Ay121: Midterm Oral Exam

S. R. Kulkarni

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Framework. The mid-term is an oral exam that lasts up to 30 minutes. This is a pilot exam (helping you become familiar for the final exam which will last an hour). The exams will be conducted on November 4 and 5. On Friday, I will provide a Google sign up sheet. Sign up for one of the slots. Please come to my office half an hour before your appointment. Pick up one of the paper slips (each of which has three questions). You then wend off to your office and work on the questions (no access to books or equivalent information). You are allowed to use calculators. Ideally, you should know a number of basic constants, but for this exam, I am providing the values for constants and even some basic formulae.[1]

You are advised to write your answers down on paper. At the appointed time, you come to my office and explain your answers to me (using the black board).

Please note that an oral exam is very different from a written exam. The discussions stop when (1) we run out of time or (2) examiner's run out of knowledge! The grading is more indicative, in my view, of the student's true understanding. In addition, an oral exam is also an opportunity for teaching and better understanding.

Please do not discuss the exam with other students until the last student has finished their exam.

Experience has taught me that it is wise for you **to print this master list of questions** before coming to the exam.

Revision history:
28/October: Added E, B transformations (Appendix).
31/October: Problem 5. Clarified that the occupation index should be computed for photons with an energy of about 7 eV.
31/October: Clarified that you are expected to reconstruct or remember the black body intensity and the definition of the photon index.
4/November: Corrected expression for Eddington luminosity.
4/November: Clarified question 1.

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1. Intensity. Consider a star with radius R and effective temperature T_*, located at distance d. Describe an operational procedure to measure intensity at frequency ν. Use the small angle approximation. [Bonus: Discuss the same for any distance d.]

2. Geo-coronal Hα. The Earth's thermosphere (located at a height of about 500 to 1,000 km) is heated by solar photo-dissociation of O_2 to temperatures between 1,000 °C and 2,000 °C. H (from H_2O and CH_4) and He escape ballistically, forming the exosphere of Earth.[2] Lit by the solar EUV radiation the exosphere fluoresces and one of the consequence is "geocoronal" Hα emission. At midnight, geocoronal Hα varies between 2 Rayleigh to 4 Rayleigh (depending on solar activity). Compute the intensity of Hα in photon/cm^2/s/arcsecond^2. What is the width of this line in Å?

3. Dark Sky Background (Terrestrial). Along with the 2DF survey on the 3.9-m Anglo-Australian Telescope (AAT), the Sloan Digital Sky Survey (SDSS) was a consequential spectroscopic survey, essentially kicking off the modern era of massively multiplexed spectroscopy. In advance of the spectroscopic

[1] I am not providing the formula for black body. You have to either remember or reconstruct it. Same, also with the photon index.

[2] The study of exospheres of extra-solar planets, particularly hot Jupiters, is a major focus area of exo-planetology.

survey, a five-band imaging survey was conducted at APO. The measured sky background (restricted to dark[3] time) expressed in as AB magnitudes per square arcsecond is shown in Figure 1. Using the mode of distribution, compute the brightness of the sky, in any SDSS band, in units of Rayleigh/Å. [Bonus: compute it in all five bands and appreciate where the sky is darkest.]

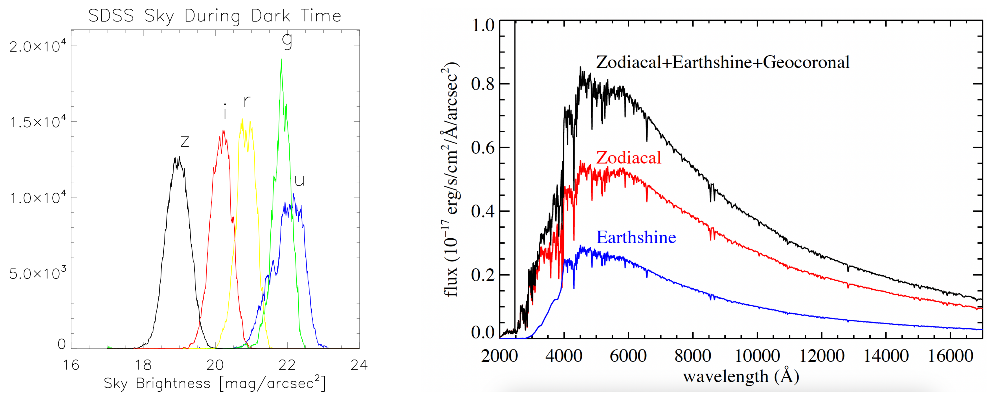

Figure 1: (Left): Histogram of sky brightness (dark time, only) at Apache Point Observatory (APO; Otero County, New Mexico) where the SDSS imaging survey was carried out. [The center wavelengths for the bands are as follows: u (3543 Å), g (4770 Å), r (6231 Å), i (7625 Å) and z (9134 Å). For the interested student: Right (sky as seen as by HST). (Bonus: How much do you gain at 6,000 Å by using HST?).

4. Ionization rate. Consider a source that puts Q ionizing photons per second. Let σ be the cross-section (cm^2) for the hydrogen atom to absorb and become ionized. Write down the expression for the time scale for ionization at distance, d.

5. Photon occupation index. At the solar circle the energy density due to the stars is $u = 0.54\,\mathrm{eV\,cm^{-3}}$. From Figure 2 we can see that the radiation is dominated by UV photons with energy, say, of 7 eV. Compute the photon occupation index for photons with this energy.

6. Wein's law. In (4) show that if stimulated emission is neglected, then consistency would require that the radiation field be the one given by Wien.

7. Solar Constant. Calculate the solar constant (the specific flux density at the top of Earth's atmosphere due to the sun's brilliance; units: $\mathrm{erg\,cm^{-2}\,s^{-1}}$). For this pedagogical purpose we assume that the Sun is a black body with effective temperature $T_{\rm eff} \approx 5,800\,\mathrm{K}$). What is the energy density (unit: $\mathrm{erg\,cm^{-3}}$).

8. Eddington Luminosity. Consider a star with radius R, mass M, blackbody temperature T, and luminosity L. For the purpose of this exam, we assume that the atmosphere is pure hydrogen. Say that T is high enough that the atmosphere is largely ionized. We review the fate of an electron in the photosphere. The force due to gravity and that due to Thompson scattering is equally reduced to r^{-2} where r is the radial distance from the center of the star. Show that the two are balanced when

$$L = L_{\rm Edd} = \frac{4\pi GM(m_p + m_e)c}{\sigma_T}$$

where the symbols have obvious meaning and σ_T is the Thompson cross-section.

9. Fourier Transform Gymnastics. What is the Fourier transform of $h_1(t)h_2(t)$ where $h_1(t) = \cos(2\pi\nu_1 t)$ and $h_2(t) = \cos(2\pi\nu_2 t)$. Deduce the answer analytically and, separately, by making clever use of the convolution theorem.

10. Gauge Choice. Maxwell's equations are centered on electric and magnetic field. These fields can be expressed as space and time derivatives of a vector potential ($\boldsymbol{A}$) and a scalar potential (ϕ). Derive

[3] When the moon is not up or very low sky in the sky

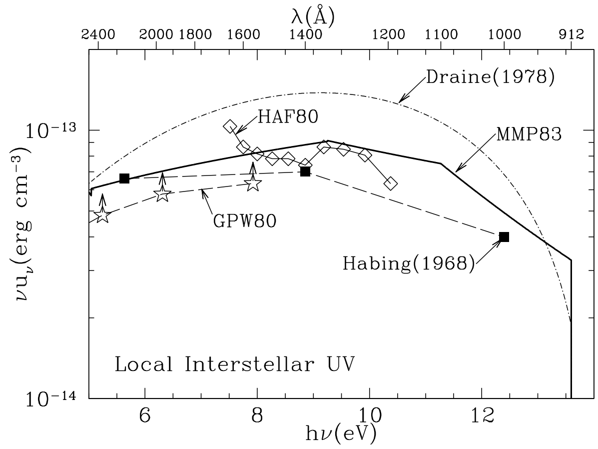

Figure 2: Measurements and models of the starlight background in the solar neighborhood. (From Draine's book; Chapter 12).

an expression for $\boldsymbol{A}$ [Equation 2.64, Rybicki & Lightman] and explain how this Equation simplifies to Equation 2.66b. [Useful vector relations are given in the Appendix].

11. Cyclotron Emission. Consider a particle with charge q and mass m moving in a homogeneous magnetic field with field strength B. Show that it undergoes a circular motion with an angular frequency $\omega_B = eB/mc$ and that it will radiate at this frequency with a power given by

$$P = \frac{2}{3} r_e^2 c (v_\perp/c)^2 B^2$$

where $v_\perp$ is the speed perpendicular to the magnetic field direction and r_e is the classical radius of the electron.

12. Polarization due to Thompson scattering. Consider an electron located at the origin of a Cartesian frame (cf. Figure 3.7 of R & L). A circularly polarized electromagnetic wave propagating in the $\hat{y}$ axis is incident on the electron. The observer is located in the x-y plane (unit vector: $\boldsymbol{n}$). Let $\cos(\theta) = \hat{x} \cdot \boldsymbol{n}$. Please be in a position to describe the scattered electric field (polarization state) at an angle θ.

13. Familiarity with four-vectors. Define 4-velocity $\vec{U}$ and relate it to the ordinary velocity $\boldsymbol{u}$. Define four-acceleration $\vec{a}$. Show that $\vec{U} \cdot \vec{a} = 0$.

14. Invariance of the radiated power. Consider a process which has back-front symmetry (i.e., with respect to the velocity, there is symmetry between emission along the velocity and opposite to the velocity; dipole emission is an example of this symmetry). Show that the radiated power, P, is Lorentz invariant.

15. Red-shifting of the CMB Show that an observer moving with velocity v, with respect to a blackbody field of temperature T, will see blackbody radiation with a temperature that depends on the angle according to

$$T' = \frac{(1-\beta^2)^{1/2}}{1+\beta\cos(\theta')} T$$

where $\beta = v/c$ and the prime refer to the observer's frame.

16. Transformation of magnetic fields. Consider a wire carrying a steady current I of electrons (velocity v). We can regard the wire as composed of a row ion at rest in the lab frame (K) with linear charge density, $\lambda_i = \lambda_0$ and electrons with charge density $\lambda_e = -\lambda_0$ and moving with velocity v so that $I = \lambda_e v$. For definiteness, we assume that the wire is along the x-axis. The net charge is zero, and so there is no electric field in the lab frame. The ions in the wire are at rest in the laboratory. The electron current produces a magnetic field that is perpendicular to $\hat{x}$, $B = 2I/(c\rho)$ where $\rho = \sqrt{y^2 + z^2}$. Now, move to a frame in which the electrons are at rest. Apply relativistic transformations and deduce the resulting electric and magnetic fields.

(Bonus: Compute the field vectors using the formal transformation equations and compare the same to your answer.).

Appendix: Constants, Formulae and Data

All constants are CGS or Astro-CGS (e.g. $\mathrm{cm}^{-3}\,\mathrm{pc}$), unless otherwise noted. $c = 2.998 \times 10^{10}\,\mathrm{cm\,s^{-1}}$
$h = 6.63 \times 10^{-27}$ CGS
$N_A = 6.02 \times 10^{23}$
$e = 4.8 \times 10^{-10}$ CGS [$e = 1.6 \times 10^{-19}$ Coulomb, SI]
$m_p/m_e = 1836$
statvolt(CGS) = 299.8 volt(SI)
1eV=1.6×10^{-12} erg, $k_B T = 0.86 T_4$ eV
$\mathrm{Ry} = R_\infty = 2\pi^2 e^4 m_e/h^3 \approx 13.6\,\mathrm{eV} \rightarrow 109,737.316\ \mathrm{cm}^{-1}$ (Rydberg)
$\sigma_T = (8\pi/3) r_e^2 = 0.66$ barn (1 barn=$10^{-24}\,\mathrm{cm}^{-2}$; $r_e = e^2/(m_e c^2)$ is the classical radius of the electron)
$a_0 = \hbar^2/(m_e c^2) = 0.53$ Å (Bohr radius, 1Å=10^{-8} cm)
$\sigma = 5.67 \times 10^{-5}\,\mathrm{erg\,cm^{-2}\,s^{-1}\,K^{-4}}$ (Stefan-Boltzman)

$G = 6.67 \times 10^{-8}$ GCS
The sun: $M_\odot = 2 \times 10^{33}$ g, $R_\odot = 7 \times 10^{10}$ cm (or 2 light seconds) and $L_\odot = 4 \times 10^{33}\,\mathrm{erg\,s^{-1}}$

radian, 20,626 arcseconds (2.1×10^5)
the astronomical unit (AU) is about 500 light seconds.
One parsec is about 3×10^{18} cm

AB magnitude: The AB magnitude is defined by the relation $m_{\mathrm{AB}} = -2.5\log(f_\nu/f_0)$ where f_ν is the flux density in Jansky (Jy) and $f_0 = 3631$ Jy. One Jansky is $10^{-23}\,\mathrm{erg\,cm^{-2}\,s^{-1}\,Hz^{-1}}$.

Maxwell's equation: These equations should be known to any graduate student of astronomy. However, for many of you, CGS is new, and so I am providing the equations of Maxwell along with some useful vector algebra relations:

$$\begin{aligned} \nabla \cdot \boldsymbol{E} &= 4\pi\rho \\ \nabla \cdot \boldsymbol{B} &= 0 \\ \nabla \times \boldsymbol{E} &= -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} \\ \nabla \times \boldsymbol{B} &= \frac{4\pi}{c}\boldsymbol{j} + \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} \end{aligned}$$

Useful vector relations:

$\nabla \times (\nabla \times \boldsymbol{A}) = \nabla(\nabla \cdot \boldsymbol{A}) - \nabla^2 \boldsymbol{A}$, $\quad \nabla \times \nabla(\psi) = 0$, $\quad \nabla \cdot (\nabla \times \boldsymbol{A}) = 0$,
$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = \boldsymbol{b}(\boldsymbol{a} \cdot \boldsymbol{c}) - \boldsymbol{c}(\boldsymbol{a} \cdot \boldsymbol{b})$ ("bac minus cab")

Larmor's formula:

$$P = \frac{2q^2\dot{u}^2}{3c^3}$$

Transformation of fields:

$$\begin{aligned} \boldsymbol{E}'_{\parallel} &= \boldsymbol{E}_{\parallel} & \boldsymbol{B}'_{\parallel} &= \boldsymbol{B}_{\parallel} \\ \boldsymbol{E}'_{\perp} &= \gamma(\boldsymbol{E}_{\perp} + \boldsymbol{\beta} \times \boldsymbol{B}) & \boldsymbol{B}'_{\perp} &= \gamma(\boldsymbol{B}_{\perp} - \boldsymbol{\beta} \times \boldsymbol{E}) \end{aligned}$$