Collisional Processes: Chapter 2, Appendix I, H

$$A + B \rightarrow \text{collision}$$

reaction rate per unit volume

$$= n_A n_B \langle \sigma v \rangle_{AB}$$

$$\langle \sigma v \rangle_{AB} \equiv \int_0^\infty \sigma_{AB}(v)\, v\, f(v)\, dv \qquad cm^3 s^{-1}$$

v is the relative velocity.

$$\mu = \frac{m_A m_B}{m_A + m_B} \qquad \text{Energy in center-of-mass} = \frac{1}{2}\mu v^2$$

$$f_v := \frac{1}{\sqrt{\pi}} e^{-\frac{1}{2}\frac{\mu v^2}{kT}} \frac{v^2 dv}{v_{rms}^3}$$

Usually σ is known as a function of energy, E:

We have $f(v)\,dv = f(E)\,dE \qquad E = \frac{1}{2}\mu v^2$

$$\int \sigma(v)\, v\, f(v)\, dv \rightarrow \int \sigma(E)\, v\, f(E)\, dE$$

$$\langle \sigma v \rangle_{AB} = \left(\frac{8kT}{\pi \mu}\right)^{1/2} \int \sigma(E) \frac{E}{kT} e^{-E/kT} \frac{dE}{kT}$$

In most instances, for ISM, we will only consider 2-body interactions.

Four types of collisions

- Long-range Coulomb interaction

  $\phi \propto \frac{1}{r}$ (potential)

  ex. ions-ions, ions-electrons, electrons-electrons

- intermediate range Induced-dipole reactions

  $\phi \propto \frac{1}{r^4}$

  ex. ions and neutral.

- electron and neutrals

  use experimental data

- short range (neutral, neutral)

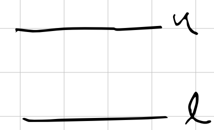
  van der Waals forces

  approximated by "hard spheres"

  $\phi \propto \frac{1}{r^6}$

Detailed balance

$$X(\ell) + Y \longrightarrow X(u) + Y$$



Here $X(u), X(\ell)$ are the two energy states of X

Assume Y does not change (internal) state.

In equilibrium the upward (excitation) rate should match the downward (de-excitation) rate.

$$n_e n_\ell k_{\ell u} = n_e n_u k_{ul}.$$ Use Thermodynamics:

$$\frac{k_{\ell u}}{k_{ul}} = \frac{n_u}{n_\ell} = \frac{\tilde{g}_u}{\tilde{g}_\ell} \exp\left(-\frac{E_{ul}}{kT}\right)$$

It is traditional to list the de-excitation coffeciant (since it does not have the strong exponential dependence).

Inelastic Scattering

$$\langle \sigma v \rangle_{lu} = \frac{g_u}{g_l} \langle \sigma v \rangle_{ul}$$

Let $\sigma_{lu} \equiv \langle \sigma v \rangle_{lu} \Rightarrow \sigma_{lu}(E) = \frac{g_u}{g_l} \sigma_{ul}(E)$

In equilibrium

$$\int_{E_{ul}}^{\infty} E e^{-E/kT} \sigma_{lu}(E) dE = \int_0^{\infty} E e^{-E/kT} \sigma_{ul}(E) dE$$

$$= \frac{g_u}{g_l} e^{-E_{ul}/kT} \int_0^{\infty} E e^{-E/kT} \sigma_{ul}(E) dE$$

Now let $E' = E - E_{ul}$ on the LHS

$$\int_0^{\infty} (E' + E_{ul}) e^{-(E' + E_{ul})/kT} \sigma_{lu}(E' + E_{ul}) dE'$$

$$= \frac{g_u}{g_l} e^{-E_{ul}/k_B T} \int_0^{\infty} E e^{-E/kT} \sigma_{ul}(E) dE$$

$$\Rightarrow (E_{ul} + E) \sigma_{lu}(E_{ul} + E) = \frac{g_u}{g_l} E \sigma_{ul}(E)$$

Neutral-neutral collisions

van der Waal's forces are dipole-dipole interactions

$$\phi \propto r^{-6}$$

The "size" of atoms varies from 0.5 Å (HI) to a few Å (high Z elements)

Hard sphere collisions require $b < R_1 + R_2$

$$\therefore \quad \sigma = \pi (R_1 + R_2)^2 \simeq 1.2 \times 10^{-15} \text{ cm}^2$$

$$\langle \sigma v \rangle = \left(\frac{8kT}{\mu} \right)^{1/2} \pi (R_1 + R_2)^2$$

$$= 1.8 \times 10^{-10}\, T_2^{1/2} \left(\frac{m_H}{\mu} \right)^{1/2} \left(\frac{R_1 + R_2}{\text{Å}} \right)^2 \text{ cm}^3 \text{ s}^{-1}$$

At low temperatures (CNM, GMC.) the neutral-neutral interactions are $\gtrsim$ order of magnitude <u>smaller</u> than ion-neutral interactions.

ion-neutral collisions:

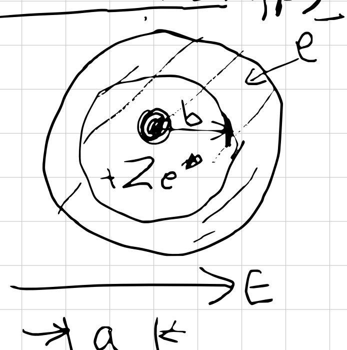
The Coulomb interactions are ~~drive~~ dominated by distant encounters.

The neutral-neutral interactions are dominated by geometrical sizes.

The ion-neutral collisions are in between

Essentially, the electric field of the ion induces a dipole in the neutral atom. The resulting dipole interacts with electric field and thereby increases the cross-section of the interaction.

BACKGROUND: Apply an external electric field, E



Electric field at radius b

$$= e\left(\frac{b^3}{a^3}\right)\frac{1}{b^2}$$

$$= \frac{eb}{a^3}$$

This is matched by external field

Thus, $E = \frac{eb}{a^3} \Rightarrow b = \frac{a^3 E}{e}$

The dipole moment of the atom is $p = eb$

but $p = eb = a^3 E$

$$\Rightarrow \quad \vec{p} = \underset{\sim}{\alpha}\, \vec{E}$$

$\uparrow$ atomic polarizability

For Hydrogen $\alpha_N = a_0^3$. An exact QM calculation shows $\alpha_N = (9/2)\, a_0^3$.

Element	H	He	Li	Be	C	Ne	Na	A	K
α_N ($10^{-24}\,cm^3$)	0.67	0.21	12	9.3	1.5	0.4	27	1.6	34

Recapitulation

For a dipole

$$\phi = \frac{\hat{r} \cdot \vec{p}}{r^2} \quad \text{potential}$$

Dipoles only feel gradients in electric field

$$\vec{F} = \vec{p} \cdot \nabla E$$

Consider an ion and atom. The ion polarizes the atom; $\vec{p}$. The resulting force is

$$F_r = \vec{p} \cdot \frac{dE}{dr} = -2\alpha_N \frac{Z^2 e^2}{r^5}$$

The corresponding potential is

$$U(r) = -\frac{1}{2}\alpha_N \frac{Z^2 e^2}{r^4}$$

Define:

$$b_0 \equiv \left(\frac{2\alpha_N Z^2 e^2}{E_{cm}} \right)^{\frac{1}{4}}$$

where $E_{cm} = \frac{1}{2}\mu v^2$ is the center-of-mass energy

We find

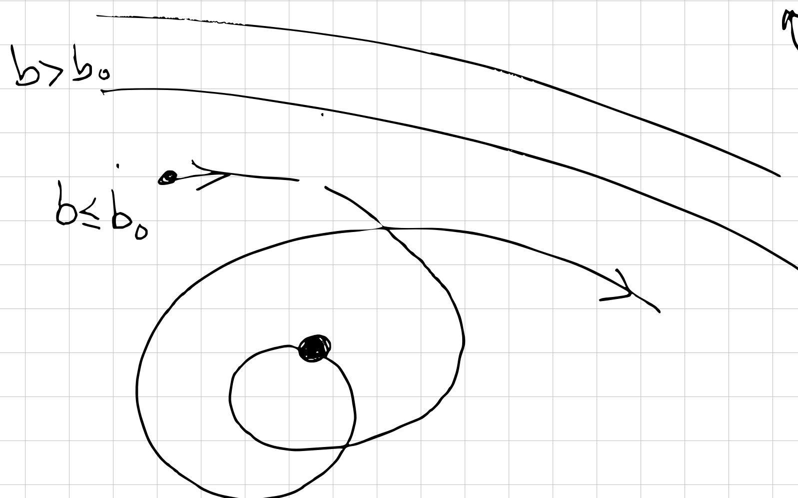
$$U(b_0) = -\frac{E_{cm}}{4}$$

For $b \leq b_0$ the projectile smashes into the target. (potential overcomes the ~~part~~ projectile)

It turns out that all encounters with $b < b_0$ smash into the target

So, $\sigma_{orb} = \pi b_0^2 = 2\pi Z e \left(\frac{\alpha_N}{\mu}\right)^{1/2} \frac{1}{v}$

↑ notice v^5



The rate coefficient is

$$\langle \sigma v \rangle_{orb} = 9 \times 10^{-10} Z \left(\frac{\alpha_N}{a_0^3}\right)^{1/2} \left(\frac{m_H}{\mu}\right)^{1/2} \text{cm}^3 \text{s}^{-1}$$

Note the lack of dependence on temperature, T. If the resulting reaction is exo-thermic then the reaction can proceed at very low temperatures.

The ion-neutral reactions are critical to organic chemistry in the ISM.

$$O^+ + H \rightarrow O(^3P_0) + H^+ \quad \text{"charge exchange"}$$

$$H_2^+ + H \rightarrow H_3^+ + H \quad \text{"exchange"}$$

$$CH^+ + H_2 \rightarrow CH_2^+ + H \quad \text{"exchange"}$$

The above reactions are exothermic and so operate at low temperatures

<u>Formation of Water</u> (in Giant Molecular Clouds)

$$n_H \sim 10^6\ cm^{-3} \quad T \simeq 10K$$

$$H_2 + \text{cosmic ray} \rightarrow H_2^+ + e^-$$

$$H_2^+ + H_2 \rightarrow H_3^+ + H$$

$$H_3^+ + O \rightarrow OH^+ + H_2$$

$$OH^+ + H_2 \rightarrow OH_2^+ + H$$

$$OH_2^+ + H_2 \rightarrow OH_3^+ + H$$

$$OH_3^+ + e^- \rightarrow H_2O + H$$

Electron–ion inelastic collisions

(Draine Appendix I)

1. Consider ion in excited state.
2. Consider low velocity electron incident on it
3. Electron attracted by ion
4. Approaches orbital speed at close encounter

A. $\frac{1}{2} m_e v_{max}^2 = \frac{1}{2} m_e v^2 + \frac{Ze^2}{r_{min}}$

B. $v_{max} r_{min} = v b$ (angular momentum)

Thus impact parameter is

$$b = r_{min} \left[1 + \frac{Ze^2/r_{min}}{m_e v^2/2} \right]^{1/2}$$

<u>Model:</u> If $r_{min} < W a_0$ then ion is de-excited

so critical impact parameter is

$$b_{crit}(v) = W a_0 \left[1 + \frac{Ze^2/W a_0}{m_e v^2/2} \right]$$

$$\sigma_{ul} = \pi b_{crit}^2$$

$$\langle \sigma v \rangle_{u \to l} = \int_0^\infty 4\pi \left(\frac{\mu}{2\pi kT}\right)^{3/2} e^{-\mu v^2/2kT} v^2 \sigma_{ul}\, dv$$

$$= \pi W^2 a_0^2 \left(\frac{8kT}{\pi m_e}\right)^{1/2} \left[1 + \frac{Ze^2}{W a_0 kT}\right]$$

$$\frac{Ze^2/a_0}{kT} \simeq \frac{15.8\, Z}{T_4}$$

For $T_4 \lesssim 1$

$$\langle \sigma v \rangle_{u \to l} = \frac{h^2}{(2\pi m_e)^{3/2}} \frac{1}{(kT)^{1/2}} 2WZ$$

$\downarrow$

$$\frac{\Omega_{ul}}{g_u}$$

Ω_{ul} = "collisional strength"

$\Omega_{ul} \simeq 1$ for $Z = 1$

n_c = number density of collidor

$$\frac{dn_1}{dt} = n_0 \left[n_c k_{01} + \bar{n}_\gamma \frac{g_1}{g_0} A_{10} \right] - n_1 \left[n_c k_{10} + (1+\bar{n}_\gamma) A_{10} \right]$$

In steady-state $\frac{dn_1}{dt} = 0$

$$\frac{n_1}{n_0} = \frac{n_c k_{01} + \bar{n}_\gamma (g_1/g_0) A_{10}}{n_c k_{10} + (1+\bar{n}_\gamma) A_{10}}$$

Limits:

In the limit of no background photons $\bar{n}_\gamma \to 0$

$$\frac{n_1}{n_0} = \frac{n_c k_{01}}{n_c k_{10} + A_{10}}$$

$$= \frac{k_{01}}{k_{10}} \frac{1}{1 + \frac{A_{10}}{n_c k_{10}}}$$

$$= \frac{k_{01}}{k_{10}} \frac{1}{1 + \frac{n_{cr}}{n_c}} \quad \text{where } n_{cr} = \frac{A_{10}}{k_{10}}$$

$n_c \gg n_{cr}$ $\quad \frac{n_1}{n_0} = \frac{k_{01}}{k_{10}} =$ Boltzman

$n_c \ll n_{cr}$ $\quad \frac{n_1}{n_0} = \frac{k_{01} n_c}{A_{10}}$ $\quad \boxed{n_1 A_{10} = n_0 k_{01} n_c}$

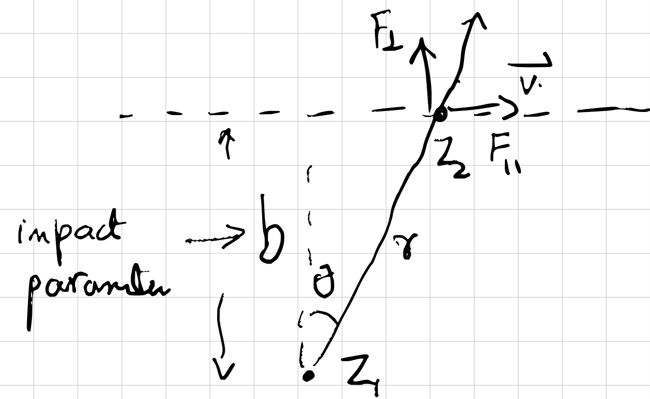
In this case every collision leads to a photon being radiated.

In general, can define

$$n_{cr} = \frac{(1+\bar{n}_\gamma) A_{ul}}{k_{ul}}$$

Thus critical density is increased by a background radiation field.

Coulomb interactions: Draine 2.2



Consider a projective with charge Z_2 moving at velocity $\vec{v}$.
The stationary target has charge Z_1.

We assume that the projectile moves at high speed.
Also the impact parameter is large so that the net impulse is small
$\Rightarrow$ trajectory of Z_2 is undeflected.

The electrostatic force $\vec{F} = \dfrac{Z_1 Z_2 e^2}{r}$

$F_\parallel$ changes sign but $F_\perp$ does not,

$$F_\perp = \left[Z_1 Z_2 e^2 / (b/\cos\theta)^2\right] \cos\theta$$

The corresponding momentum is

$$\Delta p_\perp = \int F_\perp \, dt = \int F_\perp \frac{d}{v}(\tan\theta)$$

$$\Delta p_\perp \approx \frac{2 Z_1 Z_2 e^2}{v}$$

The energy imparted to the electron is $\frac{\Delta p_\perp^2}{2m_e}$

Application: Rydberg atoms.

Target is now HI atom in high-n state

Projectile is thermal electron.

There is no net force on H atom. However $\Delta p_\perp$ of opposite sign is imparted to p^+ and $\bar{e}$. However, the corresponding energy is $\propto \frac{1}{mass}$

Let $\frac{\Delta p_\perp^2}{2m_e} > I$ (ionization energy)

then all encounters with $b_0 < \sqrt{\frac{2Z_1^2}{m_e v^2 I}}$

will ionize the atom

$$\therefore \sigma(v) = \pi b_0^2 = \frac{2\pi Z_1^2 e^4}{m_e v^2 I}$$

← notice v^2

$$\langle \sigma v \rangle = \int v \sigma v f_v(v) dv$$

$$= Z_1^2 \left(\frac{8\pi}{m_e kT}\right)^{1/2} \frac{e^4}{I} e^{-I/kT}$$

Application: Rydberg atoms

Scattering and Gradual Losses:

Projective: $Z_1 e$; velocity, v_1

Targets: $Z_2 e$, stationary

We use the impulse approximation. The projectile receives $\Delta p_\perp^2$ on each flyby.

$$\left\langle \frac{d}{dt}\left[(\Delta p_\perp)^2\right]\right\rangle = \underbrace{\int_{b_{min}}^{b_{max}} 2\pi b\, db\, n_2 v_1}_{\text{event rate}} \underbrace{\left[\frac{2 Z_1 Z_2 e^2}{b v_1}\right]^2}_{(\Delta p_\perp)^2}$$

$$= \frac{8\pi n_2 Z_1^2 Z_2^2 e^4}{v_1} \int_{b_{min}}^{b_{max}} \frac{db}{b}$$

Clearly b_{min} is given by

$$\frac{Z_1 Z_2 e^2}{b_{min}} = E = \frac{1}{2} m_1 v_1^2$$

For b_{max} we use Debye length

$$L_D = \left(\frac{kT}{4\pi n_e e^2}\right)^{1/2} = 690\,\text{cm} \left(\frac{n_e}{\text{cm}^{-3}}\right)^{-1/2} \left(\frac{T}{10^4\,\text{K}}\right)^{1/2}$$

Thus

$$\left\langle \frac{d}{dt} (\Delta p_\perp)^2 \right\rangle = \frac{8\pi n_2 Z_1^2 Z_2^2 e^4}{v_1} \ln \Lambda$$

$$\Lambda = \frac{b_{max}}{b_{min}} = 4\times10^9 \left(\frac{E}{kT}\right)\left(\frac{T}{10^4 K}\right)^{3/2}\left(\frac{cm^{-3}}{n_e}\right)^{1/2}$$

$\ln \Lambda \simeq 20-35$ for ISM conditions

The weak distant encounters appear to dominate.

The timescale for energy loss for the electron is

$$t_{loss} = -\frac{E}{\langle dE/dt \rangle}$$

$$= \frac{m_1 v_1^2}{\langle \frac{d}{dt} \Delta p_\perp^2 \rangle / m_2} = \frac{m_2 m_1 v_1^2}{\langle d/dt (\Delta p_\perp)^2 \rangle}$$

$$t_{loss} (e^- \text{ to } p^+) = 1.4\times10^7 s \; T_4^{3/2} \left(\frac{cm^{-3}}{n_e}\right)\left(\frac{25}{\ln\Lambda}\right)$$

The time scale for deflection by 90°

$$t_{defl} = \frac{(m_1 v_1)^2}{\left(\frac{d}{dt} \Delta p_\perp^2\right)}$$

$$t (e^- \text{ by } p^+) = 7.6\times10^3 s \; T_4^{3/2} \left(\frac{cm^{-3}}{n_e}\right)\left(\frac{25}{\ln\Lambda}\right)$$

These two timescales define how rapidly electrons isotropize and thermalize.

This is a big issue for young shocks at low densities..

~~At~~ In this limit of distant encounters dominating over nearby encounters the assumptions we made are equally good.