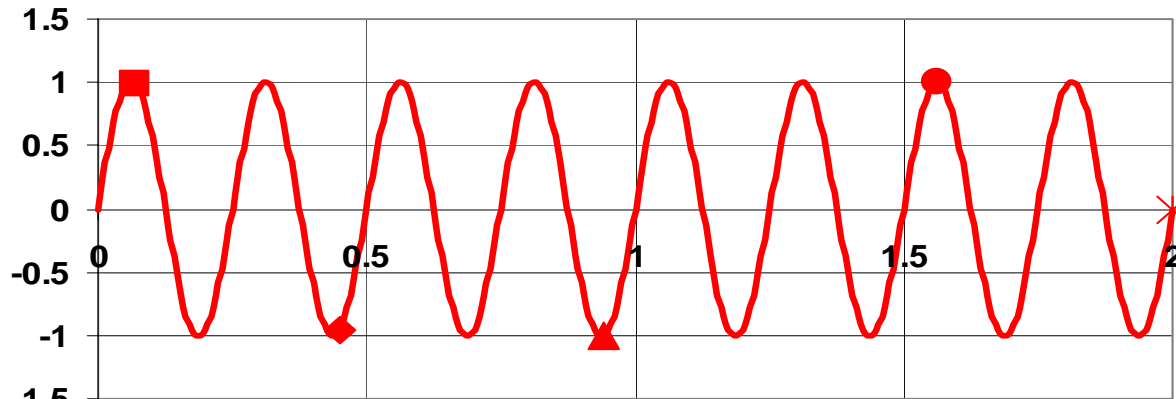
Notice that $\lambda=1.3m \sim 4/3$. So perhaps this is the $n=3$ mode. If so, then $L=2m$. Now check that this is consistent with $\lambda_2=2m$:
 $\lambda_2 = 2L/n = 2 \cdot 2 / 2 = 2m$

So the solution is $L=2m$

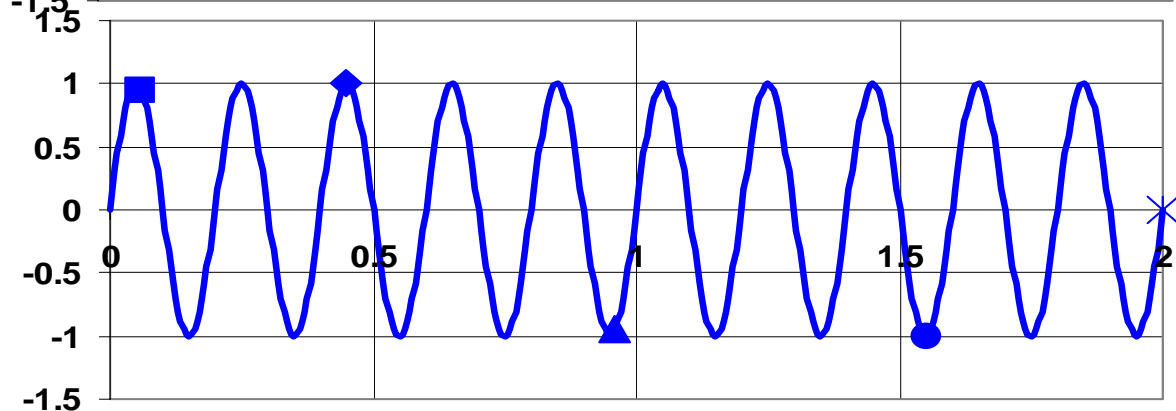
In a 50cm long flute with one end closed and one end open, how many times greater is the 4th harmonic's frequency than the 3rd's?

The 4th harmonic is not allowed with one end open only odd modes are permitted.

Beats from Interfering two similar frequencies

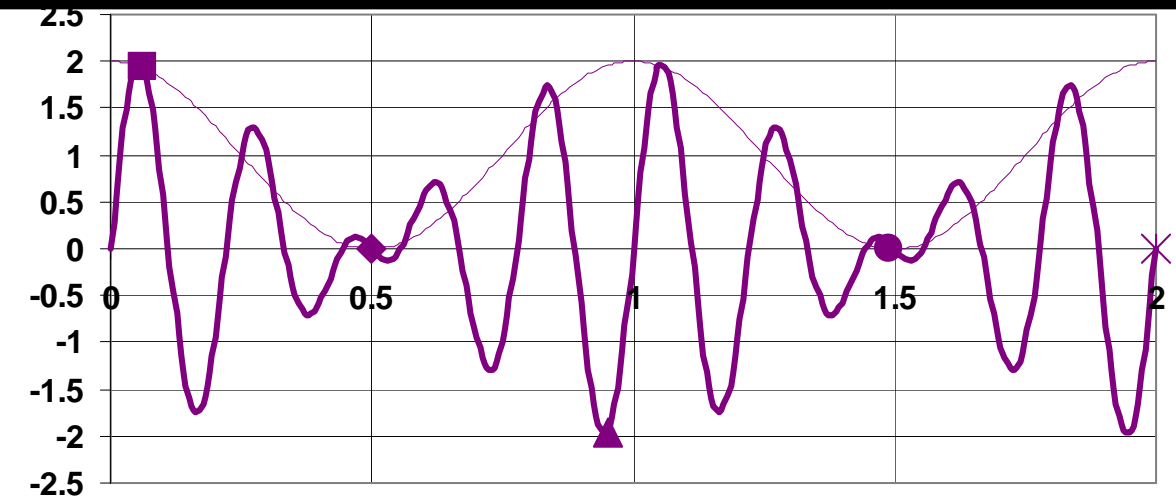


$f_1=4\text{Hz}$



$f_2=5\text{Hz}$

+



$f_{\text{beat}}=1\text{Hz}$

$$f_{\text{beat}} = |f_2 - f_1|$$

Problems with Beat Frequencies

As I tune the “A-string” of a guitar against the “E-string” (440 Hz), I hear 3 beats per second. What’s the frequency of the “A-string”?

- A) 437 Hz B) 443 HZ C) Either A or B D) Neither A nor B

$$\begin{aligned} f_{\text{beat}} &= |f_A - f_E| \\ 3 &= |f_A - 440| \text{ so } f_A = 437 \text{ or } 443 \end{aligned}$$

I’m in a parked ambulance with a siren wailing at 900 Hz. A similar ambulance passes me and as it pulls in front, I hear 300 Hz beat frequency. How fast is that ambulance moving?

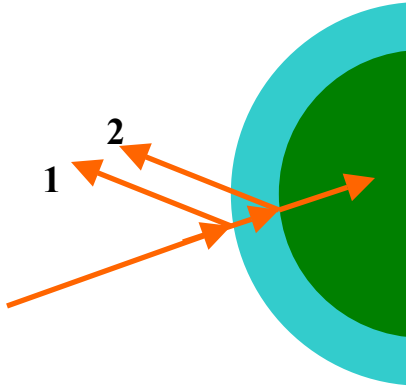
The differing frequencies arise from a doppler shift, and since the other ambulance is moving *away* from me, he’d better have a lower frequency:

$$\begin{aligned} f_{\text{beat}} &= |f_{\text{mine}} - f_{\text{other}}| \\ 300 &= 900 - f_{\text{other}} \text{ so } f_{\text{other}} = 600 \text{ Hz} \end{aligned}$$

No just plug into the Doppler shift formula and solve for the source speed. Don’t forget to pick the “+” sign in the denominator.

$$f_D = f_S \frac{c \pm v_D}{c \mp v_S} \xrightarrow{\text{Plug in}} 600 = 900 \frac{c \pm 0}{c + v_S} \xrightarrow{\text{Solve}} \boxed{v_S = \frac{c}{2} \approx 170 \text{ m/s}}$$

Thin Films



We can remove “glare” (partial reflections) on camera lenses by putting a film of some material over the lens and picking it to be $\lambda/4$ thick.

That way, when the two reflected beams shown at left recombine, they will be 180° out of phase and destructively interfere.

If I want to put an “antireflective coating” over a camera lens with a material that has an index of refraction 1.5 and I want to prevent reflection of violet light ($\lambda=200$ nm in vacuum), how thick does the coating need to be?

- A) $0.03 \mu\text{m}$ B) $0.05 \mu\text{m}$ C) $0.13 \mu\text{m}$ D) $0.20 \mu\text{m}$

The coating needs to be $\lambda/4$ thick, but this should be done with the λ in the coating with $n=1.5$, not vacuum. So don't use $\lambda=200$ nm; instead realize that:

$$\lambda f = c/n$$

tells us that if we increase n by $3/2$, we decrease λ by $2/3$. So $\lambda=133$ nm in the thin film. Hence the film is $\lambda/4=30$ nm= $0.03 \mu\text{m}$ thick

Index

Topic	Page
Absolute Pressure	51
Action Reaction Pair	14-15
Angular acceleration	41
Angular Momentum	43-46
Angular speed	40
Apparent weight	28-31
Apparent weight	49
Archimedes Principle	48
Area under graph	8
atm	50
Barrel of Fun	32
Beat Frequency	118-119
Bernoulli Effect	54
Bernoulli's Equation	53-55
Big 5 Eqns	10
Bulk Modulus	6
Buoyant Force	48
Capacitor	62
Capacitors	55, 71-75
Capacitors in Parallel	75
Capacitors in Series	75
Capacitors with dielectrics	73
Capacitors, fields energy & charge	72
Capacitors, time dependence	71

Topic	Page
Center of Mass	25-26
Centripetal Acceleration	29-32
Centripetal Force	No such thing
Completely Inelastic Collision	38-39
Constant Acceleration	10
Continuity Equation	52
Cosine	6
Coulomb's Law	57
Current	67
Decibels	93
density	48
displacement	6
distance	6
Dopler Shift	94-95
Elastic Energy	86
Electric Field from Line	61
Electric Field from Point	62
Electric Field from Sphere	60
Electric Fields	59-62
Electric Potential from Point	64
Electric Potential	63
Electric Potential from Sphere	64
Electromagnetic Wave	105
Electron	57
Electron Volt eV	63
elementary charge	57

Topic	Page
Energy	34
Energy conservation	36-37
Energy conservation with friction	36
Equilibrium Position of Mass-Spring	85
Floating	48
Flow Rate	52
flow speed	52
Forces	13-23
Free Fall	10,17
Free Fall	17
Frequency of Mass Spring Oscillator	83
graphs	7-9
Guage Pressure	50-51
Hooke's Law	82
Impulse	38
Inclined Plane	21-22
Index of Refraction	105
Inellastic Collistion	38
Interference Complete Destructive	114
Interference, Constructive	113
Interference, Destructive	113
Inverted Image	98-102
Kinematics	5-12
Kinetic Energy, linear	35
Kinetic Energy, rotational	44
Kinetic Friction	18-22

Topic	Page
Lens, converging	102
Lens, diverging	103
Lens, image position	101
Lens, magnification	101
Magnetic Fields	76-77
Magnetic Force	76
Mirror, concave	98
Mirror, convex	99
Mirror, image position	97
Mirror, magnification	97
MKS	6
Momement of Inertia	43-44
Momentum	38
Momentum Conservation, 1 D	38
Momentum Conservation, 2 D	38
Negative Work	34-35
Net Force	14
Newton's 1st Law	14
Newton's 2nd Law	14-16
Newton's 3rd Law	14-15
Newton's Law of Gravity	17
Normal Force	18

Topic	Page
Ohm's Law	67-70
Pa	51
Pendulum, frequency of	87-88
Period of Oscillation	83
Photon	106
Potential Energy GMm/r^2	37
Potential Energy mgh	35
Potential Energy of Spring	82
Potential Energy, Negative	35
Power	37
Projectile Motion	11-12
Proton	57
Pulleys	23
Range	12
Real Image	98-102
Reflection, Law of	107-108
Refraction, Law of	107-108
Resistance	67-70
Resistance of cylinder	70
Resistors in Parallel	69-70
Resistors in Series	69-70

Topic	Page
Right Hand Rule	78-80
Roller Coaster Loop	31
Rolling	45
Rotational Kinematics	41-42
Sine	6
Slope of graph	7
Snell's Law	107-108
Sound in tube	116-117
Sound, intensity of	93
Sound, speed of	92
specific gravity	48
Spring & Mass Oscillator	83
Spring Force	82
Standing Waves	115
Static Friction	18-19,21
Static Friction	32
Strain	86
Stress	86
String Wave on sliding ring	115
String Wave Speed	89
String Waves fixed at ends	91, 114-115
String Waves on two tied ropes	90
Tension	16-17,23
Tension	29-30

Topic	Page
Thin Films	120
Time of Flight	12
Torcelli's Result	55
Torque	26-28,43
Total Internal Reflection	109-110
Trigonometry	6
Units	6
Vector Components	6-12
velocity	7
Virtual Image	98-99,102-103
Voltage	66,65
Work	34
Work Energy Thm	35
Young's Modulus	86