11 Nov, 2024 **Ay 121** E.S. Phinney

Radiative Processes

Problem Set 5

Due **in class** before midnight, Monday Nov 18, 2024

Readings: Reading for this week is Chapter 4 of Rybicki and Lightman.

Homework Problems: (50 pts total)

1. **Relativistic relative velocity** (10pts) Rybicki and Lightman Problem 4.8 (very useful in computing flux factors for cross-sections!).

2. **Radiated momentum** (7pts: 2+2+3) Rybicki and Lightman Problem 4.10

3. **Poynting-Robertson drag** (10pts: 1+3+2+4) Rybicki and Lightman Problem 4.12

4. **Invariants and electromagnetic waves** (6 points: 3+3) Pulsar and magnetar winds are often modelled using either relativistic magnetohydrodynamics, or its negligible-inertia limit, force-free electrodynamics. These are defined by the existence of a frame with a time-like 4-velocity $\vec{u}$ in which $F_{\mu\nu}u^\nu = 0$ Considering the electromagnetic invariants (Rybicki & Lightman eqs 4.66 and 4.67),

   a) For these wind models, is $\det F > 0$, $= 0$ or < 0?

   b) For these wind models, is $F_{\mu\nu}F^{\mu\nu} > 0$, $= 0$ or < 0?

5. **Particle Motion in EM fields** (17 points: 3+2+10(=2+2+2+2+2)+3) Consider a particle moving in an inertial frame S under the influence of spatially and temporally constant, orthogonal $\mathbf{E}$ and $\mathbf{B}$ fields, so $\mathbf{E}\cdot\mathbf{B} = 0$.

   a) If $B > E$ in frame S, show that you can Lorentz transform to a frame S' in which $\mathbf{E}' = 0$. What is the vector 3-velocity $\mathbf{v}$ of frame S' as seen from frame S, in terms of $\mathbf{E}$, $\mathbf{B}$ and c?

   b) Describe the particle's motion in frame S' and in frame S.

   c) Now suppose that $E > B$ in frame S. Show that you can Lorentz transform to a frame S'' in which $\mathbf{B}' = 0$.

      i. What is the vector 3-velocity $\mathbf{v}$ of frame S'' as seen from frame S, in terms of $\mathbf{E}$, $\mathbf{B}$ and c?

      ii. Give a simple expression in terms of $\mathbf{E}$, $\mathbf{B}$ for the Lorentz factor $\gamma = 1/\sqrt{1-\mathbf{v}\cdot\mathbf{v}}$ of the Lorentz boost between S and S''.

      iii. What is the electric field $\mathbf{E}''$ in frame S''?

      iv. Suppose that the particle has charge q, mass m, and proper time in the particle's rest frame is measured by τ, with the particle at rest in frame S'' at $\tau = 0$. Show that the particle's Lorentz factor in frame S is given by $u^0 \equiv u^t = \gamma\cosh\frac{qE\tau}{\gamma mc}$, where $\gamma = 1/\sqrt{1-\mathbf{v}\cdot\mathbf{v}}$, the Lorentz factor of the Lorentz boost velocity between frame S and frame S''.

      v. Give similar expressions for u^x, u^y, and u^z.

d) Qualitatively describe what would be different if the orthogonal **E** and **B** were those of a vacuum electromagnetic wave. You may wish to refer to Gunn & Ostriker 1971. https://ui.adsabs.harvard.edu/abs/1971ApJ...165..523G/abstract This case is important to Fast Radio Bursts and to laser fusion.