Ay 121: Homework 6

Due COB, December 2, 2024

December 5, 2024: fixed parametric representation of the helix (Appendix; red color)

[1] Duration of Synchrotron Flash. In §6.2 of Rybicki & Lightman (RL) the authors derive Δt^A, the duration of the synchrotron flash. In my view, Figure 6.2 of RL is misleading and the approximation presented ("$|\Delta \boldsymbol{v}| \approx v\Delta\theta$") just above Equation 6.8a is also misleading. The former is misleading because the trajectory of a relativistic particle follows a helix (for pitch angle, $\alpha \neq 0$) and as such is not confined to a plane (as implied by Figure 6.2). The latter is incorrect because the proper equations are given in Equation 6.3 and the approximation made by RL does not immediately follow from these two equations.

The simplest way to derive Δt^A is to recognize that the trajectory of the electron is a helix. In the sketch below, the observer (marked by a circled cross; top panel) is located in a plane perpendicular to the page and at a great distance away from the electron. Let us focus on point "L" in the figure below. Our goal is to calculate the distance between 1 and 2 as the electron goes through point L. The bottom panel shows the view from the top (and is meant to be the same as in RL).

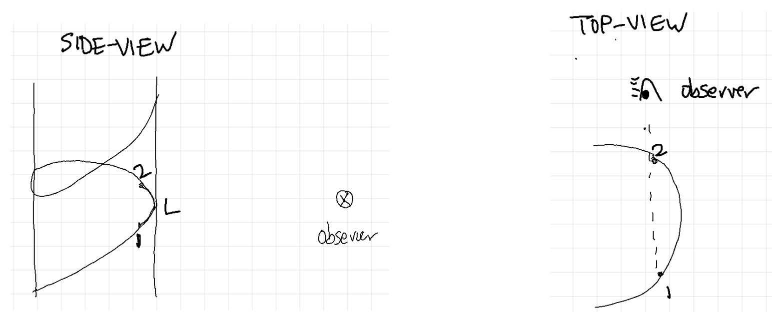

Figure 1: The helical trajectory of a relativistic electron in a magnetic field (along the vertical axis).

Calculate κ, the radius of curvature of the helix. Then the distance between 1 and 2 is simply $\kappa\Delta\theta$ where $\Delta\theta = 2/\gamma$. Show that this value agrees with that given by Equation 6.9

of RL. [15 points]

[2] Lifetime of a highly relativistic electron. Consider a highly relativistic electron with Lorentz factor $\gamma \gg 1$. It is moving in the interstellar medium with parameters typical of the case at the solar circle: the diffuse stellar photon field (characterized by a dilute blackbody with $T \approx 6,000\,\mathrm{K}$) with energy density $0.54\,\mathrm{eV\,cm^{-3}}$, the CMB ($0.26\,\mathrm{eV\,cm^{-3}}$) and magnetic field energy density of $0.9\,\mathrm{eV\,cm^{-3}}$.

For Lorentz factor, $\gamma = [10^4, 10^5, 10^6]$ compute the time on which this electron will lose half of its energy due to inverse Compton scattering (CMB, starlight) and due to synchrotron radiation. [Warning: Do not use the Thompson cross section indiscriminately. Use an approximate cross section (Klein-Nishina) as needed.] 3×10 points

Compute the characteristic frequencies for each process. 10 points

[3] Problem 6.3 of RL. 15 points

[4] Problem 6.5b of RL 15 points

Appendix

A helix can be specified by $x = a\cos(q)$, $y = a\sin(q)$, and $z = bq$ where b is the distance gained along the z axis for every turn made. The unit vector of the tangent is given by $\boldsymbol{t} = d\boldsymbol{r}/ds$ where ds is the arc length; here, $\boldsymbol{r} = (x, y, z)$. For a parametric curve, $\boldsymbol{t} = d\boldsymbol{r}/dq/|d\boldsymbol{r}/dq|$. The curvature vector is $\boldsymbol{k} = d\boldsymbol{t}/ds$ which, for a parametric curve, is given by $d\boldsymbol{t}/dq/|d\boldsymbol{r}/dq|$. The radius of curvature is given by $\kappa = |\boldsymbol{k}|^{-1}$.