Ay 121: Homework 1

Due COB, 10 October 2025 at TA's office

Notes.
1. Astronomers use CGS units. So take the plunge and consistently use CGS units!
2. You can use Mathematica for integration (but I find it is a great pleasure to integrate from scratch and also keeps my neuron circuitry in top shape)

[A] Wien's law. The blackbody specific intensity, $B_\nu(T)$, is given by Planck's formula ($\mathrm{erg\,cm^{-2}\,s^{-1}\,Hz^{-1}}$). Let $B_\lambda(T)$ be the corresponding specificity in wavelength ($\mathrm{erg\,cm^{-2}\,s^{-1}\,\AA^{-2}}$).

1. Analytically determine the frequency (ν_p) at which $B_\nu(T)$ peaks.
2. Analytically determine the wavelength (λ_m) at which $B_\lambda(T)$ peaks. What is the relationship between λ_p and ν_p?
3. Analytically determine the λB_λ peaks (λ_m) and the frequency at which νB_ν peaks (ν_m). What is the relationship between ν_m and λ_m?
4. Analytically integrate $B_\nu(T)$ into ν and compare the result with $\nu_m B_\nu(\nu_m)$.

Total number of points: 4×5.

[B] Characteristic energies for the blackbody radiation field. Compute the mean (analytically) and first through third quartile (numerically) of the photon energies of blackbody radiation field. [Express the photon energy in $k_B T$, the natural unit of energy for this problem. 10 points

[C]: Cosmic Microwave Background radiation. Work out the formula for the number density of photons. Using your formula, compute the number density of photons (photon/cm^3) of the CMB radiation field. [Why is this an interesting problem? Read up GreisenZatsepinKuzmin (GZK) limit.] 10 points

[D]: Solar Sail. Consider a solar sail made of thickness t, μm Al sheet of size A. A payload of mass M is attached. The sail is rolled up and then launched in an Earth trailing orbit (like the Spitzer Space Telescope). It unfurls and is aligned perpendicular to the solar rays. For a given M, derive[1] A which will result in the radiative force being equal to the gravitational force. [Curious: check out IKAROS/JAXA]. 10 points

[E]: Hubble Space Telescope and James Web Space Telescope. Most astronomical observations[2] are limited by the brightness (intensity) of the background. An enormous amount of effort is put into reducing the background. Hubble Space Telescope (HST; operating in the UV through Near-IR bands) is in "low-earth orbit" (LEO; 600 km altitude) whilst James Webb Space Telescope (JWST; operating in the Near-IR to Mid-IR) is located in the Lagrange 2 (L2) region. The sky intensity for the two telescopes is shown in Figure 1. The units used are technical units and do not provide useful insight to a practicing astronomer. A somewhat sensible unit is Rayleigh/Å (good for optical but a bit too fine grained for MIR).[3] Another possibility is photon $\mathrm{cm^{-2}\,s^{-1}\,ster^{-1}\,\AA^{-1}}$. This is useful when developing exposure-time calculators.

Estimate the background for HST at 1500 Å (FUV), 5000 Å (V-band) and that for JWST at 1 μm, 3 μm, 10 μm and 20 μm. [Curious? check the footnote.[4]] 20 points

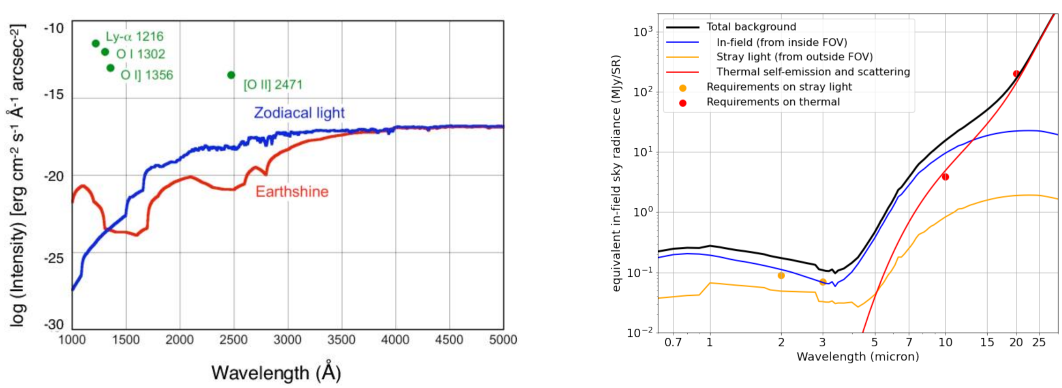

Figure 1: (Left): Sky background for HST (UV and optical). (Right): Sky background for JWST. MJy is mega Jansky or 10^6 Jy. One Jy is $10^{-23}\,\mathrm{erg\,cm^2\,s^{-1}\,Hz^{-1}}$.

[1] Look up the web for density and reflectivity of Al.

[2] The brightest stars are limited by Poisson noise

[3] One Rayleigh is $10^6/(4\pi)$ photon $\mathrm{cm^2\,s^{-1}\,ster^{-1}}$.

[4] HST: `https://hst-docs.stsci.edu/cosihb/chapter-7-exposure-time-calculator-etc/7-4-detector-and-sky-backgrounds`
JWST: `https://jwst-docs.stsci.edu/jwst-general-support/jwst-background-model#gsc.tab=0`