Fast Radio Bursts & Related Phenomena

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Propagation Through Plasma

"If you haven't found something strange during the day, it hasn't been much of a day."

– John Archibald Wheeler

An electromagnetic pulse incident on a plasma cloud will excite charged particles which in turn will radiate. These new fields add onto the propagating field and the net result is that pulse will travel with a velocity less than that of the speed of light in vacuum. High frequency signals will travel closer to the speed of light relative to low frequency signals. Our goal in this chapter is to broadly understand the propagation of electromagnetic (radio) signals through plasma.

At the most basic level, plasma physics rests on the four equations of Maxwell and the three equations of fluid mechanics (conservation of mass, momentum and energy). Below, let m_j, q_j and n_j be the mass, charge and number density of species j where j=e for electrons and j=i for ions. The corresponding generic quantity will not have a subscript. Since we are at an introductory phase in the book we list the basic equations:

$$\nabla \cdot \mathbf{E} = 4\pi(n_i q_i + n_e q_e) \tag{2.1}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \tag{2.2}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{2.3}$$

$$\nabla \times \mathbf{B} = \frac{1}{c}(4\pi\mathbf{j} + \frac{\partial \mathbf{E}}{\partial t}) \tag{2.4}$$

where

$$\mathbf{j} = n_i q_i \mathbf{v}_i + n_e q_e \mathbf{v}_e \tag{2.5}$$

is current density, with $\mathbf{v}_j$ standing for velocity of species j. Proceeding forward, we may at times, use the short hand, $\dot{X}$, for the partial time derivative of quantity X. We have dropped the energy conservation equation by assuming a constitutive relation between pressure, P_j and the corresponding particle densities, n_j. A standard approach is to use "polytrope" formulation in which $P_j \propto (m_j n_j)^\gamma$. For instance, $\gamma = 5/3$ for simple atomic gas. The remaining two equations of fluid mechanics are

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \mathbf{v}_j) = 0 \tag{2.6}$$

$$m_j n_j \Big[\frac{\partial \mathbf{v}_j}{\partial t} + (\mathbf{v}_j \cdot \nabla)\mathbf{v}_j\Big] = q_j n_j \Big[\mathbf{E} + \frac{1}{c}\mathbf{v}_j \times (\mathbf{B} + \mathbf{B_0})\Big] - \nabla P_j. \tag{2.7}$$

Here, $\mathbf{B}$ is the oscillating field and $\mathbf{B_0}$ is the static external field. There are 16 unknown scalars: n_i, n_e, P_i, P_e, $\mathbf{v}_i$, $\mathbf{v}_e$, $\mathbf{E}$ and $\mathbf{B}$ and the same number of scalar equations (E 1).

The above set of "Maxwell-Boltzmann" equations has significant limitations. The equations are based on averaged quantities of the local fluid, assuming that, apart from the bulk motion $\vec{v}$, the Boltzmann distribution function is isotropic non-relativistic Maxwellian at a given temperature T. The pressure is then $P_j = n_j k_B T$, and the polytropic equation of state gives $\nabla P_j = \gamma k_B T \nabla n_j$. This fluid picture does not take into account kinetic effects operating on a subgroup of particles such as Cherenkov wave-particle interaction, Landau damping, etc. Later in the book, we will present appropriately sophisticated plasma equations to deal with such phenomena.

E 1. Number of Scalar Equations. Nominally the four equations of Maxwell add up to 8 scalar equations. The two equations of fluid mechanics, applied to electrons and ions, and the two equations of state, add up to 10 scalar equations. Show that Gauss's law for charge (Equation 2.1) and magnetism (Equation 2.3) can be recovered by taking divergence of the remaining two equations of Maxwell. Thus the total number of independent scalar equations is 16.

Phase. The sinusoid, $A\cos(\omega t + \phi_0)$ has two free parameters, amplitude, A, and phase, ϕ_0. To infer these two values we need two measurements. These could be achieved by measuring the signal $(4\nu)^{-1}$ apart. The radio band is squarely in the "wave" limit (as opposed to the photon limit) and so the electric field can be manipulated directly, without noise penalty. One approch is to use an "I-Q" mixer. The signal is multiplied by $\cos(\omega t)$ and $\sin(\omega t)$ either in hardware or via a numerical Fourier transform. The resulting signals, "I" for inphase and "Q" for "quadrature", following rectification (retaining dc or low frequency signal), yield $I = A\cos(\phi_0)$ and $Q = A\sin(\phi_0)$. The amplitude is $\propto \sqrt{I^2 + Q^2}$ and phase is given by $\tan^{-1}(Q/I)$.

Sign convention. A standard approach in the study of waves is to seek solutions for an input function of the form, $\mathbf{E} \propto \cos(\mathbf{k}\cdot\mathbf{x} - \omega t)$. The choices of sign of k and ω are arbitrary. The tradition is to let $\omega \geq 0$ (which is why frequency meters and oscilloscopes do not display negative frequencies) and let $+k$ represent a wave moving in the positive x direction and $-k$ for a wave moving in the $-x$ direction.

2.1 Dispersion in Cold Plasma

We will consider the simple case of hydrogen plasma by assuming an unmagnetized (B_0=0) "cold" plasma. The latter assumption allows us to ignore gas pressure in the RHS of Equation 2.7. We seek solutions to an electromagnetic wave, $\mathbf{E} \propto \cos(\mathbf{k}\cdot\mathbf{x} - \omega t)\hat{\mathbf{k}}$ propagating in direction $\mathbf{k}$ through the plasma. Here, $k = 2\pi/\lambda$ and $\omega = 2\pi\nu$ where λ is the wavelength and ν is the frequency of the electric field.

Eliminating $\nabla \times \dot{\mathbf{B}}$ from the curl of Equation 2.2 and the time derivative of Equation 2.4 results in

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2\mathbf{E} = -\frac{1}{c^2}(4\pi\mathbf{j} + \ddot{\mathbf{E}}). \tag{2.8}$$

Phasor Algebra. It is tedious to deal with sines and cosines. It is convenient to switch to the "phasor" framework in which $\cos(\mathbf{k}\cdot\mathbf{x} - \omega t) \rightarrow \exp[i\mathbf{k}\cdot\mathbf{x} - i\omega t]$. Bear the following rules in mind: (1) phasors can be added; however, (2) when multiplying two phasors (e.g., mixing a local oscillator with the sky signal), the phasor with the lower frequency should be conjugated; (3) the mean signal strength of the vector product of two phasors, say A and B, is $\frac{1}{4}(AB^* + A^*B)$, which means the mean power of a single phasor is $\frac{1}{2}AA^*$; and (4) at the last step, following all vector or algebraic manipulations, the signal is the real part of the resulting phasor, Z. The amplitude of the signal is ZZ^* and the phase is $\phi = \tan^{-1}[\mathrm{Im}(Z)/\mathrm{Re}(Z)]$.

Thanks to phasor algebra, $\dot{\mathbf{E}} = -i\omega\mathbf{E}$ and $\nabla^2\mathbf{E} = -k^2\mathbf{E}$ and thus we obtain(E 2) the foundational equation for the dispersion relation:

$$(k^2 - \frac{\omega^2}{c^2})\mathbf{E} - \mathbf{k}\cdot(\mathbf{k}\cdot\mathbf{E}) \;=\; \frac{4\pi i\omega}{c^2}\mathbf{j}. \tag{2.9}$$

The magnetic field is always perpendicular to the electric field since from Equation 2.2 $\mathbf{B} \propto \mathbf{k}\times\mathbf{E}$.

In vacuum there is no current, $\mathbf{j}$=0, so the RHS is zero. There are also no charges and so Gauss's law (Equation 2.1) leads to $\mathbf{k}\cdot\mathbf{E} = 0$. Thus, propagating waves in vaccuum are transverse and obey the well known vacuum dispersion relation, $\omega^2 = k^2c^2$. The electrons and ions, upon being excited by the incident field, will radiate. We now compute the resulting electromagnetic field. For the same field strength, the ions, being heavier, will undergo smaller acceleration. Thus the radiation from the ions is much smaller than that from the electrons. Justified thus, we set $\mathbf{v}_i = 0$.

We now make the standard "small amplitude" approximation (also sometimes referred to as "linearized" approximation): we assume $v/c \ll 1$ and only retain terms which are linear in perturbed quantities In Equation 2.7, the second term in the LHS (left hand side), being $O(v_e^2)$, can be ignored. Thus the equation of motion for electrons simplifies to

$$m_e \frac{\partial \mathbf{v}_e}{\partial t} = -e\Big[\mathbf{E} + \frac{\mathbf{v}_e}{c} \times (\mathbf{B} + \mathbf{B_0})\Big] \tag{2.10}$$

where $\mathbf{v}_e$ is the velocity of the perturbed electrons. Above we have included $\mathbf{B}_0$ to set the stage for later discussion but, for now, set $B_0 = 0$.

Next, for a propagating electromagnetic wave, the strength of the electric field is comparable to that of the magnetic field (cf. Equation 2.2). However, from the RHS of Equation 2.10 we see that the force due to the magnetic field is reduced by v_e/c or in other words the force on the electron due to the fluctuating magnetic field, relative to that from the electric field, is smaller by (v_e/c). So we can ignore the force from the propagating magnetic field. With these assumptions, Equation 2.10 simplifies to $m_e\dot{\mathbf{v}}_e = -e\mathbf{E}$. The velocity function of the electron follows that of the electric field, $\mathbf{v}_e \propto \exp[i\mathbf{k}\cdot\mathbf{x} - i\omega t]$ and so $\mathbf{v}_e = e\mathbf{E}/(im_e\omega)$ which the yields the electric current, $\mathbf{j}$. Plugging (Equation 2.5) into Equation 2.9 leads to the cold plasma dispersion law:

$$\omega^2 = k^2c^2 + \omega_e^2, \qquad \omega_e^2 = \frac{4\pi n_e e^2}{m_e}. \tag{2.11}$$

ω_e is called as the electron plasma frequency. While ω is naturally well suited to physical modeling the ordinary frequency, ν is more useful when it comes to measurements. In CGS units, $\nu_e \approx 9n_e^{1/2}$ kHz where n_e has the unit of cm^{-3}.

In a dispersion relation, the frequency at which $k \to 0$ is called as a "cutoff" frequency. For cold and unmagnetised plasma the cutoff frequency is the plasma frequency (E 3). For $\omega < \omega_e$, k becomes imaginary which means the incident waves dies down exponentially as it propagates into the plasma. The corresponding exponential scale length for this "evanescent" wave is $\delta = c/\sqrt{\omega_e^2 - \omega^2}$.

E 2. Phasor calculus. Derive Equation 2.9 from Equation 2.8.

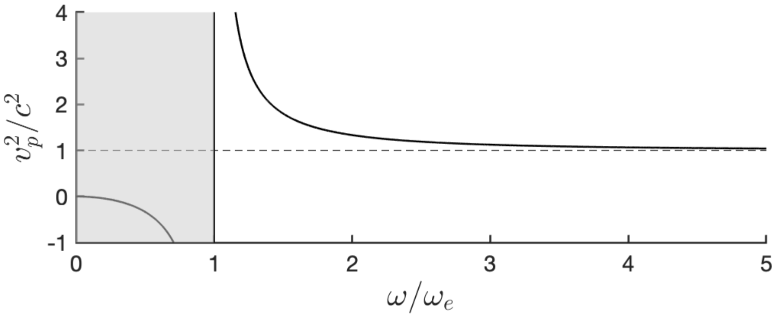


Figure 2.1: The square of the ratio of the phase velocity, v_p, to c as a function of ω/ω_e (solid black line). The dashed horizontal line marks the speed of light. The shaded region marks frequency range for which there is no propagation ($v_p^2 < 0$).

E 3. Including the ions. Show that when the motion of ions (charge Ze) is included the dispersion relation is

$$\omega^2 = k^2c^2 + \omega_p^2, \quad \text{with} \quad \omega_p^2 = \frac{4\pi n_e e^2}{m_e} + \frac{4\pi n_i Z_i^2 e^2}{m_i}. \tag{2.12}$$

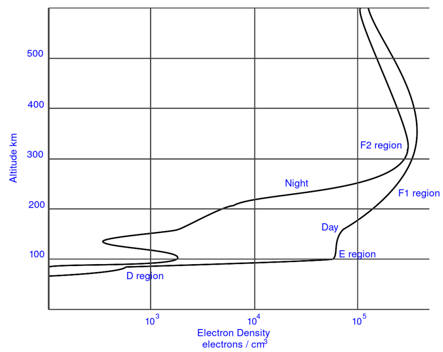
Furthermore, show that if the plasma has several ionic species (as is the case for ISM or IGM) then similar terms for these other species can be added to the above equation. Thus, each charged particle contributes to ω_p^2, the square of the "plasma" frequency, but weighted by Z^2/m.

2.1.1 The Ionosphere

The history of ionosphere and radio communications begins with Equation 2.12. The diurnal variation of the magnetic field strength (as measured at Earth's surface) had already led to the idea of large currents in the upper atmosphere. Experimental evidence of a high altitude conducting layer came serendipitously from the successful transmission of signal (the carrier frequency was around about a MHz) across the Atlantic ocean by Marconi in 1901–1902. Over the next two decades the basic properties of the ionosphere were deduced. The space era, which began in 1957, allowed aeronomers to probe beyond what is now called as the "F" layer.

Briefly, the upper atmosphere (80–1000 km) is a weakly ionized (fraction of a percent by number) plasma peaking at 300 km (Figure 2.2). The ionization in lower regions is controlled by solar extreme ultra-violet (EUV) whereas the upper layers are influenced by Earth's magnetosphere. A typical plasma frequency is 10 MHz. Solar activity has a direct impact on the ionosphere. Inversely, the magnetosphere (which includes the ionosphere) protects us from solar storms.

Given the importance of the ionosphere for ordinary life, electrical grid system and health of artificial satellites, the ionosphere is monitored extensively. For instance, in Australia, the Bureau of Meteorology provides[1] real time data.



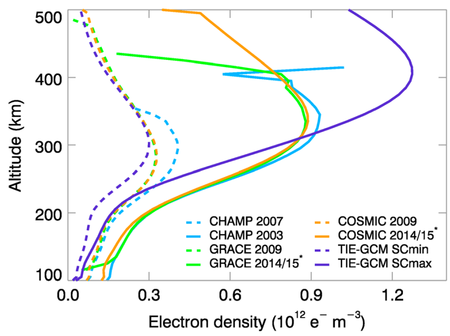


Figure 2.2: (Left) Profile of the electrons in the upper atmosphere. The D layer is thought to be ionized by cosmic rays and X-rays. The electrons within E and F1 layers combine within minutes of the sun setting. The F2 layers persists during the night-time hours and is influenced by strong circulating winds. It is the E and F layers that primarily determine the plasma frequency. The ions (the source of electrons) are, in increasing height, O_2^+, NO^+, O^+, He^+ and H^+. **I am looking for a better figure.** (Right) The influence of the last solar cycle on the structure of the ionosphere. The year follows the name of the mission which undertook the measurements. The EUV radiation increases with solar activity (which is traced by the sunspot index and radio flux at 10.7 GHz). From [llb+18].

It is important that students of astronomy, even those aspiring to be pure theorists, should be aware of the real world. In this spirit we provide a summary of the names of electromagnetic bands that matter for pulsar and FRB astronomy: Very Low Frequency (VLF; 3–30 kHz; time signals, wireless heart monitors), Low Frequency (LF; 30–300 kHz; RFID, AM, amateur radio), Medium Frequency (MF; 300–3000 kHZ; AM, amateur radio), High Frequency (HF; 3–30 MHz; citizens band, shortwave radio), Very High Frequency (VHF; 30–300 MHz; FM & TV) and Ultra-High Frequency (UHF; 0.3–3 GHz; microwaves, GPS, Bluetooth, remote control, Wireless LAN, cell phones). This brief summary should inform you of the clutter of strong artificial sources of radiation in the decimetric sky – the sky in which pulsars and FRBs are discovered and studied. Mastering “Radio Frequency Interference” (RFI) is an essential part of the FRB astronomer’s toolkit.

E 4. Reflection by the Ionosphere. Estimate the plasma frequency for the profile shown in Figure 2.2. Estimate crudely the vertical column density and express that in units favored by aeronomers, the TEC (total electron content expressed in

[1] `http://www.sws.bom.gov.au/`

unit of 10^{12} electrons cm^{-2}) and that favored by radio astronomers, DM (dispersion measure; see §2.3).

E 5. Refraction by the Ionosphere. Assume that the ionosphere can be approximated by a single layer with plasma frequency, ω_e. Apply Snell's law and write an expression for the angular displacement of a source due to refraction, θ_r. Compute θ_r, assuming $\nu_e = 10\,\text{MHz}$, for elevation angle of 30° for "typical" (survey) frequencies of leading low frequency radio telescopes.[2] (e.g., MWA, LOFAR, LWA, GMRT and VLA).

2.2 Phase & Group Velocities

In formal terms, the phase velocity, v_p is the velocity at which one travels to keep the phase constant. For a given phasor,

$$v_p = \frac{\omega}{k}. \tag{2.13}$$

The refractive index, $\tilde{n} \equiv c/v_p$.

Real signals have frequency content. Roughly speaking, a signal of duration Δt satisfies the condtion $\Delta\nu\Delta t \approx 1$. In a dispersive medium different frequency components travel at different velocities. This naturally raises the question: what is the speed of a pulse of radiation traveling through dispersive medium?

The basic physics of propagation in dispersive medium can be illustrated by the fate of a signal consisting of two frequency components, $\nu_0 \pm \Delta\nu$ (and corresponding k-vector, $k \pm \Delta k$) with the same (unity) strength, at the source (x=0, t=0). After traveling for a time t, the sum total of the electric field is

$$\begin{aligned} E_t &= e^{i(k_0-\Delta k)x - i(\omega_0-\Delta\omega)t} + e^{i(k+\Delta k_0)x - i(\omega_0+\Delta\omega)t} \\ &= 2e^{ik_0(x-v_p t)}\cos(\Delta k x - \Delta\omega t) \end{aligned} \tag{2.14}$$

We seek a velocity which keeps the amplitude of the signal constant. This means setting

$$x - \frac{\Delta\omega}{\Delta k}t = 0.$$

A Taylor expansion yields

$$x - t\left(\frac{\partial\omega}{\partial k} + \frac{\Delta k}{2}\frac{\partial^2\omega}{\partial k^2} + ...\right)\Bigg|_{k=k_0} \approx 0 \tag{2.15}$$

[2] A student aiming to be an expert observer must become familiar with the capabilities of leading facilities.

where $\Delta\omega = \omega(k_0) - \omega_0$ has been expanded around k_0. If all terms in the Taylor expansion beyond the first term can be ignored then the velocity with which one must travel to keep the amplitude constant is the "group" velocity:

$$v_g \equiv \frac{\partial\omega}{\partial k}. \tag{2.16}$$

We now generalize the propagation of a signal, $S(x,t)$ through dispersive medium. Let $f(x) = S(x,0)$. Its Fourier transform is

$$F(k) = \frac{1}{2\pi}\int e^{-ikx} f(x,0)dx.$$

Since $\int dx \exp[ik(x'-x)] = 2\pi\delta(x-x')$ the inverse transform of $F(k)$ yields the function back:

$$S(x,t=0) = \int e^{ikx} F(k)dk. \tag{2.17}$$

Given a dispersion relation, $\omega(k)$, each Fourier amplitude satisfies the underlying wave equation. Each Fourier amplitude travels as $x - v_p(k)t$ where $v_p = \omega/k$. Thus, the temporal evolution of $f(x)$ is

$$S(x,t) = \int e^{ik(x-v_p t}F(k)dk = \int e^{ikx-i\omega t}F(k)dk \tag{2.18}$$

If v_p is independent of k we find $S(x,t) = S(x - v_p t)$, as expected. If, we can make the approximation discussed above, the wave packet travels at the group velocity.

For cold plasma without any external magnetic field, given Equation 2.11), we find

$$\tilde{n}(\omega) = \frac{c}{\omega/k} = \Big(1 - \frac{\omega_e^2}{\omega^2}\Big)^{1/2}, \qquad v_g(\omega) = c\Big(1 - \frac{\omega_e^2}{\omega^2}\Big)^{1/2}. \tag{2.19}$$

Apparently, for $\omega > \omega_e$, the refractive index is less than unity, $v_p(\omega) > c$ and $v_g(\omega) < c$. For $\omega < \omega_e$ no propagation is possible because the $k = \omega/v_p$ becomes imaginary (Figure 2.1).

Most beginning students are bothered by phase and group velocities, especially if either of them exceed the speed of light. Much of the angst on this topic can be reduced by re-reading the *formal* definition of phase and group velocities and equally by noting that no system has yet been constructed in which a signal (say, a single bit) has been transmitted at a speed exceeding that of light. The agitated reader should first attend to exercises (E 6 to E 8) and, if not convinced, consult the monograph by Brillouin (1960).

E 6. Super-luminal group and phase velocities. Consider an observer restricted to a long trench. This trench is illuminated by a source plane waves of frequency, ν and wavelength λ and incident at an elevation angle of θ (Figure 2.3). Strictly applying the formal definitions of phase and group velocities, show that $v_p = c/\sin(\theta)$. Now let us assume that all even crests are higher than the odd ones. Show that the group velocity is also $v_g = c/\sin(\theta)$.

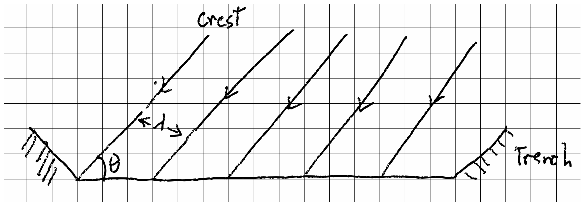


Figure 2.3:

E 7. Group Velocity: Cold Plasma. For the dispersion relation given in Equation 2.11 evaluate the second order term in the Taylor approximation (Equation 2.15) and convince yourself that the requirement for defining the group velocity is justified. Much of the grief (and alarming news that Special Theory of Relativity is wrong) comes when the Taylor expansion approximation is not justified. See for example, demonstration of super-luminal group velocity in a laboratory experiment centered on the resonance line of Cesium [wkd00] or in the 21-cm absorption profile of interstellar HI gas [jff+10].

E 8. Dispersion of a Gaussian Pulse. Consider a Gaussian pulse,

$$f(x) = e^{-x^2/(2\sigma_0^2)} e^{ik_0 x}$$

where $\omega_0 = ck_0$ is the carrier frequency and σ_0 is the spatial width of the signal at $t = 0$. Following Equation 2.15 Taylor expand ω around ω_0 and retain the first two terms. Show

$$S(x,t) = e^{-(x-v_g t)^2/(2\sigma^2(t))} e^{i\phi(x,t)} \tag{2.20}$$

where

$$\sigma(t) = \sigma_0 \sqrt{1 + \frac{\Gamma^2}{\sigma_0^4} t^2} \tag{2.21}$$

$$\phi(x,t) = k_0 x - k_0 t(v_g - v_p) - \frac{t\Gamma}{t^2\Gamma^2 + \sigma_0^4} \tag{2.22}$$

with $\Gamma = \partial^2\omega/\partial k^2$ evaluated at $k = k_0$. So the Gaussian pulse still propagates with the group velocity but the width, $\sigma(t)$ is gradually increasing – as would be expected given that the velocities are frequency dependent.

SHORT SUMMARY & EXERCISE INTRODUCING ISM/CGM/IGM.

2.3 Dispersion Measure

The frequency dependent group velocity naturally leads to a frequency-dependent arrival time,

$$t(\nu) = \int_0^L \frac{1}{v_g(\nu)} dl \approx \int_0^L \frac{dl}{c}\Big(1 + \frac{1}{2}\frac{\omega_e^2}{\omega^2}\Big), \tag{2.23}$$

$$\tau(\nu) = t(\nu) - t_0 = \frac{e^2}{2\pi m_e c}\frac{1}{\nu^2}\int_0^L n_e dl \tag{2.24}$$

where, $t_0 = L/c$ is the arrival time of the pulse at very high frequency (high enough so that v_g is close to the speed of light). The small value of $\epsilon = (\omega_e/\omega)^2 \approx 10^{-10}\nu_9^{-2} n_e$ justifies the Taylor approximation made above; here, $\nu = 10^9\nu_9\,\mathrm{Hz}$ (E 10). $\int_0^L n_e dl$ is the column of electrons between the observer and source. By tradition, astronomers use the parsec for L but cm^{-3} for n_e. This quantity is called as the "Dispersion Measure" (DM).

From Equation 2.24 we see that the DM is a product of ν^2 and the dispersive delay to that frequency:

$$\mathrm{DM} = K\nu_9^2\tau(\nu), \text{ where } a = K^{-1} \equiv \frac{e^2}{2\pi m_e c} \times \text{parsec}. \tag{2.25}$$

In the above equation "parsec"stands for the length of parsec in cm. The IAU in 2012, via Resolution B2, fixed[3] the value of AU. The same resolution defined the parsec to be the small-angle-approximation[4] distance to a star which subtends a parallax of an arcsecond across an AU-wide baseline. Next, on 2019 May 20 (the "World Metrology Day"), the Système Internationale d′Unités (SI)[5] announced permanent values for four fundamental constants[6], which amongst other things, replaced the kilogram and the ampere. With these defined constants and the latest measurement for ϵ_0

$$\begin{aligned} a &= 4.148\,806\,4239(11)\ \mathrm{GHz}^2\,\mathrm{cm}^3\,\mathrm{pc}^{-1}\,\mathrm{ms}, \\ K &= 241.033\,1786(66)\ \mathrm{GHz}^{-2}\,\mathrm{cm}^{-3}\,\mathrm{pc}\,\mathrm{s}^{-1}. \end{aligned} \tag{2.26}$$

The reader should be aware of a tradition, going back to the early days of pulsar timing, of fixing the value of K: "Because of the uncertainty in propagation constants, most accurate published dispersions are quoted as the dispersion constant, $DC = \Delta t/\Delta(1/f^2)$, which is a directly measured quantity. However, for consistency we have converted these values

[3] https://www.iau.org/public/themes/measuring/

[4] eschewing the proper trigonometric formula!

[5] `https://www.nist.gov/si-redefinition`

[6] With this announcement the seven defining constants of the SI system are: the hyperfine line frequency of Cesium-133 ($\Delta\nu_{\mathrm{Cs}}$), c, h, k_B, e, Avogadro's number (N_A) and luminous efficacy (K_{cd}). See `https://physics.nist.gov/cuu/Constants/index.html` for values.

to the more commonly quoted dispersion measures, DM, using the relation $DM(\mathrm{cm}^{-3}\,\mathrm{pc}) = 2.41000 \times 10^{-16}\, DC\,(\mathrm{Hz})$." (Manchester & Taylor [mt72]. This suggestion took traction within some precision pulsar timing groups. However, in the *Handbook of Pulsar Astronomy* [lk04], a textbook used by students and researchers entering the field of pulsars, the authors quote $a = 4.148\,808(3)\ \mathrm{GHz}^2\,\mathrm{cm}^3\,\mathrm{pc}^{-1}\,\mathrm{ms}$. Some pulsar programs even use $a \equiv 4.15\ \mathrm{GHz}^2\,\mathrm{cm}^3\,\mathrm{pc}^{-1}\,\mathrm{ms}$. As one of the authors can attest, lack of awareness of this convention can lead to wasted effort (E 12).

E 9. Infer the Dispersion Measure Figure 2.4 heralded the birth of FRB astronomy. Estimate the dispersion measure and then read the discovery paper [lbm+07].

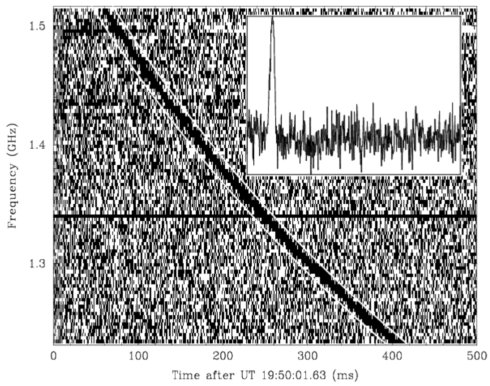


Figure 2.4: The first FRB. The x-axis is time of arrival and the y-axis is the frequency of the filter bank. From [lbm+07].

E 10. Deviations from ν^{-2} sweep. The Taylor expansion undertaken in Equation 2.24 was motivated on the grounds that for astronomical situations, $\omega_e^2/\omega^2 \ll 1$. The $\tau(\nu) \propto \nu^{-n}$ arrival with $n = 2$ to good precision arrival time constrains possible dense intervening plasma clouds. confirmation of this assumption. Soon after the discovery of pulsars in 1967, astronomers started to seek deviations from $n = 2$. Thornton et al. [tbs13] which reported four FRBs is the second most influential paper in FRB astronomy. The authors allowed n to be free for the two bright bursts and found $n = 2.003 \pm 0.006$ and $n = 2.000 \pm 0.006$. What constraints can you derive from these inferences?

E 11. Using GPS to probe ionosphere. The Global Positioning System (GPS) are a network of satellites in "semi-synchronous" orbit (radius of 20,000 km) operating in the "L"-band. Each satellite carries an atomic clock which provides a clock frequency of 10.23 MHz. The two main frequency channels are $L1 = 10.23\,\mathrm{MHz} \times 154 = 1575.42\,\mathrm{MHz}$ and $L2 = 10.23\,\mathrm{MHz} \times 120 = 1227.60\,\mathrm{MHz}$ straddle the hyperfine line of HI. Assuming that you can measure the time in each channel to, say, 30 ns, estimate the precision with which you can measure TEC. Under what conditions would the first order Taylor expansion not suffice?

E 12. Geocentric Arrival Time of FRB 200428A. UNDER CONSTRUCTION.

2.4 Relative Motion

The motion of the pulsar with respect to the intervening medium or the observer does not affect the inferred DM. However, the relative motion of the observer with respect to the intervening medium will lead the observer to infer a different value for the DM.

Consider an event that puts out a broadband pulse, radio through X-ray. We start by considering the simple case of an intervening plasma cloud that is stationary with respect to an observer located at the solar system barycenter (SSB). Owing to dispersive delay within the cloud, the radio pulse, frequency ν_0, arrives τ_i after the X-ray pulse (which we assume traveled at the speed of light). Following Equation 2.25 the observer infers

$$\mathrm{DM}_i = K\tau_i\nu_0^2. \tag{2.27}$$

Next, consider the case of an observer stationed on Earth. The orbital velocity of Earth around the SSB can range up to $\pm 30\,\mathrm{km\,s^{-1}}$. The frequency of the radio pulse as perceived by the Earth-bound observer is given by the relativistic Doppler formula:

$$\nu = \frac{1}{\gamma}\Big(\frac{1}{1+\beta_r}\Big)\nu_0 \tag{2.28}$$

where $v_r = c\beta_\mathrm{r}$ is the radial velocity[7], $\beta = v_\mathrm{orb}/c$ and $\gamma = (1-\beta^2)^{-1/2}$.

Let us assume that we have arranged for light pulses be emitted when the radio pulse enters and exits the cloud. All quantities, unless stated otherwise, are in the frame of the observer. Let the distance from the observer to the cloud at the time of entry be d. The first light pulse arrives at time $t_1 = d/c$. From time dilation we know that the second pulse will be emitted $\Delta t = \gamma\tau_i$ (observer frame) later. Thus, the observer receives the second light pulse at time $t_2 = \Delta t + (d + v_r\Delta t)/c$. As a result, the dispersive delay measured by the observer, $t_2 - t_1$, is $\tau = (1+\beta_r)\gamma\tau_i$. As before (Equation 2.25) the inferred DM is given by the product of the dispersive delay and square of the frequency, both measured in the same frame, or

$$\mathrm{DM} = K\tau\nu^2 = \frac{1}{\gamma}\frac{1}{(1+\beta_r)}\mathrm{DM}_i. \tag{2.29}$$

[7] We follow the astronomical convention in which receding radial velocities are positive.

Thus, the observer value of DM is the intrinsic value times the Doppler factor. It appears that the DM behaves like a spectral line. Since β_r is small we see that the fractional error in the inferred DM is $\pm\beta_{\rm r}$ (E 13). Precision pulsar timing programs account for this effect whereas routine pulsar and FRB searches ignore this correction.

It is tempting to extend Equation 2.29 to cosmology. However, it is conceptually incorrect to treat Hubble radial flow as a velocity. For a pure Hubble expansion (no peculiar motions) we should use the appropriate time-dilation factors for $\Delta t = (1+z)\tau_i$ and $\nu = \nu_0/(1+z)$, which will then lead to $\mathrm{DM}(z) = \mathrm{DM}_i/(1+z)$ — a well known result (and discussed in great detail, later in the book).

E 13. Variation in Dispersion Measure. Estimate the maximum variation in the inferred DM for a pulsar, due to the velocity of Earth around the Sun, which has been observed at different times of the year. Compute this delay, at say, 0.4 GHz, and compare it to the Roemer delay. Is the former readily measurable for a bright pulsar, say, PSR???.

2.5 Warm Plasma

Much of the diffuse plasma is in the inter-galactic medium (IGM) and the circumgalactic medium (CGM). The temperature in these media ranges from 10^5 to 10^8 K. So it is of some interest to know the dispersion relation for warm plasma.

Electrons with temperature T move with rms thermal speeds, v_t, which, in turn, gives rise to pressure, P_e. A proper derivation of the dispersion relation including P_e in the RHS of Equation 2.7 is given in Equation 2.31. For $(\omega/\omega_e)^2 \gg 1$,, the high frequency limit, Equation 2.31 simplifies (E 14) to

$$\omega^2 = k^2c^2 + \Big[1 - \tfrac{3}{2}\epsilon_T\Big]\omega_e^2. \tag{2.30}$$

This equation amazingly agrees exactly with the following elegant argument. The thermal motion of electrons results in an increased mass of the electrons, $\gamma_e m_e$; here, $\gamma_e = (1-\beta_t^2)^{-1/2}$ with $\beta_t = v_t/c$. As can be seen from Equation 2.11, a heavier electron leads to a smaller ω_e. The mean of the inverse mass of the electron is decreased by $\langle\gamma\rangle^{-1} \approx 1 - (1/2)\langle\beta_t^2\rangle$ where the averaging is done over the Maxwellian distribution. Since $\langle\beta_t^2\rangle = 3kT_e/(m_ec^2)$, the fractional change in ω_e^2 amounts to $-(3/2)\epsilon_T$.

E 14. Warm Plasma Dispersion. Buneman [b80] provides the following dispersion relation for warm plasma, accurate to $O(\xi)$ where $\xi = v_t^2/c^2$,

$$\frac{\omega^2}{\omega_e^2} = \frac{1}{1-\tilde{n}^2}\Big(1 - \frac{\xi}{2}\Big) - \frac{\xi}{3}. \tag{2.31}$$

Show that for $(\omega/\omega_e)^2 \gg 1$, you can recover Equation 2.30.

E 15. Interpolation of dispersion relation. The following dispersion relation is valid from warm to relativistic plasma:

$$\omega^2 = c^2k^2 + \frac{\alpha}{2}\left[1 - \frac{K_0(\alpha)}{K_2(\alpha)}\right]\omega_e^2 \tag{2.32}$$

where $\alpha = \epsilon_T^{-1}$ and K_n is the modified Bessel function of the second kind with index n. Show that in the relativistic limit, $\alpha \ll 1$, Equation 2.32 can be simplified to

$$\omega^2 = c^2k^2 + \frac{\alpha}{2}\omega_e^2. \tag{2.33}$$

This simplification shows very clearly the suppression of dispersion by relativistic plasma due to increased mass of the relativistic electrons. Next, plot the multiplier to ω_e^2 in Equations 2.30, 2.32 and 2.33 as a function of ϵ_T, say from 0 to 100. You will get a good sense of the usable range for the two asymptotic relations (Equations 2.30 and 2.33).

2.6 Magnetized Plasma

In this section we assume that the plasma is threaded by an external field of strength $\mathbf{B_0}$. With no loss generality the external field is oriented along the z axis. The novelty here is that electrons are not only excited by the electric field but also gyrate due to the magnetic field. The two principal geometries, $\mathbf{k} \perp \mathbf{B_0}$ and $\mathbf{k} \parallel \mathbf{B_0}$, are discussed below.

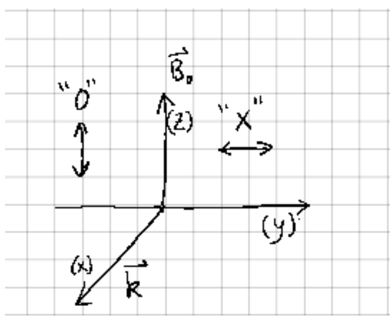


Figure 2.5: The geometry of the external magnetic field, $\mathbf{B_0}$ (along z axis) and $\mathbf{k}$ vector (along x axis). Ordinary mode ("O") refers to electromagnetic field oscillating parallel to $\mathbf{B_0}$ whereas the extra-ordinary mode ("X") is when the field is oscillating perpendicular to $\mathbf{B_0}$ (along the y axis).

2.6.1 $\mathbf{k} \perp \mathbf{B_0}$: Ordinary and Extra-ordinary waves

As can be seen from Figure 2.5 the two cardinal cases are the electric field is parallel to $\mathbf{B_0}$ or perpendicular to $\mathbf{B_0}$. For the former case, there is no magnetic Lorentz force and so the dispersion relation remains the same as that for unmagnetized plasma (Equation 2.11), hence the name "ordinary" waves.

In contrast, when the electromagnetic field is oscillating perpendicular to $\mathbf{B_0}$, the electrons, in addition to oscillating, gyrate with angular frequency $\omega_c \equiv eB_0/(m_e c)$ (electron gyro-frequency or electron synchrotron frequency). The radiated fields will have both E_x and E_y components. The resulting field thus *develops a longitudinal component*, hence the term "Extraordinary" wave.

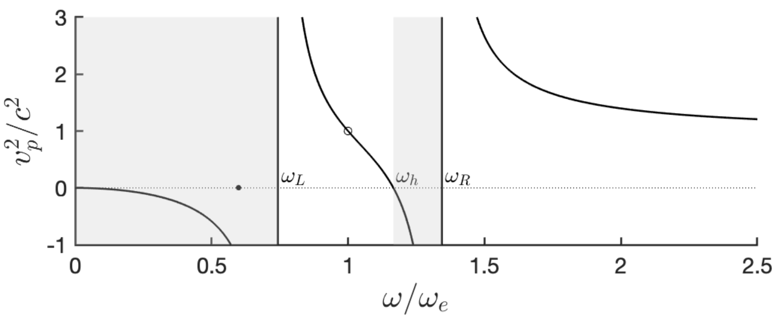


Figure 2.6: The square of the ratio of the phase velocity (v_p) to c as a function of ω/ω_e for the extra-ordinary mode (solid black line). The dotted line marks $v_p = 0$. The solid circle marks ω_c/ω_e and the open circle shows that at $\omega = \omega_e$, $v_p = c$. The shaded region marks frequency range for which there is no propagation ($v_p^2 < 0$). The following inequality always holds: $\omega_L \leq \omega_e \leq \omega_h \leq \omega_R$.

The small-amplitude approximation equation of motion (Equation 2.10) now becomes:

$$m_e \frac{\partial \mathbf{v}_e}{\partial t} = -e\Big[\mathbf{E} + \frac{\mathbf{v}_e}{c} \times \mathbf{B_0}\Big]. \tag{2.34}$$

Given the development of the longitudinal component we cannot set $\mathbf{k} \cdot \mathbf{E} = 0$ in Equation 2.10. After some algebra (E 16), the following dispersion relation can be obtained:

$$\frac{c^2k^2}{\omega^2} = \frac{c^2}{v_p^2} = 1 - \frac{\omega_e^2}{\omega^2}\frac{\omega^2 - \omega_e^2}{\omega^2 - \omega_h^2} \tag{2.35}$$

where $\omega_h^2 = \omega_e^2 + \omega_c^2$. ω_h is the so-called "upper hybrid" angular frequency. Recall that for a dispersion relation the cutoff frequencies are defined by setting $k = 0$. Equation 2.35 admits two cutoffs:

$$\omega_R = \tfrac{1}{2}\Big[(\omega_c^2 + 4\omega_e^2)^{1/2} + \omega_c\Big], \tag{2.36}$$

$$\omega_L \quad = \quad \tfrac{1}{2}\Big[(\omega_c^2 + 4\omega_e^2)^{1/2} - \omega_c\Big]. \tag{2.37}$$

While a cutoff frequency is defined by $k \to 0$, a resonance frequency is defined by $k \to \infty$. From Equation 2.35 we see that the extraordinary wave has a resonance at $\omega = \omega_h$. As can be seen from Figure 2.6 the extra-ordinary wave has two bands for propagation: between ω_L and ω_h and above ω_R.

E 16. Refractive Index for Extra-ordinary Wave. As discussed in the text, E is restricted to the x-y plane which, when applied to Equation 2.34, generates currents in the x-y plane, In turn, these currents form the source term in Equation 2.9. You will find two equations containing E_x and E_y, each with RHS equal to 0. Consistency[8] between the resulting equations leads to Equation 2.35.

2.6.2 k ‖ B₀: Circular Polarization & Whistler Waves

Let **k** and $\mathbf{B_0}$ be along the z axis. As in the case for the extra-ordinary wave, we must let **E** have both an x and y component. Applying Equation 2.34 to a phasor we obtain

$$v_x = \frac{-ie}{m_e\omega}(E_x + v_y B_0), \qquad v_y = \frac{-ie}{m_e\omega}(E_y - v_x B_0) \tag{2.38}$$

where $\mathbf{v_e} = v_x\hat{\mathbf{x}} + v_y\hat{\mathbf{y}}$. By construction, $\mathbf{k}\cdot\mathbf{E} = 0$. Applying the current implied by Equation 2.38 to Equation 2.9, followed by some algebra, leads to

$$\begin{aligned}(\omega^2 - c^2k^2 - \alpha)E_x + i\alpha\frac{\omega_c}{\omega}E_y &= 0\\ (\omega^2 - c^2k^2 - \alpha)E_y - i\alpha\frac{\omega_c}{\omega}E_x &= 0\end{aligned} \tag{2.39}$$

where, $\alpha = 1 - \omega_p^2/(1-\omega_c^2/\omega^2)$. Consistency of these two equations leads to the following dispersion relation:

$$\omega^2 - c^2k^2 = \frac{\omega_e^2}{1 \pm (\omega_c/\omega)}. \tag{2.40}$$

The mode with "+" is called the L wave and that with "−", the R wave. These two modes correspond to circularly polarized signals and the R and L convention refers to the handedness.

There are two possibilities for naming the handedness of circular polarization and unfortunately both opposing conventions are in use! Radio astronomers

[8]Mathematically, the condition for consistency of linear equations with RHS of 0 is to require the determinant to be zero.

follow the IEEE (Institute of Electrical and Electronics Engineers) convention in which the handedness is determined from the point of view of the source. A signal is called as Right Handed Circular (RCP) polarization if, when viewed along the $\mathbf{k}$ axis, the electric vector traces a clockwise circle.[9] The other convention, used by optical physicists and blessed by SPIE (Society of Photo-Optical Instrumentation Engineers), is based on viewing the signal along the direction to the source (along $-\mathbf{k}$). Thus the RCP signal of IEEE is the Left Handed Circular (RCP) of SPIE. In the past (and perhaps even continuing today) this vexing situation led to failed VLBI (Very Long Baseline Interferometry) runs and missed opportunities to communicate with satellites.

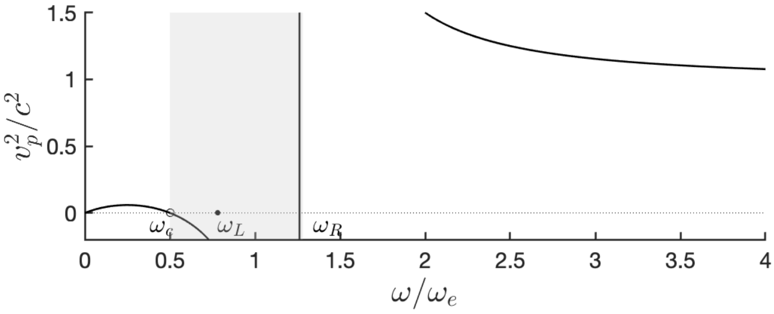


Figure 2.7: The square of the ratio of phase velocity (v_p) to the speed of light as a function of ω/ω_e for right-handed wave or R wave (solid line). The dotted horizontal line represents $v_p = 0$. The open circle marks the electron cyclotron frequency, ω_c. The phase velocity for $\omega = \omega_c$ is zero. The R wave does not propagate in the shaded region bracketed by ω_c and ω_R. In other words, the R wave propagates for $\omega > \omega_R$ and $\omega < \omega_c$. The latter is the domain of whistlers. In contrast, the left-handed wave (L wave), marked by a bullet, propagates only for $\omega > \omega_L$.

With the RCP/LCP convention explained in some detail we now turn to investigating the two modes. The dispersion relation for the L wave is

$$\frac{c^2k^2}{\omega^2} = 1 - \frac{\omega_e^2}{\omega(\omega + \omega_c)}. \tag{2.41}$$

This dispersion relation has a single cutoff at $\omega = \omega_L$ (Equation 2.37) and admits of no resonance. Essentially it behaves the same way as unmagnetized plasma (cf. Figure 2.1) but with a cutoff at ω_L instead of ω_e. Equation 2.41 can be combined with Equation 2.39 to yield

$$\alpha\frac{\omega_c}{\omega}(E_x - iE_y) = 0. \tag{2.42}$$

For $\omega > \omega_L$ the pre-factor in the above equation is not zero and so $E_x - iE_y$ must be zero. Thus, $E_y = -iE_x$. Letting $E_x = \mathrm{Re}[\exp(-i\omega t)] = \cos(\omega t)$] we

[9] Curl your right hand and point the thumb along the wave vector. The four fingers are curled clockwise.

find $E_y = \mathrm{Re}[-i\exp(-i\omega t)] = -\sin(\omega t)$ and, so, $\mathbf{E} = [\cos(\omega t), -\sin(\omega t)]$. Looking along the $-\mathbf{k}$ is the same as looking towards the source. Viewed thus, the tip of the electric vector traces a circle which is clockwise. Looking towards the observer is the same as looking along $\mathbf{k}$. Viewed thus, the tip of the electric vector traces an anti-clockwise circle. Thus, for radio astronomers, this mode is an LCP which jives with "L" wave notation.

As can be seen from Equation 2.40 the dispersion relations for L and R wave is determined by k^2 and not $\mathbf{k}$. Thus, the sense of rotation of the electric vector is independent of the direction of $\mathbf{k}$. In other words, flipping the magnetic field does not change the sense of rotation of the vector.

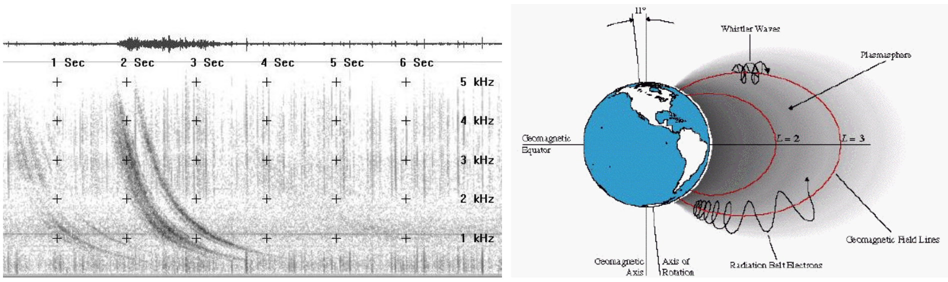


Figure 2.8: (Left) Dynamic spectra (spectra as a function of time) showing ionospheric Whistlers (gray scale). Time runs along the x-axis and frequency along the y-axis. Whistlers correspond to curved traces. (Right) Lightning strikes are sources of broad-band emission. A lightning strike say close to the Southern magnetic pole acts as a source of broad-band radio emission. The very low frequency RCP waves, guided by the plasma in ionosphere, travel along the field lines to the North where they can be heard, for instance, in Canada. The observations were made by a radio amateur.

The dispersion relation for the R wave is

$$\frac{c^2k^2}{\omega^2} = \tilde{n}^2 = 1 - \frac{\omega_e^2}{\omega(\omega - \omega_c)}. \tag{2.43}$$

In this case $E_x + iE_y = 0$ and the mode is Right Circular Polarization (RCP).

The R wave has a resonance at $\omega = \omega_c$ and a cutoff at $\omega = \omega_R$ (Equation 2.36). As can be seen from Figure 2.7, transmission is possible for very low frequencies, from near dc to ω_c! It is this unusual property that neatly explains the phenomena called as whistlers and discussed below. We next address the resonance frequency. Consider an RCP signal with frequency equal to ω_c. An electron with some velocity in x-y plane will gyrate anti-clockwise when viewed from the top ($-\hat{z}$ or $-\mathbf{k}$ axis). Thus, in the frame of the electron, an incident RCP wavefront will become stationary. The radial and constant electric field will accelerate the electron. Thus there can be significant transfer of energy from the wave to the electron – hence the term "resonance". In contrast, an LCP signal has no resonance with electrons but can have a resonance at Ω_c (E 18).

Whistler Waves. Whistler waves were discovered with the onset of low frequency radio communications. These waves span the audio frequency range and beyond (Figure 2.8). So one can actually "listen" to whistler waves by simply connecting a headphone directly to a (square law) detector which is fed by a low frequency antenna. The group velocity for whistler waves, $v_g \propto \omega^{1/2}$ for $\omega \ll \omega_c$. As a result, at low frequencies, the higher frequency signal arrives ahead of the lower frequency signal. This decrease in tone is similar to that of a whistle, hence the name.

Up until the late fifties the ionosphere was probed primarily by radio reflection techniques (which laid the foundation for radar) and so the investigations stopped at the F-layer. As can be seen from Figure 2.8 whistler waves allowed aeronomers to sense the ionosphere at distance of several Earth radius. Whistler waves haves been detected in other planets (e.g. Jupiter, Venus, Saturn) and provided the first indication of lightning on these planets.

Faraday Rotation. A linearly polarized signal can be decomposed into two an RCP and an LCP signal with a phase difference which defines the linear polarization vector. As can be inferred from Equations 2.41 and 2.43, the two modes, LCP and RCP, have different group and phase velocities. As a result, an incident linearly polarized signal is rotated. This effect – "Faraday rotation" – is discussed later in the book.

E 17. Faraday Rotation in e^+-e^- plasma. Do you expect to see Faraday rotation for a ray going through e^+-e^- plasma threaded by an external magnetic field?

E 18. Ion Cyclotron Frequency. The L wave, unlike the R wave, does not have a resonance. However, if you include ions (mass m_i) then LCP will also have a resonance but at the ion gyro-frequency, $\Omega_c = eB/m_i c$. Derive the dispersion relation that allows for motion of ions:

$$\frac{c^2k^2}{\omega^2} = 1 - \frac{\omega_p^2}{(\omega \mp \Omega_c)(\omega \pm \omega_c)}. \tag{2.44}$$

2.7 Other Waves in Plasma

So far we have considered the modification of an electromagnetic wave incident upon a plasma cloud. There are many other waves and oscillations that are seen in plasma. Hundreds of theses and entire books have been written on this topic! The richness of waves from plasma is perhaps best illustrated by the rich Jovian radio spectrum (Figure 2.9). Motivated by this figure, here, with a simple overview, we set the stage for discussions later in the book.

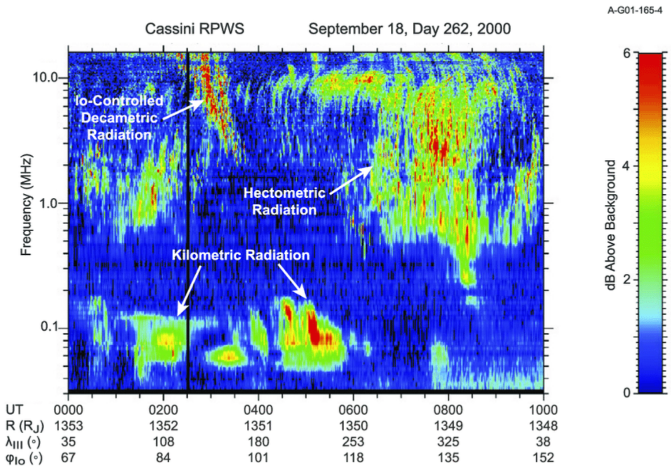


Figure 2.9: A frequency time spectrogram of common Jovian radio emissions acquired by the Cassini mission.

The simplest oscillation is an oscillation resulting from electrostatic forces. Say you displace a clump of electrons. As before, we assume that the ions are fixed. Then the displaced clump is attracted by the positively charged hole left behind. The restoring force moves the clump back but owing to its inertia it overshoots but is then attracted back and so on and so forth. If, in turn, an external magnetic field is present then the restoring force will include $\mathbf{v} \times \mathbf{B_0}$ force. It can be shown (E 19) that the oscillation frequencies correspond to ω_e (B_0=0) and ω_h ($\mathbf{B_0} \perp \mathbf{k}$). Note that these two modes, as implied by the name "oscillations", are not traveling waves.

The plasma oscillation (aka Langmuir oscillation) and upper hybrid oscillation are the simplest members of electrostatic modes. By definition, electrostatic waves do not have oscillating magnetic fields. Magnetic fields do not play a role in electrostatic modes. So we can set $\dot{\mathbf{B}} = 0$ which means, thanks to Equation 2.2, $\nabla \times \mathbf{E} = 0$ or $\mathbf{k} \times \mathbf{E} = 0$ Thus, electrostatic modes are longitudinal.

Next, the plasma, being gas, has two other restoring forces arising from gas pressure (which leads to "acoustic" waves) and magnetic pressure (which leads to "magnetosonic" waves). That the phenomenon is rich is evidenced by a rich set of modes: upper & lower hybrid oscillations, ion acoustic waves, electrostatic ion cyclotron waves, hydromagnetic waves (aka Alfvén waves) and magnetosonic waves.

E 19. Electron Plasma Oscillation and Upper Hybrid Oscillations. The only relevant equation of Maxwell for this phenomenon is Equation 2.1 (electrostatic force). Linearize this equation and the two equations of fluid mechanics (Equations 2.6 and 2.7) and show that the oscillation frequencies of these two electrostatic modes is ω_e ($\mathbf{k} \parallel \mathbf{B_0}$ or $B_0 = 0$) and ω_h ($\mathbf{k} \perp \mathbf{B_0}$).

E 20. Asymptotic Limit of Extraordinary Wave. In the course of solving E 16 you would have derived the electric fields generated by the electrons (the equivalent of Equations 2.39 but for the extraordinary wave). Show that as $\omega \to \omega_h$ the transverse electric field, E_y vanishes, leaving only the longitudinal field, E_x. Thus at resonance the extra-ordinary wave becomes a purely electrostatic oscillation.

Notes: Much of the material in this chapter is based on "Introduction to Plasma Physics[10]" by F. C. Chen [c74] which is also an excellent self-study book for a first course in plasma physics. An advanced student will benefit from reading "Modern Classical Physics" by K. Thorne & R. Blandford [tb17]. Lecture notes of "Physics 15C" by Matthew Schwarz (Harvard University) are very useful for students rusty on Waves. See Kulkarni [k20] and references therein for §2.3, 2.4, 2.5.

Exercises: Useful references for exercises: E 10 [t14], E 6 [w54]. E 8 [tb17].

Figures: The left side of Figure 2.8 is from URL:`http://www.auroralchorus.com/wr3gde.htm` and the right side is from the website of Prof. Koeppen of Kiel University, `https://portia.astrophysik.uni-kiel.de/~koeppen/VLF/Whistlers.html`. However, for the latter, upon inquiry with Prof. Koeppen I found that he himself does not know the antecedents of the cartoon.

[10] The edition of the seventies or eighties but not the recent one.

Table 2.1: Glossary

α	$m_e c^2/kT$ (ϵ_T^{-1}
$\mathbf{B}$	Magnetic field vector
$\mathbf{B_0}$	Magnetic field vector (ambient)
β	beta factor, v/c
c	speed of light
e	unit chargea (proton) or index for electron
ϵ	ω_e^2/ω^2
ϵ_T	$kT/m_e c^2$
$\mathbf{E}$	Electric field vector (E_x, E_y, E_z)
γ	Lorentz factor, $(1-\beta^2)^{-1/2}$ and also polytropic index
i	$\sqrt{-1}$ or index for ion
j	index
k	wave vector
K_n	modified Bessel function of the second kind and degree n
$\mathbf{j}$	charge current vector
m_j	mass of particle, (j=e for electrons, j=i for ions)
$\tilde{n}$	refractive index
n_j	number density of electrons (j=e) and ions (j=i)
ν	frequency of wave
ν_e	electron plasma frequency
$\tilde{n}$	refractive index
ω	angular frequency
ω_c	electron-gyro frequency
ω_e	electron plasma frequency
ω_h	upper hybrid frequency
ω_p	plasma frequency
Ω_c	ion gyro-frequency
P_j	pressure (j=e for electrons, j=i for ions)
q_j	charge of particle, (j=e for electrons, j=i for ions)
T	temperature
$\mathbf{v}_j$	velocity vector (j=e for electrons, j=i for ions)
v_g	group velocity
v_p	phase velocity
ξ	$\langle v^2\rangle/c^2$
z	redshift

Table 2.2: Acronyms

AM	Amplitude Modulation
AU	Astronomical Unit
CGM	Circumstellar Medium
CGS	Centimeter-gram-second system
DC (dc)	Direct Current
DM	Dispersion Measure
DS	Dispersion Slope
EUV	Extreme Ultra Violet
FM	Frequency Modulation
FRB	Fast Radio Bursts
GPS	Global Positioning System
HF	High Frequency
IAU	International Astronomical Union
IEEE	Institute of Electrical and Electronics Engineers
IGM	Intergalactic Medium
ISM	Interstellar Medium
LAN	Local Area Network
LCP	Left-handed Circular Polarization
LF	Low Frequency
LHS	Left-hand side
MKS	Meter-kilogram-second
MF	Medium Frequency
PSR	Pulsating Radio Source
RHS	Right-hand side
RCP	Right-handed Circular Polarization
RFI	Radio Frequency Interference
RFID	Radio Frequency Identification
SI	Systéme International (dúnités)) [MKS]
SPIE	Society of Photo-Optical Instrumentation Engineers
SSB	Solar System Barycenter
TEC	Total Electron Content
TV	TeleVision
UHF	Ultra High Frequency
URL	Uniform Resource Locator
VLF	Very Low Frequency