

Ay 121: Homework 4

Due no later than midnight of November 11

November 5, 2024

[1] **Doubly Ionized Helium.** There will be some overlap of the optical spectrum (say $0.3\text{--}1\ \mu\text{m}$) of doubly ionized helium with that of hydrogen. Compute, in the Bohr model, with attention to precision to the wavelengths of the lines of hydrogen. Express the difference in km/s. 10 points

[2] **Deuterium.** A major program in the early days of the Keck Observatory was to measure, using bright quasars as background light, the abundance of deuterium in the early Universe (high redshift) with the goal of using it as input for Big Bang models. Compute the wavelength difference between corresponding key line(s) of hydrogen and deuterium and express this difference in km/s. 5 points

[3] θ^1 **Ori C.** H II regions are ionized nebulae and particularly prominent when powered by young O stars in star-forming regions. Within the H II region the ionization fraction is essentially close to unity. Their primary cooling is via metastable lines (fine structure) of metals. The temperature of H II regions varies from 6,000 K (metal rich) to 9,000 K (metal poor). θ^1 Ori C is one of the bright stars in Orion, is embedded in an H II region with diameter of 0.5 pc and a mean electron density of $3200\ \text{cm}^{-3}$. Assume a mean temperature of 8,000 K and mean distance to Orion of 400 pc. Compute the emission measure ($\text{cm}^{-6}\ \text{pc}$) for a line-of-sight through the center of the spherical H II region.

Compute the flux (Jansky) due to free-free emission at frequencies of interest to Caltech radio astronomers: 50 MHz (LWA), 1400 MHz (DSA-110), OVRO 40-m (12GHz) and 30 GHz (COMAP). [Hint: Compute the intensity along a line of sight through the center of the H II region and multiply the solid angle; Bonus: do it correctly!] 20 points

[4] **The Galactic Warm Ionized Medium.** The Galactic WIM was inferred from low-frequency absorption seen towards the diffuse synchrotron emission from the inner Galaxy (Ellis & Hoyle 1962). Wisconsin H α Mapper (WHAM) observed recombination line (H α) from this medium. Looking towards the Galactic pole, the inferred emission measure is approximately, $2\ \text{cm}^{-6}\ \text{pc}$. Compute the brightness temperature at the frequencies listed in the previous problem. [Compare your estimates to the estimates discussed in class from

Galactic synchrotron, anomalous microwave emission, heated dust etc]. 10 points

[5] **Spectroscopic Terms: Pair of Electrons.** In the class we worked out the spectroscopic terms for carbon, $\text{He}2s^2p^2$. Here, we will focus on Titanium, $\text{Ar}4s^23d^2$ which, as you can see, has two d valence electrons.

Consider first excited Ti with one valence electron in the $3d$ state and the other in $4d$ state. How many pairs of distinct micro states does this configuration admit? List the resulting 10 spectroscopic terms. 10 points

Next, we focus on the ground configuration with the two electrons in $3d$ state. How many pairs of distinct micro states does this configuration admit? Apply “Trick 1” (see SRK notes) to the previous answer and show that the ground state has five spectroscopic terms: three singlets (1S , 1D and 1G) and two triplets (3P and 3F). 10 points

Finally, either via a computer program or working by hand, reproduce the above result. 15 points