Ay 121: Mid-term Exam

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Framework. The mid-term is an oral exam lasting up to 30 minutes. The exams will conducted on October 31 and November 1. On Monday I will provide a Google sign up sheet. Sign up to one of the slots. Please come to my office half an hour before your appointment. Pick up one of the paper slips (each of which has three questions). You then wend off to your office and work on the questions (no access to books or equivalent information). You will be graded on the best two answers.

You are advised to write down your answers on paper. At the appointed hour you come to my office and explain your answers to me (using the black board).

Please note that an oral exam is very very different from a written exam. The discussions stop when (1) we run out of time or (2) examiner's runs out knowledge! The grading is more indicative, in my view, of the student's true understanding. Also, an oral exam is also an opportunity for teaching and better understanding.

You are allowed to use calculators. Ideally, you should know a number of basic constants but for this exam I am providing the values for constants and even basic formulae.

Please do not discuss the exam with other students until the last student has finished their exam.

Experience has taught me that it is wise for you to **please print this master list of questions**.

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1. Intensity. Consider a star of radius R located at distance d. Derive, the observed intensity, in the small angle limit. [Bonus: Derive the same, for any distance d but assuming the star emits isotropically].

2. Geo-coronal Hα. The Earth's thermosphere (located at a heigh of about 500 to 1,000 km) is heated by solar photo-dissociation of O_2 to temperatures between 1,000 °C to 2,000 °C. H (from H_2O and CH_4) and He escape ballistically, forming the exo-sphere of

Earth.[1] The exosphere fluoresces, lit by solar EUV, and one of the consequence is "geo-coronal" Hα emission. At midnight, geocoronal Hα varies between 2 Rayleigh to 4 Rayleigh (depending on solar activity). Compute the intensity of Hα in photon/cm^2/s/arcsecond^2. What is the width of this line in Å? How does this compare with the dark sky from, say, Mauna Kea.

3. Ionizing rate. Consider a source which is putting Q ionizing photons per second. Let σ be the cross-section (cm^2) for hydrogen atom to absorb and get ionized. Write down the expression for the time scale for ionization at distance, d.

4. Einstein A and B coefficients. Derive the relation between Einstein A and B coefficients. [You need to be able to construct Planck's formula from basic principles].

5. Wein's law. In (4) show that if stimulated emission is neglected then consistency would require that the radiation field be the one given by Wien.

6. Dimming of galaxies. Galaxies are extended objects and their apparent angular size depends on the sensitivity of the instrument to surface brightness. Using the fact that I_ν/ν^3 is invariant explain why cosmologically located galaxies become hard to detect as extended objects.

7. Group Velocity. The dispersion relation for waves in the ocean is given by

$$\omega^2 = gk \tanh(kh).$$

where g is the acceleration due to gravity and h is the depth of the ocean floor. Derive the group and phase velocity in the limiting case of shallow and deep ocean.

8. Gauge Choice. Maxwell's equations are centered on electric and magnetic field. These fields can be expressed as space and time derivatives of a vector potential ($\boldsymbol{A}$) and a scalar potential (ϕ). Derive an expression for $\boldsymbol{A}$ [Equation 2.64, Rybicki & Lightman] and explain how this Equation simplifies to Equation 2.66b. [Useful vector relations are given in the Appendix].

9. Eddington Limit. Consider a star with luminosity, L (erg s^{-1}) of radius R and mass, M. Assume that the photosphere is dominated by hydrogen, most of which is ionized. Consider an electron in photosphere of the star. Due to Thompson scattering the electron will experience a force pushing it away from the star. Gravity attracts it. At what luminosity will the radiative force be balanced by the gravitational force? [tip: do not forget the protons].

10. Conservation of mass. Derive the conservation of mass for a fluid (density, ρ; velocity, $\boldsymbol{v}$) in both Eulerian and Lagrangian framework.

[1] The study of exo-spheres of extra-solar planets, particularly hot Jupiters, a major focus area of exo-planetology.

11. Dispersion Measure. Assuming the dispersion relation for cold plasma derive the formula which gives the delay of a signal at frequency ν relative to infinite frequency. Compute the delay at 100 MHz and at 1 GHz for a fast radio burst with dispersion measure of $200\,\mathrm{cm}^{-3}\,\mathrm{pc}$.

12. Stoke's Parameters. Write down the normalized Stokes parameters ($[I, Q, U, V]/I$ for a linearly polarized signal (e.g. a pulsar), a circularly polarized signal (e.g., Jupiter-Io system, stellar radio flares from late type stars, e.g., GJ 1151) and an incandescent lamp.

13. Polarization due to Thompson scattering. Consider an electron located at the origin of a Cartesian frame. A circularly polarized electromagnetic wave propagating in the $\hat{y}$ axis is incident on the electron. The observer is located in y-z plane (unit vector: $\boldsymbol{n}$). Let $\cos(\theta) = \hat{x} \cdot \boldsymbol{n}$. You should be able to describe the parameters as a function of θ.

Appendix: Constants, Formulae and Data

All constants are CGS or Astro-CGS (e.g., $\mathrm{cm}^{-3}\,\mathrm{pc}$), unless otherwise noted.

$c = 2.998 \times 10^{10}\,\mathrm{cm\,s^{-1}}$
$h = 6.63 \times 10^{-27}\,\mathrm{CGS}$
$N_A = 6.02 \times 10^{23}$
$e = 4.8 \times 10^{-10}\,\mathrm{CGS}$ [$e = 1.6 \times 10^{-19}$ Coulomb, SI]

$m_p/m_e = 1836$
$G = 6.67 \times 10^{-8}\,\mathrm{GCS}$
statvolt(CGS) = 299.8 volt(SI)
1eV=1.6×10^{-12} erg, $k_B T = 0.86 T_4\,\mathrm{eV}$
Ry = $R_\infty = 2\pi^2 e^4 m_e/h^3 \approx 13.6\,\mathrm{eV} \rightarrow 109,737.316\ \mathrm{cm}^{-1}$ (Rydberg)
$\sigma_T = (8\pi/3) r_e^2 = 0.66\,\mathrm{barn}$ (1 barn=$10^{-24}\,\mathrm{cm}^{-2}$; $r_e = e^2/(m_e c^2)$ is the electron radius)
$a_0 = \hbar^2/(m_e c^2) = 0.53\,\text{Å}$ (Bohr radius, 1Å=10^{-8} cm)

radian, 20,626 arcseconds (2.1×10^5)
parsec, 3×10^{18} cm
Mass of Sun, $M_\odot = 2 \times 10^{33}$ gram
Radius of Sun, $R_\odot = 7 \times 10^{10}$ cm (or 2 light seconds)
Luminosity of Sun, $L_\odot = 4 \times 10^{33}\,\mathrm{erg\,s^{-1}}$
Astronomical unit is 500 light seconds

Sky brightness: On a clear dark night at Mauna Kea the sky brightness, in the optical band, is roughly 2 Rayleigh per Å.

Maxwell's equation: These equations should be known to any graduate student of astronomy. However, for many of you, CGS is new and so I am providing the equations of Maxwell along with some useful vector algebra relations:

$$\begin{aligned}
\nabla \cdot \boldsymbol{E} &= 4\pi\rho \\
\nabla \cdot \boldsymbol{B} &= 0 \\
\nabla \times \boldsymbol{E} &= -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} \\
\nabla \times \boldsymbol{B} &= \frac{4\pi}{c}\boldsymbol{j} + \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t}
\end{aligned}$$

Useful vector relations:
$\nabla \times (\nabla \times \boldsymbol{A}) = \nabla(\nabla \cdot \boldsymbol{A}) - \nabla^2 \boldsymbol{A}$, $\quad \nabla \times \nabla(\psi) = 0$, $\quad \nabla \cdot (\nabla \times \boldsymbol{A}) = 0$,
$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = \boldsymbol{b}(\boldsymbol{a} \cdot \boldsymbol{c}) - \boldsymbol{c}(\boldsymbol{a} \cdot \boldsymbol{b})$ ("bac minus cab")

Plasma Frequency:

$$\omega_p^2 = \frac{4\pi n_e e^2}{m_e} \tag{1}$$