Hyperfine structure:

Fine-structure is due to interaction of the magnetic field due to orbit of proton with the magnetic mometum of the electron. on

We now consider the interaction of the magnetic field of the proton (which is magnetized) with the magnetic moment of the electron

The proton itself is magnetized

$$\vec{\mu}_p = g_p \frac{e\hbar}{2m_p} \frac{\vec{I}}{\hbar} \qquad I: \text{spin of prot}$$

$$g_p = 2.675\ldots \quad \mu_N = e\hbar/2m_p \ (\text{nuclear magneton})$$

Incidentally $\vec{\mu}_n \equiv g_n \frac{e\hbar}{2m_p} \frac{I}{\hbar}, \quad g_n = -1.832\ldots$

This magnetic moment creates a B-field

$$\vec{B} = \frac{3\vec{r}(\vec{r}\cdot\vec{\mu}) - r^2\vec{\mu}}{r^5} + \frac{8\pi}{3}\mu\,\delta^3(\vec{r})$$

The resulting "hyper" fine structure is $\frac{m_e}{m_p}$ smaller than fine structure.

We focus on $l = 0$.

$$H_{fs} = \frac{8\pi}{3} \frac{g_p e}{2m_p} \vec{I} \cdot \frac{g_e e}{2m_e} \vec{S}$$

$$\langle H_{fs} \rangle = H_{fs} |\psi_{1s_{\frac{1}{2}}}(0)|^2$$

As before, let $\vec{F} = \vec{I} + \vec{S}$

$$\frac{I \cdot S}{\hbar^2} = \frac{1}{2}\left(\bar{F} \cdot \bar{F} - \bar{S} \cdot \bar{S} - \bar{I} \cdot \bar{I} \right)$$

$$= \frac{1}{2}\Big(F(F+1) - S(S+1) - I(I+1) \Big)$$

$F = 0$, $F = 1$

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1 Rydberg is $\frac{1}{2} m_e c^2 (Z\alpha)^2$

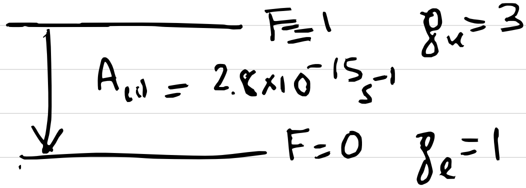
Bohr levels: $\frac{Ry}{n^2}$

fine structure: $Ry \left(\frac{Z\alpha}{n}\right)^2$

hyperfinestructure: $Ry \frac{(Z\alpha)^2}{n} \times \frac{m_e}{m_N}$ nucleus

21-cm line: $\nu = 1420.4$ MHz

$$\frac{h\nu}{k_B} = 68\,\mathrm{mK}.$$



$$\frac{n_u/g_u}{n_l/g_l} = \exp\left(-\frac{h\nu_{ul}}{kT}\right)$$

for any reasonable T $\quad \exp\left(-\frac{h\nu}{kT}\right) = 1 - \frac{h\nu}{kT} \simeq 1$

$$n_u \simeq \frac{3}{4} n_{HI}, \quad n_l \simeq \frac{1}{4} n_{HI}$$

The emissivity is

$$j_\nu = \frac{n_u A_{ul}}{4\pi} h\nu_{ul}\,\phi(\nu) = \frac{3}{4}\frac{n_{HI} A_{ul} h\nu_{ul}\phi_\nu}{4\pi}$$

The absorption coefficient is

$$\begin{aligned} K_\nu &= n_l \sigma_{lu} - n_u \sigma_{ul} \\ &= n_l \frac{g_u}{g_l}\frac{A_{ul}}{8\pi}\lambda_{ul}^2 \phi_\nu \left[1 - \frac{n_u/g_u}{n_l/g_l}\right] \\ &\simeq \frac{3}{32\pi} A_{ul} \frac{hc\lambda_{ul}}{kT} n_{HI}\,\phi_\nu \end{aligned}$$

T → "spin" temperature (Salpeter)

Thus $\kappa_\nu \propto \frac{1}{T_{spin}}$

$$\tau_\nu = 2.19 \left(\frac{N_{HI}}{10^{21}\,cm^{-2}}\right)\left(\frac{100K}{T_{spin}}\right)\left(\frac{km\,s^{-1}}{\sigma_v}\right) e^{-\frac{u^2}{2\sigma_v^2}}$$