Ay102: Oral Mid-term Exam

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a) Closed Book.
b) Constants are supplied (though you should make an attempt to memorize all constants listed at the end of this question paper). You are allowed to use calculators ranging from abacus to HP RPN and that on your lap top.
c) Please come to my office 60 minutes before your exam appointment time and pick up the Chinese cookie. Inside it you will find a piece of paper with 5 lucky numbers. The five questions will span the course and will be a mix of short (factual) and longish[1] *(you need to work through the question).*
d) Following that you have 60 minutes to cogitate. I suggest that you write down your answers. Then you have to explain your answer on the board (in my office).
e) The duration of the oral exam is 50 minutes. We need to go through all the five questions and so I will keep it moving along.
f) The Spirit: I am of the opinion that the oral exam has great pedagogical value and represents yet another opportunity to learn. It is best to over answer the question rather than stick to the apparent legal scope of the question. Thus when I say "please write down ..." then what I am really asking you is to explain your answer.

Suggestions for preparing for the Exam. The best way to prepare for the exam is to review the Lectures, read the notes and associated chapters (i.e. Draine's ISM book). I encourage you to discuss amongst yourselves, set up practice sessions and go through each and every question. *This preparation alone will be a learning experience (you will get different perspective from your friends).*

Constants: *You are expected to know the following physical constants by heart: c, G, h, k_B, m_H (or inversely Avogadro's number), m_p/m_e.* For the mid-terms I have provided the essential constants, in addition to All other constants and parameters that you may possibly need are given in the Appendix.

[1] set in italics

You should be able to convert eV to temperature, wavelength, frequency and ergs. Thus $1\,\mathrm{eV}{=}1.6 \times 10^{-12}\,\mathrm{erg}$, $1\,\mathrm{eV}{\rightarrow}\ 12345\,\text{Å}$ etc. and 1 eV (in ergs or Joules). It would be helpful to your own career that you have developed your own list of formulae that allow you to make estimates rapidly, e.g. the rms velocity of an H-atom is $\sqrt{T/121}\,\mathrm{km\,s^{-1}}$ and the gyro frequency is $3\,\mathrm{Hz}/\mu\mathrm{G}$, $10\,\mathrm{keV}/10^{12}\,\mathrm{G}$. In the same spirit I note: approximately $3 \times 10^7\,\mathrm{s}$ in a year, 84600 s in a day, 1 parsec=$3.1 \times 10^{18}\,\mathrm{cm}$, a radian is approximately 2×10^5 arc seconds and there are 40,000 square degrees in the sky.

Should you find yourself in the situation where you are not able to recall the value of a constant and the constant is not included at the end of this list then please make an educated guess (that is a part of research) and inform me of the adopted value during the exam.

Formulae. I have provided useful formulae (and more). Review the Appendix before panicking.

A: Introducing the Galactic ISM

1. Provide a reasonable accounting of the cosmic pie-chart (atoms, photons, dark matter, dark energy) at the birth of the Universe and today.

2. What are the sources and sinks (infall, star-formation, stellar death) for the interstellar medium in our Galaxy at the current epoch?

3. Provide a rough estimate in $\mathrm{eV/cm^3}$ for stellar light, magnetic field energy density, cosmic rays and CMB radiation at the solar circle.

4. List elements with abundance relative to hydrogen, by number, greater than 100 ppm (parts per million).

B: The Bohr Atom & Schrodinger Solution

1. Consider D I (atomic deuterium). *Derive* the formula to compute to high precision the wavelength of the Balmer series. Compute that for Balmerα and compare that to Hα.

2. Derive and compute the wavelength of the Lyman alpha line of Fe XXVI in the context of the Bohr model. If you want a higher precision what corrections would you be making? Please comment on the expected splitting of the $n = 2$ state.

3. List the 10 most abundant elements. Write down their electronic configurations (e.g. Helium is $1s^2$). Which of them have first ionization potential higher than that of hydrogen.[2] In regard to this discussion what is so special about oxygen?

4. *Use the trick I mentioned in the class to determine the spectroscopic terms for the ground electronic configuration of O I. You will find three spectroscopic terms. Use Hund's rule to order them in energy.*

C: Multi-electron Atoms

1. Using the selection rules given in §6.7 of Draine's book to classify the list of lines shown in Figure 1 as (1) *Permitted*, (2) *Inter-combination*, or *Forbidden* and give your reason.

2. *Explain the origin of the 1.4 GHz hyperfine line of H I. Compute the integrated absorption coefficient for density of 1 atom cm*$^{-3}$ *and at temperature,* T. *The* A_{10} *coefficient for this line is* $2.88 \times 10^{-15}\,\mathrm{s^{-1}}$.

3. From which abundant atoms or ions do you expect to see hyperfine lines?

D: Warm Ionized Medium

1. Describe various observational probes of the Warm Ionized Medium.

[2] group elements as follows: He, C, O, N (1:10^4) and group 2 (1:10^5).

(a) C III : $1s^2 2s 2p\ ^3\mathrm{P}^o_1 \rightarrow 1s^2 2s^2\ ^1\mathrm{S}_0$ 1908.7 Å

(b) O III : $1s^2 2s^2 2p^2\ ^1\mathrm{D}_2 \rightarrow 1s^2 2s^2 2p^2\ ^3\mathrm{P}_2$ 5008.2 Å

(c) O III : $1s^2 2s^2 2p^2\ ^1\mathrm{S}_0 \rightarrow 1s^2 2s^2 2p^2\ ^1\mathrm{D}_2$ 4364.4 Å

(d) O III : $1s^2 2s 2p^3\ ^5\mathrm{S}^o_2 \rightarrow 1s^2 2s^2 2p^2\ ^3\mathrm{P}_1$ 1660.8 Å

(e) O III : $1s^2 2s^2 2p^2\ ^3\mathrm{P}_1 \rightarrow 1s^2 2s^2 2p^2\ ^3\mathrm{P}_0$ 88.36 μm

(f) C IV : $1s^2 2p\ ^2\mathrm{P}^o_{3/2} \rightarrow 1s^2 2s\ ^2\mathrm{S}_{1/2}$ 1550.8 Å

(g) Ne II : $1s^2 2s^2 2p^5\ ^2\mathrm{P}^o_{1/2} \rightarrow 1s^2 2s^2 2p^5\ ^2\mathrm{P}^o_{3/2}$ 12.814 μm

(h) O I : $1s^2 2s^2 2p^3 3s\ ^3\mathrm{S}^o_1 \rightarrow 1s^2 2s^2 2p^4\ ^3\mathrm{P}_2$ 1302.2 Å

Figure 1: List of Lines.

2. *The sweep of the brightest FRB to date (Bochenek's STARE2 burst) in the frequency-time plane is displayed Figure 2. Assuming that the sweep is due dispersion in cold plasma analytically derive the time difference between two frequencies. Apply your result to the Bochenek burst and derive the dispersion measure.*

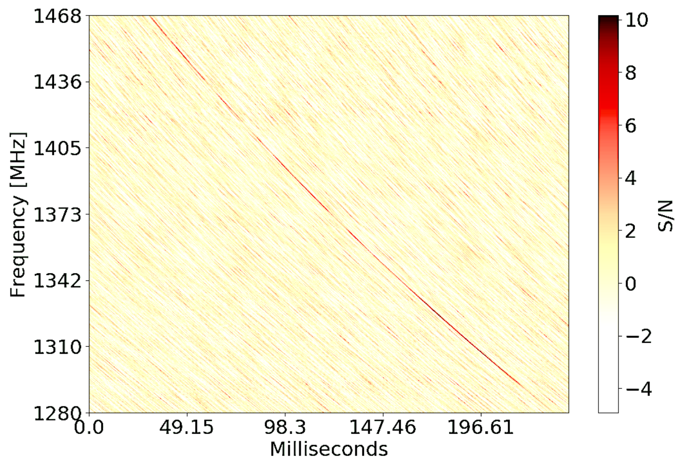

Figure 2: Sweep of Bochenek's STARE2 burst in the time-frequency plane.

3. Consider a WIM sheet with the following physical parameters: temperature of 7500 K, hydrogen density, $n_H = 0.5\,\mathrm{cm}^{-3}$, essentially fully ionized. The sheet is perpendicular to the line of sight and has a thickness of 4 pc. Compute the surface brightness[3] for

[3] The usual unit for surface brightness is Rayleigh which is surface brightness but in units of photons $\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ steradian^{-1}. However, in this sub-field the unit Rayleigh which is $4\pi \times 10^{-6} I$ phot $\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ is frequently quoted.

the resulting Hα and Hβ emission (units: $\mathrm{erg\,cm^{-2}\,s^{-1}steradian^{-1}}$).[4]

E: CNM and WNM

1. State (and be prepared to explain and as many as possible) methods by which the CNM and/or WNM can be studied.

2. Rydberg states refer to levels of an atom with energy quantum number greater than say 100. The transition of $n+1 \to n$ is called as "$n\alpha$". The HI 166α is a well known tracer. Compute the frequency for this transition. Next, the lowest frequency radio recombination line observed to date[5] is CI 843α. Can you comment on the density of the region in which this line arises? [Bonus: What other physical effects may be important?]

3. *The 157μm line is the fine structure line arising in the ground spectroscopic term of the CII ion. Write down the spectroscopic designations for the two levels. We will restrict to excitation by collisions with H (or He). For convenience we simplify and use the "hard sphere" approximation (!) and derive the de-excitation rate, k_{10}. Then derive the expression for n_1/n_0 where n_1 is the number density of C+ atoms in the excited state and n_0 in the lower state. Apply your formula for $T = 100\,K$ with $n_H = 100\,\mathrm{cm^{-3}}$ and $n_H = 10^3\,\mathrm{cm^{-3}}$.*
   You will find that the critical density is higher than that given in Draine's book. Explain why this is the case. Data: The A-coefficient for the line is $2.4 \times 10^{-6}\,\mathrm{s^{-1}}$. [See Appendix for helpful formulae].

4. The rotation curve gives v, radial velocity of a parcel of gas, with respect to the Sun (after we correct for Sun's peculiar velocity with respect to local gas), in a circular orbit around the center of the Galaxy. Show that

$$v(R,l) = R_0 \sin(l)[\Omega(R) - \Omega(R_0)] \tag{1}$$

   where $R_0 = 8.5\,\mathrm{kpc}$ is the galactocentric radius of Sun's orbit, R is the galactocentric radius and l is the Galactic longitude. Assuming a "flat" rotation compute the radial velocity for $R = 2, 5, 10, 15\,\mathrm{kpc}$.

F: HII regions

1. *What are the primary features of the spectrum of an HII region (UV, optical, IR, radio). Explain which processes "cool" HII regions.* [This is an open ended question and so be prepared to go into some details with some specificity. Waxing eloquent should be avoided].

[4] Necessary recombination coefficients are given in the Appendix. I have deliberately not given the recombination coefficient for Hβ.

[5] `https://arxiv.org/pdf/1701.08802.pdf`

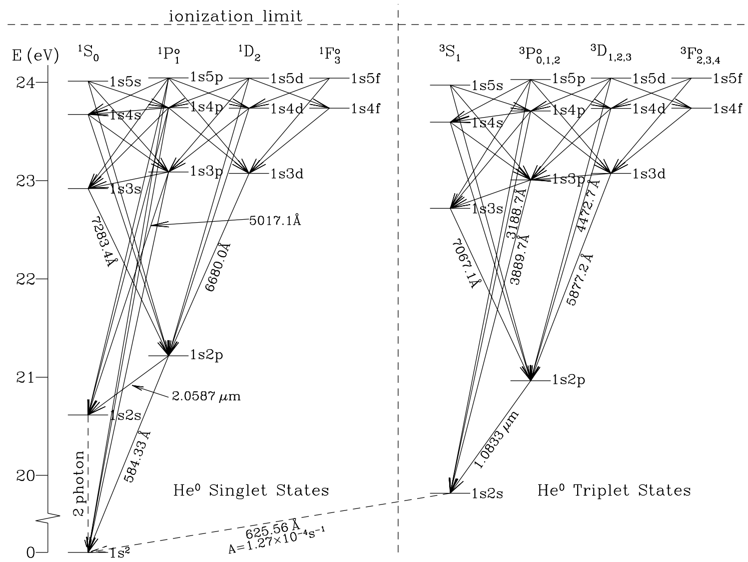

Figure 3: Grotrian diagram for Helium.

2. The peak emission measure of Messier 42 (aka The Orion Nebula) is $\mathrm{EM} = 5 \times 10^6\,\mathrm{cm}^{-6}\,\mathrm{pc}$. If M42 is approximated as a uniform density sphere of diameter 0.5 pc calculate the rate of H recombinations. At what (radio) frequency will M42 become optically thick?

3. *The Grotrian diagram for Helium is shown in Figure 3. Describe transitions into and out of $1s2s^3S_1$ state. Why is the $1.0833\,mum$ line so bright even at low densities and why does the line become brighter with increasing density and temperature.*

G: Global Model for Atomic ISM

1. List characteristic density, temperature and filling factor for the Galactic ISM (at the solar circle) for the four atomic phases: HIM, WNM, WIM and CNM.

2. *Review the cooling curve shown in Figure 4. Be prepared to walk me through the primary features of the curve (which cooling process/species dominate at various temperatures). Compute the cooling timescale (in years, normalized to particle density of 1 cm^{-3}) at $T = 10^5, 10^6, 10^7$ K.*

3. In Figure 5 we display the equilibrium curve for a two-phase model. The typical pressure of the ISM (solar circle) is $3000\,\mathrm{cm}^{-3}\,\mathrm{K}$. Determine the temperature of the two stable phases. Explain why intermediate phase is unstable.

H: Molecular Gas:

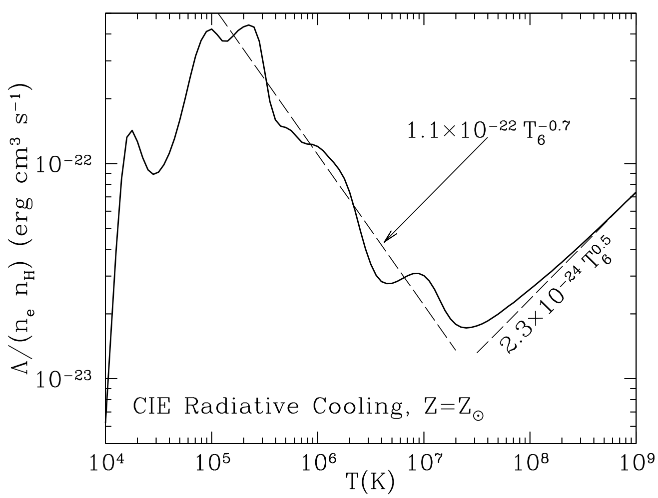

Figure 4: Cooling curve of interstellar plasma (from §34.1 of Draine's book).

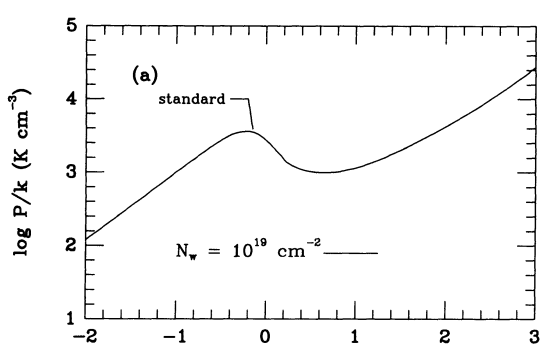

Figure 5: Two-phase equilibrium (cooling=heating) curve (from Wolfire et al. 1995). The x-axis is the log of the number density in cm^{-3}, $\log(n)$.

1. Write down the classical expression for the rotational energy of a diatomic molecule in terms of moment of inertia and angular momentum, $\mathbf{J}$. Apply the rules of QM: $\mathbf{J} \cdot \mathbf{J} = J(J+1)\hbar^2$. Assume diatomic separation of 1 Å and compute the observed first rotational transition for CO and for H_2.

2. The potential curves for the ground and first two bounded electronic states of molecular hydrogen are displayed in Figure 6. What is the dissociation energy of H_2 in eV? In the ISM how is hydrogen destroyed? [Be prepared to state your answer in nanometer or wavelength].

3. What is the estimated mass of molecular gas in our Galaxy? State typical masses and sizes of giant molecular clouds .

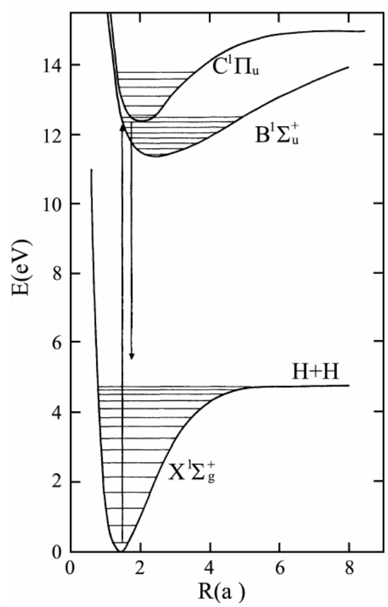

Figure 6:

Appendix: Constants, Formulae & Parameters

Essential constants that a serious student should memorize (to appropriately useful precision

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$c = 2.998 \times 10^{10}\,\mathrm{cm\,s^{-1}}$
$h = 6.63 \times 10^{-27}$ CGS
$G = 6.67 \times 10^{-8}$ GCS
$e = 4.8 \times 10^{-10}$ statcoulombs (CGS). In MKS, $e = 1.6 \times 10^{-19}$
1eV=1.6×10^{-12} erg, $k_B T = 0.86 T_4$ eV, $hc/\lambda = 1.24(\lambda/1\,\mu\mathrm{m})^{-1}$
Ry $= R_\infty = 2\pi^2 e^4 m_e/h^3 \approx 13.6\,\mathrm{erg} \to 109,737.316\ \mathrm{cm}^{-1}$ (Rydberg)
$\sigma_T = (8\pi/3) r_e^2 = 0.66\,\mathrm{barn}$ (1 barn=$10^{-24}\ \mathrm{cm}^{-2}$; r_e is the electron radius)
$a_0 = 0.53\,$Å (Bohr radius, 1Å=10^{-8} cm)

radian, 20,626 arcseconds (2.1×10^5)
parsec, 3×10^{18} cm
Mass of Sun, $M_\odot = 2 \times 10^{33}$ gram
Radius of Sun, $R_\odot = 7 \times 10^{10}$ cm (or 2 light seconds)
Luminosity of Sun, $L_\odot = 4 \times 10^{33}\,\mathrm{erg\,s^{-1}}$
Astronomical unit is 500 light seconds
Chandrashekar Mass, $1.4\,M_\odot$

The Gaussian 1-D velocity distribution is

$$\phi(v) = \frac{1}{\sigma_u \sqrt{2\pi}} \exp\Big(-\frac{u^2}{2\sigma^2}\Big) \tag{2}$$

The mean speed for a Maxwellian velocity distribution is $\sqrt{8k_BT/\pi\mu}$.
You may need this family of integrals

$$D = \int_0^\infty \frac{x^n dx}{\exp(x) - 1} dx. \tag{3}$$

For $n = 3, 4, 5$ we have $D = 6.5, 24.9, 122$.

Einstein A and B coefficients, Oscillator Strength

$$B_{lu} = \frac{g_u}{g_l} B_{ul} \tag{4}$$

$$B_{ul} = \frac{c^3}{8\pi\nu^3} A_{ul} \tag{5}$$

$$\sigma_{lu}(\nu) = \frac{g_u}{g_l} \frac{c^2}{8\pi\nu_{lu}^2} A_{ul}\phi(\nu) \tag{6}$$

$$= \frac{e^2}{m_e c} f_{lu}\phi(\nu) \tag{7}$$

Here, $\sigma_{lu}(\nu)$ is the cross-section for photon of frequency ν to $\nu + d\nu$ to be absorbed by an atom with the resulting $l \rightarrow u$ transition, $\phi(\nu)$ is the normalized as $\int \phi(\nu)d\nu = 1$ and f_{lu} is the oscillator strength.

For a column with Gaussian velocity distribution, σ_v, the optical depth at the line center is

$$\tau_0 = \sqrt{\pi}\frac{e^2}{m_e c}\frac{N_l f_{lu}\lambda_{lu}}{b}$$

$$= 1.497 \times 10^{-2}\frac{\mathrm{cm}^2}{\mathrm{s}}\frac{N_l f_{lu}\lambda_{lu}}{b} \tag{8}$$

where N_l is the column density and $b = \sqrt{2}\sigma_v$.

Photoelectric absorption

$$\sigma_{pi} \approx \sigma_0\left(\frac{h\nu}{Z^2 I_H}\right)^{-3} \tag{9}$$

where $I_H = 13.6\,\mathrm{eV}$ is the ionization potential of Hydrogen and $\sigma_0 = 6.3 \times 10^{-18} Z^{-2}\,\mathrm{cm}^2$.

Recombination Coefficients: For temperature $[5, 10, 20] \times 10^3\,\mathrm{K}$:
$\alpha_A = [6.82, 4.18, 2.51] \times 10^{-13}\,\mathrm{cm}^3\,\mathrm{s}^{-1}$ Case A (all levels)
$\alpha_B = [4.54, 2.59, 1.43] \times 10^{-13}\,\mathrm{cm}^3\,\mathrm{s}^{-1}$ Case B (all levels)
$\alpha_{H\alpha} = [2.2, 1.17, 0.6] \times 10^{-13}\,\mathrm{cm}^3\,\mathrm{s}^{-1}$ Case B Recombination Hα Coefficient

Free-free absorption:

$$\frac{\kappa_{\mathrm{ff}}}{n_i n_e} = 1.1 \times 10^{-25} Z_i^{1.882} T_4^{-1.323} \nu_9^{-2.118}\,\mathrm{cm}^5 \tag{10}$$

Plasma Frequency:

$$\omega_p^2 = \frac{4\pi n_e e^2}{m_e} \tag{11}$$

Table 1: A-coefficients for $n = 2, 3$ lines of H

lower (l)	E_l cm^{-1}	upper, u	E_u cm^{-1}	A_{ul} (s^{-1})	trans
1s $^2S_{1/2}$	0	2p $^2P_{1/2}$	82,258.919	6.26×10^8	
1s $^2S_{1/2}$		2s $^2S_{1/2}$	8.954	2.5×10^{-6}	M1
1s $^2S_{1/2}$		2p $^2P_{3/2}$	9.285	6.26×10^8	
1s $^2S_{1/2}$		3p $^2P_{1/2}$	97,492.211	1.67×10^8	
1s $^2S_{1/2}$		3s $^2S_{1/2}$	2.221	1.11×10^{-6}	M1
1s $^2S_{1/2}$		3p $^2D_{3/2}$	2.319	5.94×10^2	E2
1s $^2S_{1/2}$		3p $^2P_{3/2}$	2.320	1.67×10^8	
1s $^2S_{1/2}$		3d $^2D_{5/2}$	2.356	5.94×10^2	E2
2p $^2P_{1/2}$	82,258.919	3d $^2D_{3/2}$	2.319	5.39×10^7	
2s $^2S_{1/2}$	8.954	3p $^2P_{3/2}$	2.320	2.25×10^7	
2p $^2P_{1/2}$	8.919	3s $^2S_{1/2}$	2.221	2.10×10^6	
2s $^2S_{1/2}$	8.954	3p $^2P_{1/2}$	2.211	2.25×10^7	
2p $^2P_{3/2}$	9.285	3d $^2D_{5/2}$	2.356	6.46×10^7	
2p $^2P_{3/2}$	9.285	3d $^2D_{3/2}$	2.319	1.08×10^7	
2p $^2P_{3/2}$	9.285	3s $^2S_{1/2}$	2.221	4.21×10^6	