Ay102: Homework 4

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[1a] Hyperfine Lines. List fine structure lines of the four most abundant elements (including isotopes, if it makes sense) and their approximate frequencies and A coefficients. [5 points]

[**1b**] Research the literature and write down the most accurate value for the 21-cm line. Who else, other than astronomers, are interested in this transition? [5 points]

[**2**]**Metal Free H regions**]. JWST is pushing on the youngest galaxies. The first generation stars will be born from pristine material (H, He). The resulting H II regions thus will not be cooling via the forbidden lines of "metals". For this homework we will ignore helium and so $n_e = n_p$ and heating is from photo-electrons ejected from hydrogen and cooling is due to (1) radiative recombination (rr), (2) free-free (ff) emission and (3) collisional losses include electron line excitation of H^0 and electron impact ionization of H^0. From discussion in the class we can assume that ionization balance is established rapidly and that pressure equilibrium takes a much longer time.

In the class (and also see Chapter 27.1 of the textbook) we showed that, in ionization equilibrium, the heating rate per unit volume is

$$\mathcal{H} \approx n_e n_p \alpha_B k_B T_c \tag{1}$$

where T_c is the "color" temperature obtained by fitting the Wein formula to the tail of the observed spectrum. The cooling power loss per unit volume is

$$\mathcal{C} = n_e n_p \alpha_B [f_{\rm rr} k_B T + f_{\rm ff} k_B T] + n_e n_{\rm HI} \Lambda_{\rm HI} \tag{2}$$

where T is the temperature of the H II region. For the losses via radiative recombination and free-free see Ch 27.3 (but the Kulkarni & Shull paper has a better treatment). The electron collisional losses is described in detail in Kulkarni & Shull.

The ionization fraction is determined by the recombination time and the photo-ionization timescale and the latter is proportional to the intensity of the star. For now, set $x_e = n_e/n_{\rm H}$

to be 0.01; here n_H is the density of H nuclei (ionized or neutral).

Set $T_c = 35,000$ K. Plot the heating rate and the cooling rate as a function of T and determine the temperature at which the two rates match. Redo assuming $x_e = 0.03$. [20 points]

[**3**] **Collisional Ionization Equilibrium.** Compute the equilibrium ionization fraction of hydrogen plasma as a function of temperature. Assume that the plasma is optically thin to Lyman continuum. [10 points]

[**4**] l-v **diagrams.** The purpose of this purely pedagogical exercise is to help you get comfortable with l-v diagrams. We assume that gas clouds are on circular orbits. [Why is this a reasonable assumption?].

Consider a line-of-sight (los) starting from the Earth and along Galactic longitude l and latitude $b = 0$. The radial velocity of a cloud, under this assumption, is given by

$$v_r = R_0 \Big[\Omega(R) - \Omega_0\Big] \sin(l). \tag{3}$$

Here R is the galacto-centric radius of the cloud, R_0 is the radius of the solar circle (the distance from the Sun to the center of Galaxy) and Ω_0 is the local angular speed. $R_0 = 8.5$ kpc and $V_0 = R_0\Omega_0 = 220\,\mathrm{km\,s^{-1}}$. Assume that the rotation curve is flat, that is, $V(R) = V_0$.

1. Derive the result stated in Equation 3. [5 pts]
2. For $l = 45°$ plot the run of v_r (in $\mathrm{km\,s^{-1}}$) as a function of distance from us, d (kpc), all the way to edge of the H I disk (say 20 kpc). [5 pts]
3. Equation 3 offers a ready way to estimate distances to H II regions or giant molecular clouds (in the absence of other distance measures, which is almost always the case). However, there are deviations in the velocity field of the Galaxy due to a triaxial bulge or spiral density waves.

   As before we set $l = 45°$. We choose the following points: (i) the tangent point,[1] (ii) a point on the solar radius (where the los intersects the solar circle; $d \approx 12$ kpc) and say at (iii) $d = 20$ kpc. Perturb the velocity fields (to keep it simple, just the radial part) by $10\,\mathrm{km\,s^{-1}}$ and $30\,\mathrm{km\,s^{-1}}$ and derive the corresponding uncertainty in the inferred distances. [5 pts]
4. Inspect l-v diagrams in HI and CO that were presented in the class (April 27) . You will see deviations from that predicted by the formula. List the deviations and

[1] For a given l there is a minimum galactocentric radius that can be reached. The tangent point is the intersection between circle and the line of sight.

offer plausible explanations. [Feel free to discuss this particular problem with other colleagues and postdocs etc]. [10 points]

[**5**] **Inferring Column Density.** Consider two clouds with spin temperature and net column density of T_i and $N_H(i)$ for $i = 1, 2$. Assume no other motions other than that expected from thermal effects. A pulsar is conveniently located behind the two clouds and you are thus able to obtain an optical depth, $\tau(v)$ as well as the emission brightness spectrum, $T(v)$. Consider $N_H(1) = N_H(2)$ (with values that is indicative of typical conditions) and T_1 to be CNM and T_2 to be WNM. What are the pitfalls you will be making in deducing the total column density using the emission spectrum, $T(v)$ and $\tau(v)$? [10 points]