Ay 121: Homework 2

Due no later than midnight of October 18, 2024

October 15, 2024

[1] The Great Dipole in the Sky. The Universe is isotropic and is homogeneous on length scales larger than the Baryon Acoustic Oscillation scale of 150 Mpc. Locally, galaxies can be located in clusters, groups or occasionally voids. Galaxies experience "peculiar" velocities relative to the expansion of the Universe. For galaxies in clusters the peculiar velocity can be large, up to several thousand of km/s. For field galaxies it is typically several hundreds of km/s. These peculiar velocities can be measured by careful measurement of the brightness of the Cosmic Microwave Background radiation (CMB). This was first detected by ground-based telescopes and exquisitely measured by space-based missions (Figure 1).

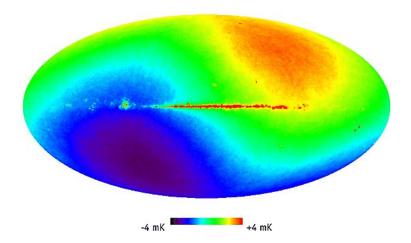


Figure 1: The brightness temperature map of the CMB in Galactic coordinates (from WMAP). The thin horizontal stripe is the Galactic plane which is teeming with HII regions and hence bright. At radio wavelengths, Planck formula is indistinguishable from Rayleigh-Jeans formula and so it is traditional to use brightness temperature $I_{\nu}/\nu^2 \propto T$ instead of I_{ν} . The color coding (horizontal thermometer) is the excess or deficit wrt $T_0 = 2.7254 \pm 0.00057 \,\mathrm{K}$.

- a) Noting the invariance of I_{ν}/ν^3 and the data given in Figure 1 infer the peculiar velocity of the Sun wrt to CMB. [5 points]
- b) Estimate the Earth's orbital velocity around the Sun. Consult a senior graduate student working in CMB and determine whether with current technology the Earth's orbital velocity can be measured? [5 points]
- [2] Allowed, Semi-Forbidden and Forbidden Lines. This is a simple exercise aimed at introducing some of the key lines in astronomy. Your primary tool is the Atomic Spectra Database (ASD)¹ maintained by National Institute of Standards & Technology (NIST). At the ADS site you will find the energy levels of atoms (H I which is H⁰) and ions (e.g. C IV which is C⁺³). For a beautiful pictorial summary of the energy levels see Appendix E of "Physics of the Interstellar & Intergalactic Medium" by Bruce Draine (available for download through our library; cf. discussion in class).

Spectral lines are classified as "allowed", "semi-forbidden" and "forbidden" lines depending on the value of A coefficients. Allowed lines whose lower level is the ground state are, at times, referred to as "resonance" lines.

- a) Let us start with allowed lines. Lyman- α of hydrogen (transition from the first excited 2p state to the ground state 1s) is both an allowed and resonance line. Obtain the A-coefficient for Lyman- α from ADS. The natural broadening width of the line is A (in Hz). What is the width, if converted to velocity (km/s)? [5 points]
- b) Query ADS for A-coefficient of the equivalent Lyman- α of Fe⁺²⁵ (Fe XXVI). Convert this to km/s. [5 points]
- c) Semi-forbidden lines have A coefficients smaller than those of permitted lines. Retrieve the A-coefficients of O V] $\lambda\lambda 1214,1218\,\text{Å}$ doublet. [There are four A-coefficients. Which matter?] [5 points]
- d) Forbidden lines have very small A-coefficients. Their origin was first recognized by Ira Bowen who later went on to become the first Director of the Palomar Observatory (cf. class discussion; see my write up). The most famous line in this group is [OIII] $\lambda\lambda4959,5007\,\text{Å}$ doublet (erstwhile "nebulium"). Obtain the A-coefficient of this line from ASD. [5 points]
- e) Be careful to note that some authors state wavelengths as measured in air while others give wavelengths in vacuum (as would be obtained from theoretical calculations). For instance, Draine's Appendix E quotes vacuum wavelengths whereas the default for ASD is air for $0.2-2\,\mu\mathrm{m}$ and vacuum otherwise! If you certify that you have read https://classic.sdss.org/dr7/products/spectra/vacwavelength.php. and promise to be

https://www.nist.gov/pml/atomic-spectra-database

wary of this issue in the future then you will be awarded 5 points!

[3] Problem 1.4 in Rybicki & Lightman.

[15 points]

[4] Problem 1.8 in Rybicki & Lightman.

[15 points]

[5] Problem 1.9 in Rybicki & Lightman.

[15 points]