Post-Lecture: Bohr-Schrodinger Model for Hydrogen

Background & Motivation. *The background for this home work is an understanding of the Bohr and Schrodinger model for hydrogen. The primary motivation is to expose the student to observations related to hydrogen or hydrogen-like atoms. Next, the student, through this homework is introduced to on-going facilities and new missions.*

All problems carry 10 points. The homework is due COB, Jan 17.

[1] Air wavelengths. The refractive index of air is slightly larger than one due to diatomic molecules and particularly water vapor. As a result wavelengths measured in air will be smaller than those reported in vacuum (or computed theoretically). The definitive paper on this topic is by P. E. Ciddor[1]. On-line tools can be found here[2]. Compute the *precise* air wavelength of Hα and Hβ at (i) sea-level and (ii) atop Mauna Kea.[3]

[2] Schrodinger orbitals. Even in the Schrodinger model for hydrogen the energy level values depend only on n. Consider a 2s electron. Lacking angular momentum it has a penetrating orbit (with finite probability at $r = 0$). In contrast, a 2p electron has no presence at $r = 0$. So, at first blush, you would expect that in the Schrodinger model the 2s, being deeper in the potential well (for some fraction of the orbit) will have a larger binding energy relative to the 2p electron. In order to resolve this puzzle, plot the radial probability distribution of a 2s and 2p electron, $p(r)dr$ and resolve this puzzle in a qualitative manner.

[2] **Positronium.** In 1932 Carl D. Anderson discovered positron, the first anti-particle.[4] There are many celestial sources of positrons: beta-decay of unstable nuclei (stellar flares, novae and supernovae), pulsars, interaction of very high energy photons with low energy photons and collisions of cosmic rays with matter. Energetic positrons are slowed down by Coulomb scattering and ionization (more on this, later in the course). They can "recombine" with interstellar electrons forming a positronium, Ps=e^+e^-. Assume that the recombination leads to the positronium in the $n = 2p$ state. The system will then radiate the equivalent of "Lyα" and reach the $n = 1s$ level.[5] Compute the energy of the Lyα photon. Is this observable from the Palomar Observatory?

[3] **Muonic Hydrogen.** A muon, μ^- is a lepton with a charge of $-e$, spin of $1/2$ and mass of 105.6 MeV. It decays in 2.2 μs. In some ways, you can regard a muon as a massive electron. Muons were discovered at Caltech by Seth Neddermeyer & Anderson in 1936. The anti-muon, μ^+, is sometimes called as "positive muon". In muonic atoms, an electron

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[1] Applied Optics, 35, 1566–1573, 1996

[2] `https://refractiveindex.info/` or `https://emtoolbox.nist.gov/Wavelength/Ciddor.asp`

[3] Note that NIST (`https://physics.nist.gov/PhysRefData/ASD/lines_form.html`) quotes air wavelengths in the range 0.2 μm to 2 μm, and vacuum otherwise.

[4] Scout around Caltech campus for a plaque which marks the discovery site of the first positron.

[5] and then annihilate producing a pair of 511-keV photons, oppositely directed

is replaced by a muon. Let us consider muonic hydrogen, $p^+\mu^-$, formed in the ground state ($n = 1$). Compute, in the Bohr framework, the orbital speed and the orbital size. How many orbits will the muon go around before it decays?

[4] **Transitions in muonic atoms.** A beam of muons produced in an accelerator is directed towards a lead brick. The muons lose energy by Coulomb scattering, bremsstrahlung and ionization losses.[6] Eventually the muon is captured by a lead atom and it trickles down the energy levels. Approximate this state by a $Z = 89$ nucleus and a muon. Apply Bohr model and compute the energy of the photon that is radiated when the muon goes from $n = 2$ state to the ground state. Compare this value to the measured energy of 5 Mev. Account for the large discrepancy.

Incidentally, the best calculations for the energy levels for muonic hydrogen are -2528.494 ± 0.0013 eV ($n = 1$) and -632.1235 ± 0.00032 eV ($n = 2$); see E. Milotti, Atomic Data & Nuclear Data Tables (1998).

[5] **Lyman-α of highly ionized ions.** The National Institute of Standards & Technology hosts "critically evaluated" atomic physics data. Consider "Lyman-α" of ions which are "iso-electronic"[7] with H I. Using this gateway[8] compute the energy of the Lyα doublet ($2\,^2\mathrm{P}_{1/2,3/2} \rightarrow 1\,^2\mathrm{S}_{1/2}$) for H I, C VI, Ca XX and Fe XXVI. Compare with that expected from the Bohr & Dirac models.[9] [If you want to know why this homework is interesting then read up the science objectives and capabilities of the XRISM mission[10] (expected launch date is 2023).[11]]

[5] Rydberg Atoms.[12] Low-frequency radio astronomy is resurgent. We have LOFAR (Netherlands), LWA (New Mexico & California), MWA (Australia) and upgraded GMRT (India). Update yourself on these new facilities.

One of the nice results from LOFAR is the detection of Carbon recombination line, C548α at about 40 MHz [A. Askegar et al. A&A 551, L11 (2013)]. Compute to necessary precision the wavelength of this line from first principles. Compare your answer to the measurements. What is the physical size of the recombined atom? Place a limit to the density of the gas.

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[6] These processes will be covered later in the course. Assume for now these are basically channels by which muon loses it energy.

[7] this means that ions have, like hydrogen, only one electron.

[8] `https://physics.nist.gov/PhysRefData/ASD/levels_form.html`. Note the chemistry notation. H I is neutral hydrogen while H II is hydrogen, once ionized. Note that NIST requires a spacing between element name and ionization stage.

[9] For the Dirac iso-electronic hydrogen solution please look up an advanced QM book of Wikipedia.

[10] `https://heasarc.gsfc.nasa.gov/docs/xrism/`

[11] The mission aims to provide very high spectral resolution studies of hot astrophysical plasma.

[12] The convention for radio recombination lines is that emission of photon from $n + \Delta n \rightarrow n$ from species X is X$n\alpha$ ($\Delta n = 1$), X$n\beta$ ($\Delta n = 2$) and so on. Thus, for instance, C590α is the recombination line due to electron recombining with C$^+$ with $n = 590$ and $\Delta n = 1$.