Ay 121: Homework 4

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Due COB, November 1, 2025

[1] **Acceleration in special relativity.** Problem 4.3 (R & L). [15 points]

[2] **Super-luminal expansion** (now routinely seen in BL Lac objects).[1]
Problem 4.7 (R & L). [15 points]

[3] **A Puzzle.**
Problem 4.10 (R& L). [10 points]

[4] **Poynting-Robertson[2] Effect**.
Problem 4.12 (R & L). [20 points]

[5] **The Great Dipole in the Sky.**
Problem 4.13a (R & L). [5 points]

[6] **Particle Motion in EM fields.**[3] [25 points]
Consider a particle moving in an inertial frame S under the influence of spatially and temporally constant, orthogonal **E** and **B** fields, so $\mathbf{E}\cdot\mathbf{B} = 0$.

a) If B > E in frame S, show that you can Lorentz transform to a frame S′ in which E′ = 0. What is the vector 3-velocity $\mathbf{v}'$ of frame S′ as seen from frame S, in terms of E, B and c?

[1] Pioneered by the Caltech VLBI group of M. Cohen and A. Readhead.

[2] An amazing mathematician, physicist and astronomer. A Caltech PhD who returned to Caltech as a faculty member. This is an important process for astronomers studying the formation of planets and debris disks.

[3] Contributed by E. S. Phinney

b) Describe the particle's motion in frame S$'$ and in frame S.

c) Now suppose that E > B in frame S. Show that you can Lorentz transform to a frame S$''$ in which B$'$ = 0.

   i. What is the vector 3-velocity **v** of frame S$''$ as seen from frame S, in terms of **E**, **B** and c?

   ii. Give a simple expression in terms of **E**, **B** for the Lorentz factor $\gamma = 1/\sqrt{1-\beta^2}$. of the Lorentz boost between S and S$''$. Here, $\beta = v/c$.

   iii. What is the electric field E$''$ in frame S$''$?

   iv. Suppose that the particle has charge q, mass m, and proper time in the particles rest frame is measured by τ, with the particle at rest in frame S$''$ at $\tau = 0$. Show that the particles Lorentz factor in frame S is given by $u^0 \equiv u^t = \gamma \cosh(qE\tau/\gamma mc)$

   v. Give similar expressions for u^x, u^y, and u^z.

d) d) Qualitatively describe what would be different if the orthogonal E and B were those of a vacuum electromagnetic wave. You may wish to refer to Gunn & Ostriker (1971).[4] This case is important to Fast Radio Bursts and to laser fusion.

[4] https://ui.adsabs.harvard.edu/abs/1971ApJ...165..523G/abstract