Ay 121: Homework 3

Due no later than midnight of 25 October 2024

October 19, 2024

[1] Cyclotron Radiation. Consider a non-relativistic electron, mass m, of energy $E = {}^1\!/\!{}_2 mv^2$ which is injected into a region threaded by a magnetic field of uniform strength, B_0. For simplicity, assume that the electron is injected with velocity $\vec{v}$ that is perpendicular to $\vec{B}$. The electron will gyrate perpendicular to the field.

A. Show that the angular frequency of the gyration is $\omega_B = eB/mc$ and the total emitted power is

$$P = {}^2\!/\!{}_3 r_0^2 c(v/c)^2 B^2 \tag{1}$$

[5 points]

B. Describe qualitatively and quantitatively the polarization of the radiation as a function of θ (the polar angle as measured with respect to the magnetic field). (Please do not minimize your answer to a few lines). [10 points]

C. In the Larmor approximation, how do you convince yourself that the radiation is at frequency ω_B. [5 points]

D. The radiation field is given by (cf. Equation 3.10 of Rybicki & Lightman)

$$E_{\rm rad}(\mathbf{r}, t) = \frac{q}{c}\left[\frac{\mathbf{n}}{\kappa^3 R} \times \{(\mathbf{n} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}\}\right] \tag{2}$$

where $\kappa = 1 - \mathbf{n} \cdot \boldsymbol{\beta}$ and the symbols have the usual meaning. The Larmor approximation sets $\beta = 0$ in both the denominator and the numerator. Here, we set $\beta = 0$ in the denominator but keep it in the numerator. Describe the radiation field in this framework (orientation and frequency). [10 points]

[2] Pulsars. Problem 3.1 of Rybicki & Lightman. [15 points]

[3] Two Adjacent Dipoles. Problem 3.3 of Rybicki & Lightman. [10 points]

[4] Rayleigh Scattering. Materials can be either non-polar (e.g., air) or polar (made of molecules which have large dipole moments, e.g., HCl). For non-polar material, an incident electric field, $\mathbf{E}$ induces dipole, $\mathbf{d} = \alpha\mathbf{E}$ where α is the atomic polarizability[1] and a natural normalization for α is $1\,\text{Å}^3$=$10^{-24}\,\text{cm}^3$. The data for a smattering of atomic and molecular species are given in the table[2] below. As you can see, the noble gases with their complete shells have low polarizability coefficients while alkali elements with a single valence electron have large coefficients.

species	H	He	Li	Be	C	Ne	Na	A	K	N_2	O_2	H_2O
$\alpha/10^{-24}\,\text{cm}^3$	0.66	0.21	12	9.3	1.5	0.4	27	1.6	34	1.71	1.56	1.50

Consider an electromagnetic wave, $E = E_0 \sin(\omega t)$ incident on any of the atoms (in gaseous form) listed above. We assume that the wavelength of the light is much larger than the size of the atoms (thus optical light would qualify as such). This wave will induce a time-variable dipole in the atom which will then radiate ("Rayleigh Scattering"). Compute the resulting differential and total cross section as a function of frequency. [20 points]

[5] The Blue Sky.

In problem **3** we learnt that two dipoles, if separated widely, radiate as $\propto 2d^2$ (here, we have set $d_1 = d_2 = d$) whereas, if sufficiently close, radiate as $4d^2$. The former is "incoherent" and the latter is "coherent". In our atmosphere, the moleculees and atoms Rayleigh scatter sunlight. Is the scattered light coherent or incoherent (a simple yes or no is not adequate; analyze the problem broadly). [10 points]

[1] The polarizability coefficient is different in CGS and MKS.

[2] data from `https://cccbdb.nist.gov/pollistx.asp`