On the strengths of nebular lines

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Synopsis. *Why are forbidden lines seen easily in the heavens but not in laboratory? It is usually (loosely) stated that forbidden lines are suppressed by high density of lab gases. However this explanation is incorrect. It is the case that permitted lines in the lab are brighter because of higher density. In the heavens, the density is low and so permitted lines do not enjoy the huge density advantage as they do in the lab. Furthermore, the forbidden lines, being fine structure lines of the ground electronic configuration, are of lower energy relative to most permitted lines (most of which lie in the UV). Early astronomers lacking access to space-based UV spectrographs naturally focused on forbidden lines. With space-based facilities nebula can be profitably studied in brighter UV permitted lines.*

Background. Ira Sprague Bowen was the first Director of the Palomar Observatory (1948–1964). He came to Caltech along with Millikan in 1921. Strong lines were seen in spectra of astronomical nebulae obtained at Lick Observatory. In 1927 Bowen explained that these mysterious lines not due to new elements but were fine structure lines of O^+, N^+ and O^{++}; see §A for historical details. The Bowen (1927) paper laid the foundation of the field of nebular astrophysics.

Unfortunately, Bowen (1927) made an incorrect statement in his otherwise seminal paper: "*If the mean life of the excited state before spontaneous emission is very long, as in the case of a metastable state, and if the mean time between impacts is short (in the most rarified terrestrial sources this is never more than 1/1000 second) practically all of the atoms will return by the second process and no radiation will take place.*" This statement has propagated a misconception in the astronomical community, namely that under laboratory conditions forbidden emission goes to zero.

Last year (2023) I wrote up this elementary exposition following a similar statement made by a graduate student at his candidacy exam. In a recent Physics Research Colloquium, the speaker (D. Schlegel), when introducing the key role of [OII] $\lambda\lambda$ 3726, 3729 doublet in the DESI project, essentially repeated this incorrect statement. For this reason I am (re?)circulating this write up to selected neophytes and tyros in Cahill.

1 Line Emission from Forbidden & Resonance Lines

Consider a two-level particle. Let n_c be the density of colliding particles (say, electrons) and let n_0 be the density of atoms (or ions) with two states. In equilibrium,

$$n_1 n_c k_{12} = n_2 n_c k_{21} + n_2 A_{21} \tag{1}$$

where n_i is the density of atoms in state i, A_{21} is the A-coefficient and k_{21} is the collisional *de-excitation* rate coefficient (and traditionally quoted in tables). From detailed balance in thermodynamic equilibrium, assuming only collisions, we find

$$k_{12} = \frac{g_2}{g_1} k_{21} \mathrm{e}^{-E/k_B T} \ .$$

Using this relation Equation 1 can be expressed as

$$\frac{n_2}{n_1} = \frac{g_2}{g_1} \frac{\mathrm{e}^{-E/k_B T}}{1 + n_{\mathrm{crit}}/n_c}$$

where

$$n_{\mathrm{crit}} \equiv A_{21}/k_{21}$$

is the "critical density". Usually we are in the situation where the exponential is much smaller than one. So, we can assume $n_1 \approx n_0$. If so, the rate of radiative decys per atom is

$$l \equiv \frac{n_2}{n_0} A_{21} \approx \frac{g_2}{g_1} \frac{A_{21} \mathrm{e}^{-E/k_B T}}{1 + n_{\mathrm{crit}}/n_c} \ . \tag{2}$$

1.1 Low Density Limit, $n_c \ll n_{\mathrm{crit}}$

In this limit, Equation 2 simplifies to

$$l = n_c k_{12} \ .$$

Essentially, every collisional excitation leads to emission and so $l \propto n_c$. The luminosity is independent of the value of A_{21}.

1.2 High Density, $n_c \gg n_{\mathrm{crit}}$

In this case, Equation 2 simplifies to

$$l = A_{21} \frac{g_2}{g_1} \mathrm{e}^{-E/k_B T} \ .$$

There is a ready availability of excited atoms and the rate of radiation is limited by the A-coefficient. The luminosity per atom is simply set by the Boltzmann equation. In particular, the luminosity does not increase with increasing density of colliding particles. It only depends on the temperature of the gas.

Bowen approached this problem probably along the following lines. Consider an atom in an excited state. It has two options to relax to ground state: by spontaneous decay (characteristic time scale of A_{21}^{-1}) and collisional de-excitation (characteristic time scale is $[n_c k_{21}]^{-1}$). Thus the branching ratio for emission is $B = A_{21}/[n_c k_{21}]$. Bowen reasoned that $B \to 0$ as n_c is increased. So, he concluded that there would be little radiation.

However, the rate of excitation to the upper state is $n_c k_{12}$ and so increases with n_c. The emissivity is $B n_1 k_{12} = A_{21} k_{21}/k_{12}$. Thus, although B decreases with increasing n_c, the flux of excited atoms increases with n_c and the emissivity is independent of n_c. Note that the emissivity favors lines with larger values of A.

Given similar k_{12}, permitted lines will always be brighter than forbidden lines at any density; see Figure 1. In dense gas, forbidden lines are weaker relative to permitted lines because the forbidden line intensity saturates at the Boltzmann value while that of the permitted lines may still be in the low density limit and not yet saturated.

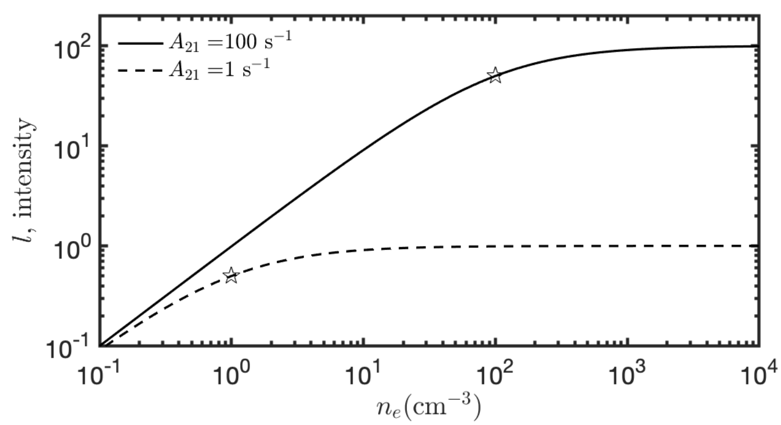

Figure 1: The luminosity per atom as a function of density of colliding particles for permitted and forbidden lines. In this example, k_{21} is fixed to $1\,\mathrm{cm}^3\,\mathrm{s}^{-1}$, $g_1 = g_2 = 1$ and the Boltzmann factor is also set to unity. As proxies for permitted and forbidden lines we have set $A_{21} = 1\,\mathrm{s}^{-1}$ for forbidden lines and $A_{21} = 10^2\,\mathrm{s}^{-1}$. The critical density in each case is marked by a pentagram. At low densities the permitted lines do not have significant advantage over permitted lines. At high densities the permitted lines are brighter.

Incidentally the A-coefficients[1] for the [OIII] doublet are $1.8 \times 10^{-2}\,\mathrm{s}^{-1}$ and $6.2 \times 10^{-3}\,\mathrm{s}^{-1}$

[1] `https://physics.nist.gov/PhysRefData/ASD/lines_form.html`

respectively while that for [OII] $\lambda\lambda$ 3276,3729 are $1.6 \times 10^{-4}\,\mathrm{s}^{-1}$ and $2.9 \times 10^{-5}\,\mathrm{s}^{-1}$, respectively. These can be compared to $10^8\,\mathrm{s}^{-1}$, typical of resonance (permitted) lines. The radiative lifetimes of the nebular lines range from minutes to a day.

A Nebulium

In 1864 astronomer William Huggins discovered strong green emission lines in Cat's Eye, a planetary nebula (NGC 6543). At the turn of the century the prevailing view was that these lines arose from new elements (inspired perhaps by helium being found in the Sun). He clarified that the then mysterious $\lambda\lambda 5007, 4959$ doublet which were variously attributed to a new element(s) were fine structure (split by $\mathbf{L}\cdot\mathbf{S}$ interaction) line of the ground state of O^{++} (Bowen 1927a,b). In the same paper(s) he showed N^{+} and O^{+} produced doublets now identified as as forbidden lines that strongly shine in H II regions.

Bowen published his discovery twice: in *Nature* (Bowen 1927a) and, perhaps as a backup, a longer article in PASP (Bowen 1927b). Later on he went on to explain why lines of O^{++} and N^{++} were strong in planetary nebulae (and some novae), namely a resonance between He II Ly-$\alpha\lambda$ 303.782 and energy levels of O^{++} and N^{++} (Bowen 1934).

Bowen recruited Horace Babcock (BS Caltech 1934; PhD UC Berkeley 1938) to the staff of Mt. Wilson. Babcock proved to be an amazing astronomer/inventor (including inventing Adaptive Optics). He served as the second director of Palomar Observatory (1964–1978). Babcock wrote up an excellent summary[2] of Bowen's career.

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[2] `https://nap.nationalacademies.org/read/576/chapter/4`