Ay 121: Homework 4

Due no later than midnight of November 11

November 5, 2024

[1] Doubly Ionized Helium. There will be some overlap of the optical spectrum (say 0.3–1 μm) of doubly ionized helium with that of hydrogen. Compute, in the Bohr model, with attention to precision to the wavelengths of the lines of hydrogen. Express the difference in km/s. 10 points

[2] Deuterium. A major program in the early days of the Keck Observatory was to measure, using bright quasars as background light, the abundance of deuterium in the early Universe (high redshift) with the goal of using it as input for Big Bang models. Compute the wavelength difference between corresponding key line(s) of hydrogen and deuterium and express this difference in km/s. 5 points

[3] θ^1 Ori C. H II regions are ionized nebulae and particularly prominent when powered by young O stars in star-forming regions. Within the H II region the ionization fraction is essentially close to unity. Their primary cooling is via metastable lines (fine structure) of metals. The temperature of H II regions varies from 6,000 K (metal rich) to 9,000 K (metal poor). θ^1 Ori C is one of the bright stars in Orion, is embedded in an H II region with diameter of 0.5 pc and a mean electron density of 3200 cm^{-3}. Assume a mean temperature of 8,000 K and mean distance to Orion of 400 pc. Compute the emission mesure (cm^{-6} pc) for a line-of-sight through the center of the spherical H II region.

Compute the flux density (Jansky) from this HII region, resulting from free-free emission, at frequencies of interest to Caltech radio astronomers: 50 MHz (LWA), 1400 MHz (DSA-110), OVRO 40-m (12$\dot{\mathrm{G}}$Hz) and 30 GHz (COMAP). [Hint: You can accurately estimate the flux in the limit of low and high optical depth. For intermediate case you can take the ready-n-rough approach of approximating the HII region by a cylinder or (BONUS) do it correctly using spherical calculus.] 20 points

[4] The Galactic Warm Ionized Medium. The Galactic WIM was inferred from low-frequency absorption seen towards the diffuse synchrotron emission from the inner Galaxy (Ellis & Hoyle 1962). Wisconsin Hα Mapper (WHAM) observed recombination line (Hα) from this medium. Looking towards the Galactic pole, the inferred emission measure is approximately, $2\,\mathrm{cm}^{-6}\,\mathrm{pc}$. The vertical scale height of this gas is 1 kpc. Assume a mean temperature of 8,000 K. The WIM, via thermal bremss, provides a foreground at radio wavelengths. Compute the brightness temperature towards the Galactic poles at the frequencies listed in the previous problem. [Compare your estimates to the foregrounds – Galactic synchrotron, anomalous microwave emission, heated dust – discussed in the class.]. 10 points

[5] Spectroscopic Terms: Pair of Electrons.

Ca I. The ground electronic state of Ca I is Ar4s^2. One of the excited configurations is Ar3d4p. How many pairs of distinct micro states does this configuration admit? List the resulting six spectroscopic terms. 10 points

Ti I. Next, consider Ti I whose ground state is Ar4$s^2$3d^2. How many pairs of distinct micro states does this configuration admit? Apply "Trick 1" (see SRK notes) to the previous answer and show that the ground state has five spectroscopic terms: three singlets (^1S, ^1D and ^1G) and two triplets (^3P and ^3F). 10 points
Either via a computer program or working by hand, reproduce the above result. 10 points
Apply Hund's rule and order the five spectroscopic terms. Compare your ordering with that given at the NIST portal. 10 points