Dynamics of HII regions:

Let us turn on the central star

$Q_0 =$ photon ionizing rate

- Assume case B (optically thick)
- almost unity ionization

$$Q_0 = n_H \, 4\pi R(t)^2 \frac{dR}{dt}$$

Thus the ionization "front" increases as

$$R(t) = \left(\frac{3Q_0}{4\pi n_H} t\right)^{1/3}$$

However, as the volume of ionized sphere increases and recombination becomes important

$$Q_0 = n_H 4\pi R(t)^2 \frac{dR}{dt} + \alpha_B n_H^2 \frac{4\pi}{3} R^3$$

The solution to this equation is

$$R(t) = R_s \left(1 - e^{-t/t_{rec}}\right)^{1/3}$$

$$R_s = \left(\frac{3Q_0}{4\pi\alpha_B n_H^2}\right)^{1/3}$$

ex. $n_H = 40 \, cm^{-3}$, $Q_0 = 10^{49} \, s^{-1}$, $T = 8{,}000 \, K$

$R_s = 5.5 \, pc$

$$t_{rec} = \frac{1}{\alpha_B n_H} = 2600\,\mathrm{yr}\left(\frac{n_H}{40\,\mathrm{cm}^{-3}}\right)^{-1}$$

Thus the ionization front reaches Stromgren radius on time scale of t_{rec}.

The speed of the front:

$$\frac{dR}{dt} = v_s \frac{e^{-t/t_{rec}}}{\left(1-e^{-t/t_{rec}}\right)^{2/3}}$$

where

$$v_s = \frac{R_s}{3t_{rec}}$$

$$= 680\,\mathrm{km\,s^{-1}}\left(\frac{Q_0}{10^{49}\,\mathrm{s}^{-1}}\right)^{1/3}\left(\frac{n_H}{40\,\mathrm{cm}^{-3}}\right)^{1/3}$$

For $t \ll t_{rec}$

$$\frac{dR}{dt} \simeq v_s\left(\frac{t}{t_{rec}}\right)^{-2/3} \qquad t \ll t_{rec}$$

For $t \gg t_{rec}$

$$\frac{dR}{dt} \simeq v_s e^{-t/t_{rec}}$$

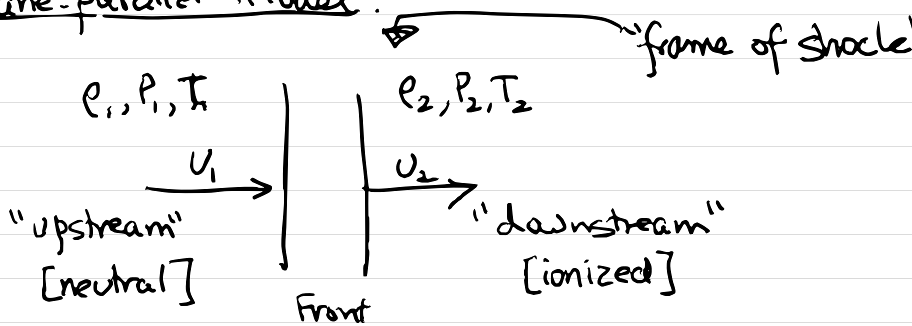
Compare the velocity of the ionization front to the sound speed

$$c_s \simeq \left(\frac{2kT}{m_H}\right)^{1/2} = 11.5\,\mathrm{km\,s^{-1}}\left(\frac{T}{8000}\right)^{1/2}$$

The timescale for nebula to double in its size:

$$t \simeq \frac{R_S}{c_S} \approx 0.5\,\mathrm{Myr}$$

<u>Plane-parallel Model</u>:



$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho \vec{u}) = 0$$

$$\rho \frac{\partial \vec{u}}{\partial t} + \rho(\vec{u}\cdot\vec{\nabla})\vec{u} = -\vec{\nabla} P$$

Evolution proceeds on long timescales

$$\frac{\partial}{\partial t} \to 0$$

(1) $\frac{d}{dx}(\rho u) = 0$

(2) $\rho u \frac{du}{dx} + \frac{dP}{dx} = 0$

$$\frac{d}{dx}(\rho u^2) = u \frac{d}{dx}(\rho u) + \rho u \frac{du}{dx}$$

$$\frac{d}{dx}(\rho v^2) - v\frac{d}{dx}(\rho v) + \frac{dP}{dx} = 0$$

$$\frac{d}{dx}(\rho v^2 + P) = 0 \qquad (3)$$

$\therefore$ $\rho v^2 + P$ is constant across front

(1) and (3) lead to

(A) $\rho_1 v_1 = \rho_2 v_2$

(B) $\rho_1(v_1^2 + c_1^2) = \rho_2(v_2^2 + c_2^2)$

$$c_1 = \left(\frac{kT_1}{m_H}\right)^{1/2} \quad \text{neutral}$$

$$\simeq 0.81\,\text{km/s}\left(\frac{T_1}{80\,K}\right)^{1/2}$$

$$c_2 = \left(\frac{2kT_2}{m_H}\right)^{1/2} \quad \text{ionize}$$

$$= 11\,\text{km/s}\left(\frac{T_2}{8{,}000\,K}\right)^{1/2}$$

So $c_2 \gg c_1$

Combine (A) and (B) to yield

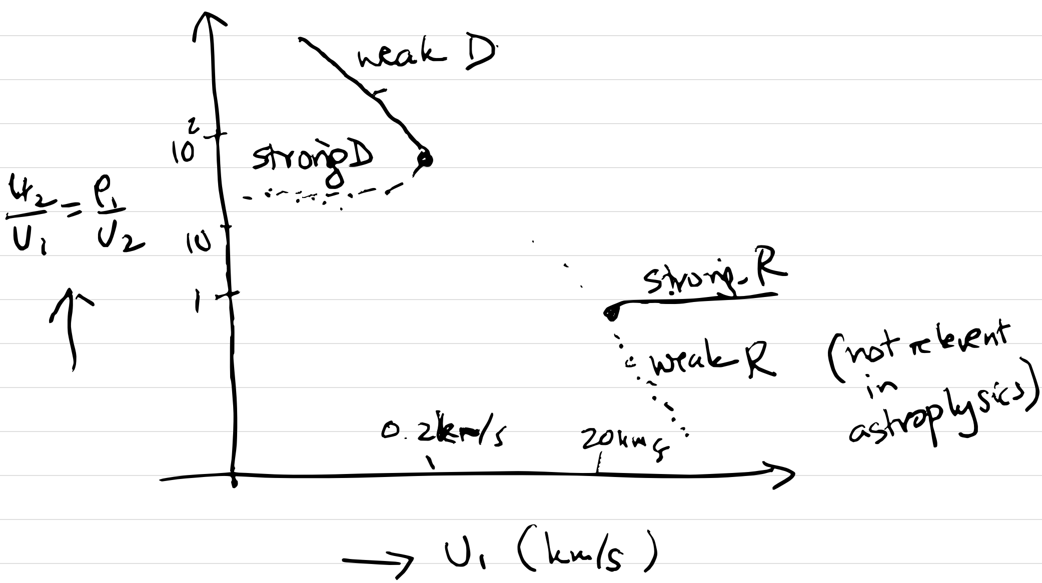
$$\frac{\rho_2}{\rho_1} = \frac{1}{2c_2^2}\left[c_1^2 + u_1^2 \pm \sqrt{f(u_1)}\right]$$

$$f(u_1) = (u_1^2 - v_R^2)(u_1^2 - v_D^2)$$

$$v_R \equiv c_2 + \sqrt{c_2^2 - c_1^2} \simeq 2c_2$$

$$v_D \equiv c_2 - \sqrt{c_2^2 - c_1^2} \simeq \frac{1}{2}\left(\frac{c_1}{c_2}\right)^2 c_2 \ll c_2$$

Figure 37.1 from Draine

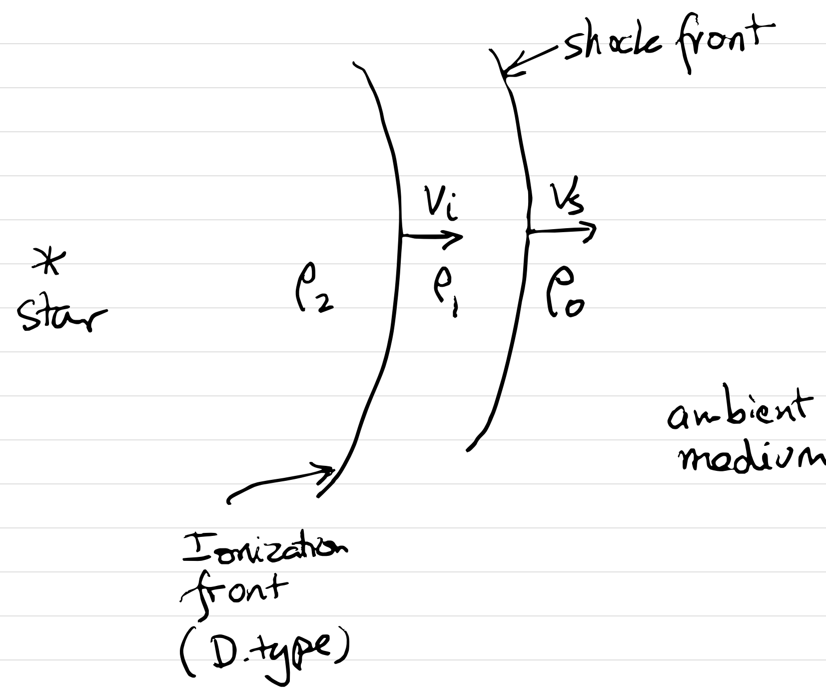


The ionization front starts as "weak R-type" front

$$\frac{\rho_2}{\rho} = 1 + \frac{c_2^2}{u_1^2}$$

Transition from R to D type fronts

Once U_1 falls below $2c_2$ there is no stable solution (see figure). A shock front develops



The final state is one of pressure equilibrium

$$2 n_{final} kT_2 = n_1 kT_1$$

↑
ionization

$$R_{final} = \left(\frac{3 Q_0}{4\pi \alpha_B n_{final}^2} \right)^{1/3}$$

$$\therefore \quad R_{final} = \left(\frac{2T_2}{T_1} \right)^{2/3} R_{SO}$$

for our values $R_{final} \simeq 200\,pc.$

However, OB stars have short life-time.